Axially Loaded Piles

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Contents

1 t-z Curve Method using Finite Element Analysis

The stress-strain relationship for an axially loaded pile can be described through three loading mechanisms: axial deformation in the pile, soil skin friction along the shaft, and soil end-bearing (Figure 1-1a). Using a spring-mass model in which springs represent material stiffness, numerical techniques can be employed to conduct the load-settlement analysis (Figure 1-1b).

As per the assumed sign convention, the x-axis typically corresponds to the distance along the pile, while the z-axis corresponds to the distance below the ground surface. However, since the pile length above the ground surface does not affect the stress distribution of the pile below ground, the z-axis has been traditionally used to denote the distance along the embedded pile length and will be used as such for this manual.

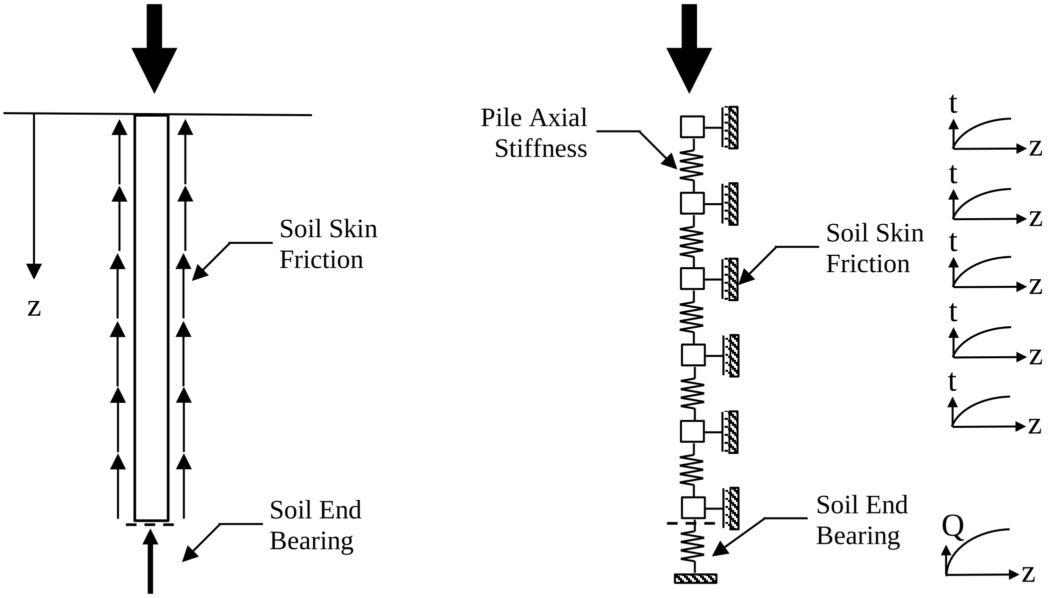

Figure 1-1: (a) Load transfer mechanisms in an axially loaded pile and (b) spring mass model

The t-z curve method using finite element analysis was employed to solve the governing differential equation. The t-z curve method allows for simulation of the non-linear stress-strain behavior in soil by employing non-linear stiffness curves denoted as t-z curves for soil skin friction and Q-z curve for the soil end bearing resistance. The stiffness is computed at each iteration based on the solved displacement values.

2 Governing Differential Equation

From force equilibrium of the free body diagram shown in Figure 2-1 , we obtain an equation for the load transfer of the externally applied loads to skin friction and pile deformation.

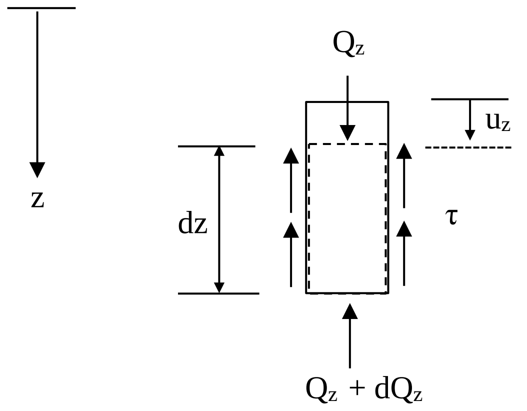

Figure 2-1: Free body diagram of pile segment

$$Q_z = Q_z + dQ_z + \tau \cdot C \cdot dz$$

$$-\frac{dQ_z}{dz} = \tau \cdot C \qquad \text{Equation 1}$$

where Q_z = internal pile force at depth z
τ = soil unit skin friction at depth z
C = circumference of pile segment at depth z

For axially loaded beams, the following equation can be used to describe the development of internal forces in the pile due to axial deformation of the pile segment.

$$Q_z = -EA\frac{du_z}{dz} \qquad \text{Equation 2}$$

Where E = pile segment modulus of elasticity at depth z
A = pile segment cross sectional area at depth z
u_z = pile segment displacement at depth z due to applied loads

Differentiating Equation 2 by z and substituting into Equation 1 yields the following governing differential equation for the pile and soil.

$$EA\frac{d^2u_z}{dz^2} = \tau \cdot C$$

$$-EA\frac{d^2u_z}{dz^2} + \tau \cdot C = 0 \qquad \text{Equation 3}$$

For an axially loaded pile, the body force produced by the pile unit weight is negligible compared to the applied loads and is therefore neglected.

3 Finite Element Discretization

The pile was discretized into segments consisting of two pile elements and one soil shear element, as shown in Figure 3-1. Note the use of the term pile segment to describe the configuration of two pile elements and one soil shear element. Each soil shear element characterizes the effect of skin friction between the pile and soil. The element configuration was selected to ensure that the soil displacement used to calculate the skin friction is based on the midpoint of each pile segment. As such, the length of each pile element to calculate pile axial stiffness is half the length of the chosen segment length since there are two pile elements per segment. The length of each soil shear element is equal to the full length of each segment since there is one shear element per segment.

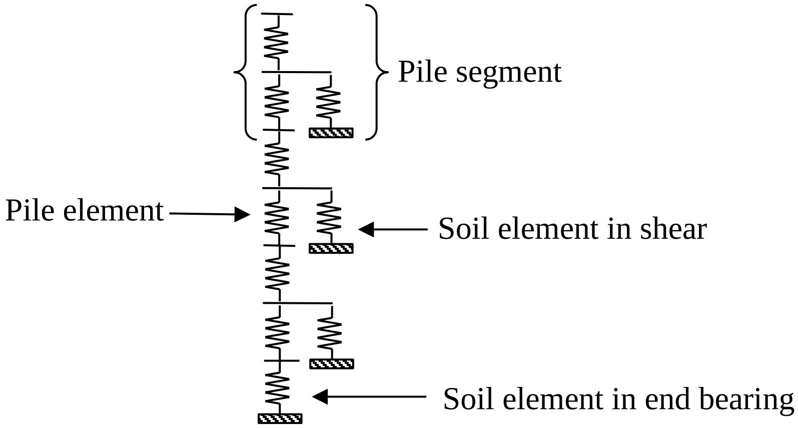

Figure 3-1: Pile segment discretization into pile elements and soil elements

The assumptions made for axial load analysis by solving the governing differential equation using the finite element method are as follows:

1. The pile is geometrically straight such that second order effects are not considered
2. Eccentric loads are not considered
3. The pile will not deform significantly throughout the simulation as to alter the original pile geometry
4. The pile material is isotropic

4 Pile Axial Stiffness

The pile is assumed to be linear elastic and perfectly plastic under axial compression. The stiffness of each pile element ($k_{pile,axial}$) in the elastic range is calculated using the following equation.

$$k_{pile,axial} = \frac{E_{pile} A_{pile}}{L_{pile}}$$

where E_{pile} = Pile modulus of elasticity
A_{pile} = Cross sectional area of the pile element
L_{pile} = Length of pile element

5 Soil Load Transfer Mechanisms

5.1 Skin Friction

The stiffness of each soil skin element $k_{soil,shear}$ is found using the t-z curve to obtain the unit skin friction corresponding to the soil displacement of the current iteration.

$$k_{soil,shear} = T/L_{segment}$$

Where $L_{segment}$ = length of each pile segment
T = total skin friction of each pile segment in units of force

The total skin friction (T) for a pile segment is calculated from the following equation.

$$T = \tau A_{s,ext}$$

where τ = soil unit skin friction in units of force per area
$A_{s,ext}$ = surface area of the pile segment exterior in contact with soil in shear

Recommended t-z curves are presented later in this document. Currently, the exterior surface area for pre-defined, standard sections, such as those found in the AISC shape database, are calculated based on the equivalent diameter. The equivalent diameter is the diameter that would produce a circular area that is the same as the section's cross-sectional area. The exterior surface area of a pile segment for a standard section is the length of the segment multiplied by the theoretical circumference based on the equivalent diameter. For a User Defined Section, the equivalent diameter defined by the user is used to compute the theoretical exterior surface area.

5.2 End Bearing Resistance – Typical Cross Section

The end bearing resistance (Q) for a pile segment is calculated from the following equation.

$$Q = qA_{toe}$$

where q = unit end bearing resistance in units of force per area
A_{toe} = cross sectional area of pile toe

5.3 End Bearing Resistance – Plugged and Unplugged Condition

For an open tube pile, the ultimate end bearing resistance is the minimum of the plugged and unplugged conditions. The plugged condition is the end bearing resistance when the open tube pile is compacted with soil such that the load transfer mechanism is the soil against the full cross-sectional area ($A_{plugged}$), as shown in Figure 5-1a. The unplugged condition occurs if we consider the load transfer mechanism at the pile toe to consist of soil against the pile cross sectional area ($A_{unplugged}$) and internal skin friction ($\tau_{internal}$) from soil moving inside the pile shaft, as shown in Figure 5-1b. In *RSPile*, the total embedment length is considered in the calculation of internal skin friction. For cohesive soils, the remolded shear strength is used to calculate the internal skin friction since the soil inside the shaft has been disturbed from its in-situ condition.

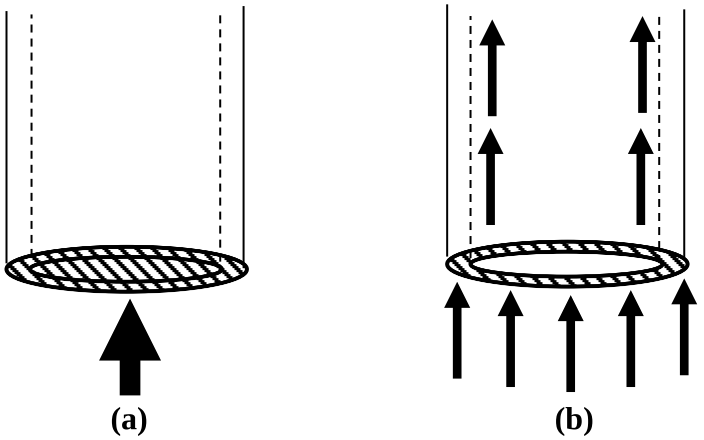

Figure 5-1: End bearing resistance considering (a) plugged condition and (b) unplugged condition

The following equation is used to calculate the end bearing resistance considering plugged condition ($Q_{plugged}$).

$$Q_{plugged} = qA_{plugged}$$

where q = unit end bearing resistance in units of force per area
$A_{plugged}$ = full or plugged cross sectional area at the pile toe

The following equation is used to calculate the end bearing resistance considering unplugged condition ($Q_{unplugged}$).

$$Q_{unplugged} = qA_{unplugged} + \tau_{internal}A_{s,int}$$

where q = unit end bearing resistance in units of force per area
$\tau_{internal}$ = internal skin friction in units of force per area
$A_{unplugged}$ = unplugged cross-sectional area at the pile toe

$A_{s,int}$ = surface area of the pile segment inside the shaft interior in contact with soil in shear

Plugged and unplugged conditions are considered for any cross section that may potentially accumulate compacted soil, such as hollow shapes, and thus change the cross-sectional area considered for end bearing resistance. Typical piles are governed by plugged condition for end bearing resistance since the embedment length is much larger than the cross-sectional area.

For H piles in stiff clays, it is possible that soil becomes trapped and compacted between flange and web causing a plugged condition. Currently, *RSPile* does not consider plugged and unplugged conditions for pre-defined, standard sections, such as those found in the AISC shape database. The standard cross-sectional area is used instead. This also applies to User Defined Sections.

6 Soil Models

6.1 American Petroleum Institute (API)

API (2002) provides recommendations for calculating ultimate skin friction, ultimate end bearing resistance, skin friction (t-z) load transfer curves and end bearing (Q-z) load transfer curves for driven piles in sand and clay.

For driven piles in clay, the ultimate unit skin friction (τ_{ult}) in units of force per area is calculated from the following equation.

$$\tau_{ult} = \alpha\, c_u$$

where α = dimensionless factor
c_u = undrained shear strength of soil at calculation point

The dimensionless factor α is calculated from the following equation.

$$\alpha = 0.5\psi^{-0.5}, \qquad \psi \leq 1.0, \qquad \alpha \leq 1.0$$

$$\alpha = 0.5\psi^{-0.25}, \qquad \psi > 1.0, \qquad \alpha \leq 1.0$$

where ψ = c_u/σ_v
σ_v = effective overburden pressure at calculation point

The ultimate unit end bearing resistance for clay (q) in units of force per area is calculated from the following equation.

$$q = 9c_u$$

where c_u = undrained shear strength at the pile tip

For driven piles in sand, the ultimate unit skin friction (τ_{ult}) in units of force per area is calculated from the following equation.

$$\tau_{\mathrm{ult}} = \mathrm{K}\sigma_{\mathrm{v}} \tan \delta$$

where K = coefficient of lateral earth pressure

σ_{v} = effective overburden pressure at calculation point

δ = friction angle between pile and soil interaction defined as ($\varphi - 5^{o}$), where φ is the soil friction angle

The ultimate end bearing resistance (q) in units of force per area is calculated from the following equation.

$$q = \sigma_{\mathrm{v}} N_q$$

where σ_{v} = effective overburden pressure at the pile toe

N_q = dimensionless bearing capacity factor

API (2002) provides recommendations for estimating the bearing capacity factor (N_q), pile-soil friction angle (δ), maximum unit skin friction (τ_{max}) and maximum end bearing resistance (q_{max}).

API (2002) recommends a linear elastic perfectly plastic skin friction transfer curve for sand as shown in Table 6-1 and Figure 6-1.

Table 6-1: API Sand Skin Friction (t-z) Load Transfer Curve

Soil Displacement (z) (in)	**Unit Skin Friction/ Ultimate Unit Skin Friction ($\tau / \tau_{\mathrm{ult}}$)**
0	0
0.1	1
∞	1

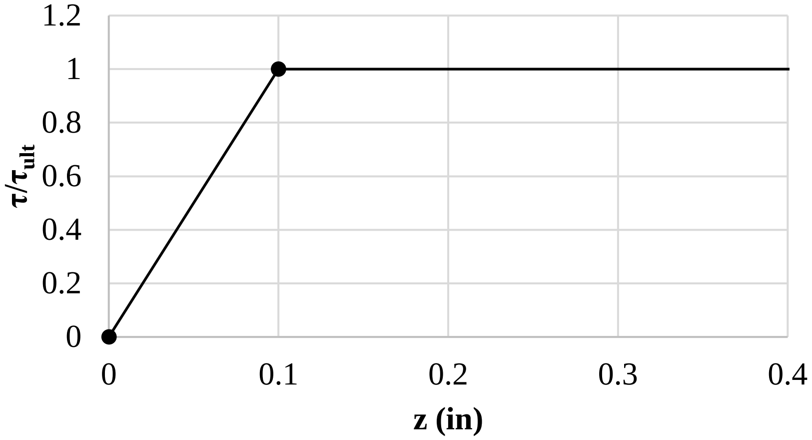

Figure 6-1: API Sand Skin Friction (t-z) Load Transfer Curve

API (2002) recommends the following skin friction transfer curve for clay as shown in Table 6-2 and Figure 6-2.

Table 6-2: API Clay Skin Friction (t-z) Load Transfer Curve

Soil Displacement/Pile Diameter (z/D)	Unit Skin Friction/ Ultimate Unit Skin Friction (τ/τ_{ult})
0	0
0.0016	0.3
0.0031	0.5
0.0057	0.75
0.0080	0.9
0.0100	1
0.0200	0.7 to 0.9*
∞	0.7 to 0.9*

* The residual strength of API Clay is assumed to be 0.9 times the ultimate unit skin friction

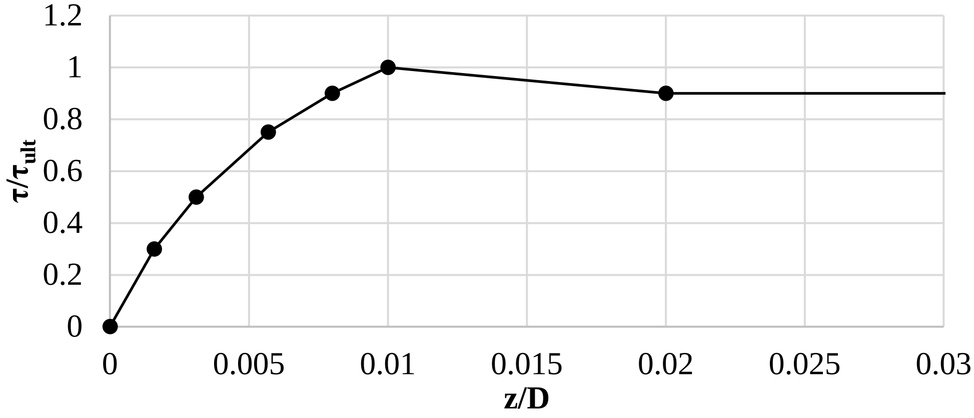

Figure 6-2: API Clay Skin Friction (t-z) Load Transfer Curve

For non-circular pile cross sections, **an equivalent diameter** is used by equating the cross-sectional area with the area of a circle and solving for the diameter.

API (2002) recommends the following end bearing transfer curve for sand and clay as shown in Table 6-3 and Figure 6-3.

Table 6-3: API Sand and Clay End Bearing (Q-z) Load Transfer Curve

Soil Displacement/Pile Diameter (z/D)	**Unit End Bearing/ Ultimate End Bearing (Q/ Q_{ult})**
0	0
0.002	0.25
0.013	0.5
0.042	0.75
0.073	0.9
0.1	1
∞	1

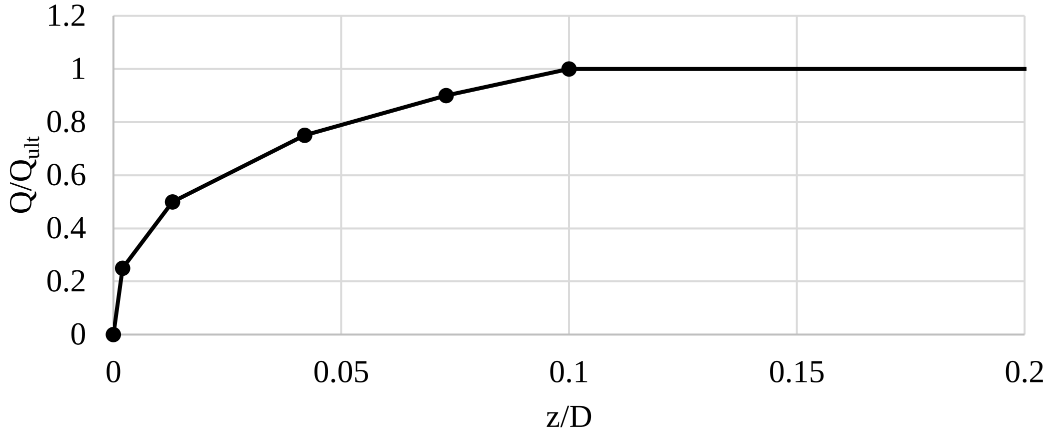

Figure 6-3: API Sand and Clay End Bearing (Q-z) Load Transfer Curve

API (2002) provides the following recommendations in Table 6-4 for cohesionless soil properties.

Table 6-4: Design Parameters for Cohesionless Siliceous Soil* (API, 2002)

Density	Soil Description	Soil-Pile Friction Angle (δ)	Max Skin Friction Values in kPa (kips/ft^2)	Bearing Capacity Factor, N_q	Max Unit End Bearing Values in MPa (kips/ft^2)
Very Loose Loose Medium	Sand Sand-Silt** Silt	15	47.8 (1.0)	8	1.9 (40)
Loose Medium Dense	Sand Sand-Silt** Silt	20	67.0 (1.4)	12	2.9 (60)
Medium Dense	Sand Sand-Silt**	25	81.3 (1.7)	20	4.8 (100)
Dense Very Dense	Sand Sand-Silt**	30	95.7 (2.0)	40	9.6 (200)
Dense Very Dense	Gravel Sand	35	114.8 (2.4)	50	12.0 (250)

*The parameters listed in this table are intended as guidelines only. Where detailed information such as in situ cone tests, strength tests on high quality samples, model tests, or pile driving performance is available, other values may be justified.

**Sand-Silt includes those soils with significant fractions of both sand and silt. Strength values generally increase with increasing sand fractions and decrease with increasing silt fractions.

6.2 Elastic Soil Model

An elastic soil material has infinite strength and can easily be defined using a constant stiffness. For elastic soil, the user must define the following parameters:

- Unit Skin Friction Stiffness - The change in unit skin friction per unit of soil displacement at each depth.
- Unit End Bearing Stiffness - The change in unit end bearing resistance per unit of soil displacement at the pile toe.

The unit skin friction stiffness is multiplied by the surface area of the pile segment to obtain the skin friction per unit of soil displacement. The unit end bearing stiffness is multiplied by the plugged cross-sectional area at the pile toe to obtain the end bearing resistance per unit of soil displacement.

6.3 User Defined Soil Model

A user defined soil model allows the user to input the t-z and q-z curves based on the ultimate skin friction or end bearing resistance and the curve shape. The curve shape is defined by entering stress to max stress ratios for various soil displacement. The stress to max stress ratio is multiplied by the ultimate skin friction or end bearing resistance for a depth to calculate the assumed soil response due to soil displacement. The stress to max stress ratios for each displacement value allows the user to vary the ultimate resistance value along the depth while maintaining the same t-z or q-z curve shape for the material. When the soil displacement exceeds the last (maximum) entered displacement value, the soil resistance is assumed to be the last entered resistance.

6.4 Coyle Reese Clay (Driven Piles)

The Coyle and Reese (1966) method is based on instrumented field test results. Note that the curve is not dependent on pile diameter.

Table 6-5: Coyle Reese Clay Skin Friction (t-z) Load Transfer Curve

Soil Displacement (z)	Unit Skin Friction/ Ultimate Unit Skin Friction (τ/ τ_{ult})
0	0
0.1	0.18
0.2	0.38
0.4	0.79
0.6	0.97
0.8	1
0.12	0.97
0.16	0.93
0.2	0.93

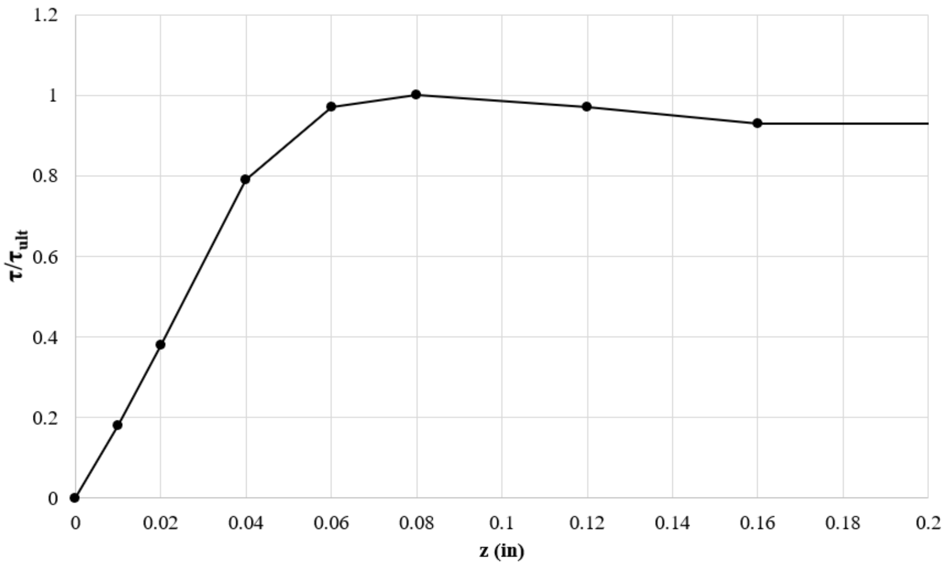

Figure 6-4: Coyle Reese Clay Skin Friction (t-z) Load Transfer Curve

Table 6-6: Coyle Reese Clay End Bearing (Q-z) Load Transfer Curve

Soil Displacement/Pile Diameter (z/D)	Unit End Bearing/ Ultimate End Bearing (Q/ Q_{ult})
0	0
0.1	0.35355
0.2	0.5003
0.3	0.61217
0.4	0.70690
0.5	0.79043
0.6	0.86580
0.7	0.93558
0.8	1.0026
1	1.00000

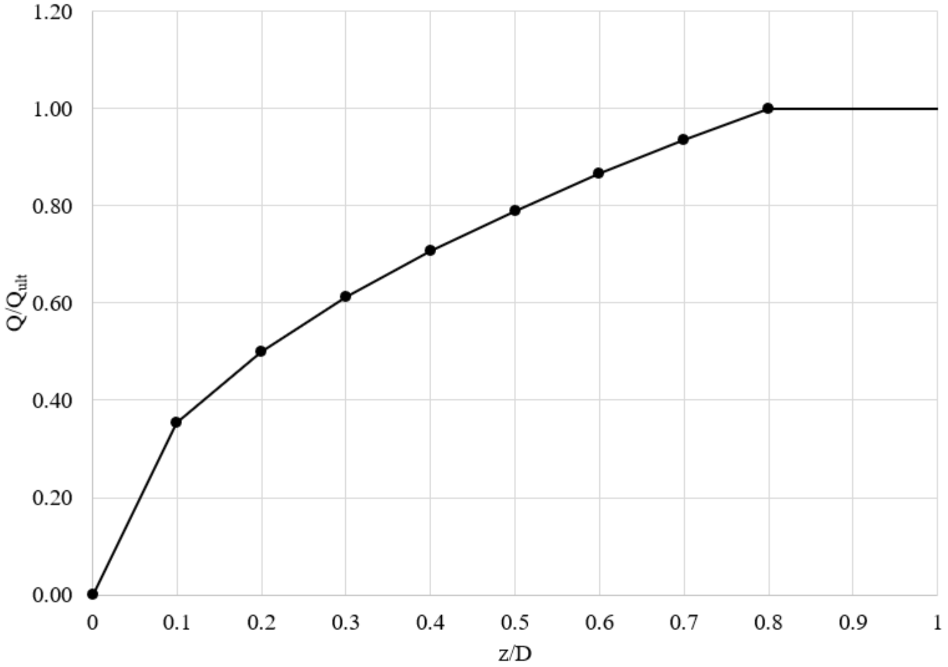

Figure 6-5: Coyle Reese Clay End Bearing (Q-z) Load Transfer Curve

6.5 Mosher Sand

Mosher (1984) proposed approaches for calculating side resistance and the maximum tip resistance in sand, based on literature review and conducting experiments.

Table 6-7: Mosher Sand Skin Friction (t-z) Load Transfer Curve

Soil Displacement (z)	**Unit Skin Friction/ Ultimate Unit Skin Friction (τ/τ_{ult})**
0	0
0.000254	0.00386223
0.000508	0.00770288
0.001016	0.0153141
0.001524	0.02284982
0.002032	0.03029118
0.003048	0.04493125
0.004064	0.05926126
0.0127	0.16931788
0.254	1
0.3	1

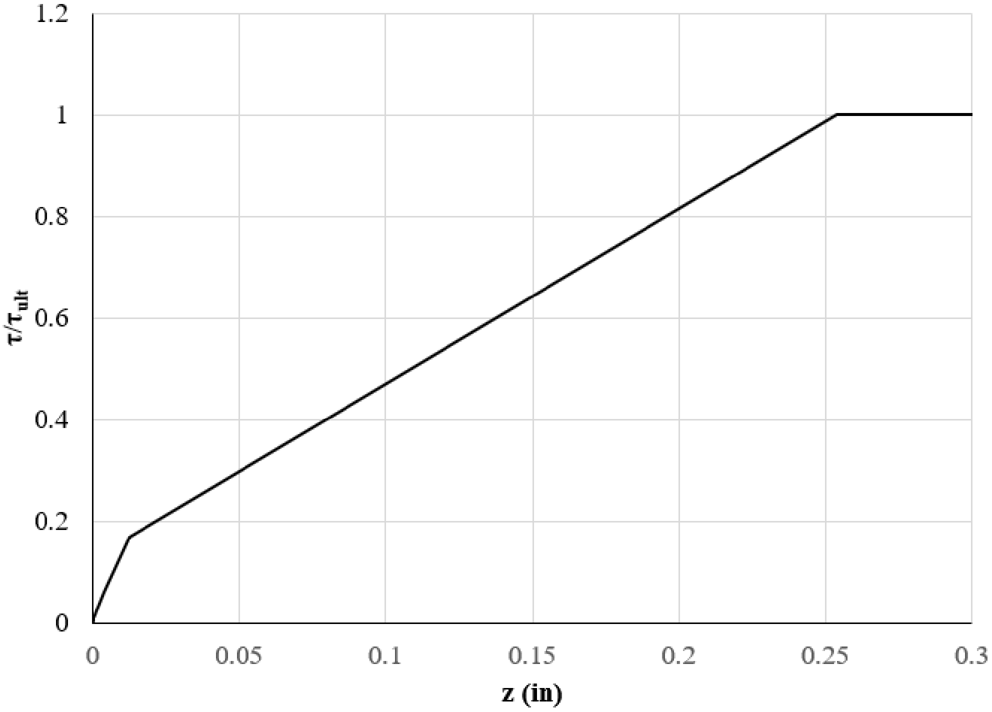

Figure 6-6: Mosher Sand Skin Friction (t-z) Load Transfer Curve

Table 6-8: Mosher Sand End Bearing (Q-z) Load Transfer Curve

Soil Displacement (z)	Unit End Bearing/ Ultimate End Bearing (Q/ Q_{ult})
0	0
0.0001	0.01
0.008	0.070711
0.01	0.1
0.005	0.223607
0.1	0.316228
0.2	0.447214
0.5	0.707107
0.75	0.866025
1	1
1.2	1

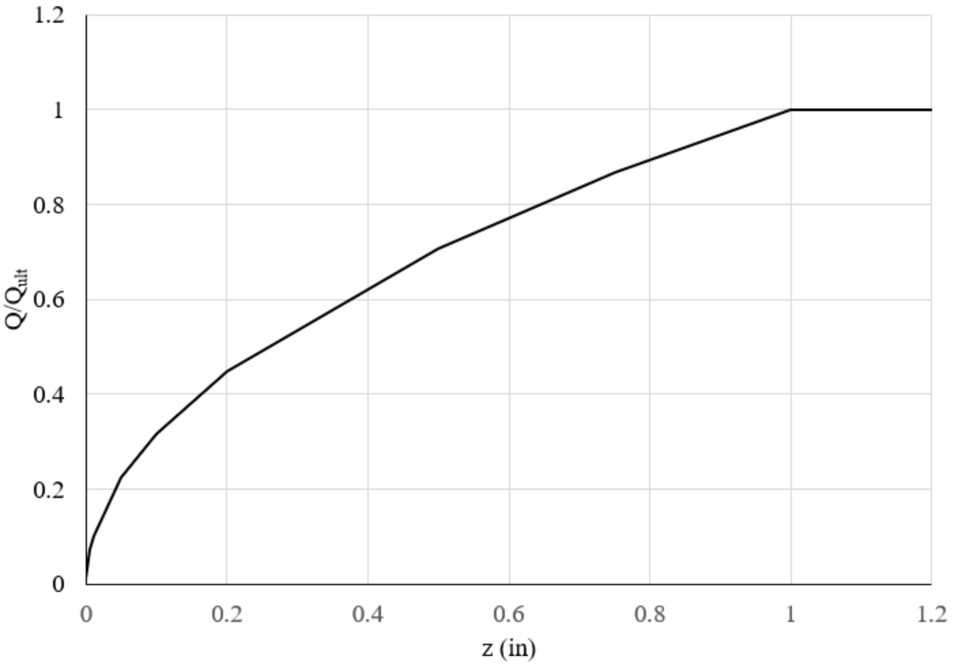

Figure 6-7: Mosher Sand End Bearing (Q-z) Load Transfer Curve

6.6 Drilled Clay

Reese and O'Neill (1988) used the results of field-load tests of drilled shafts to develop the curves shown below.

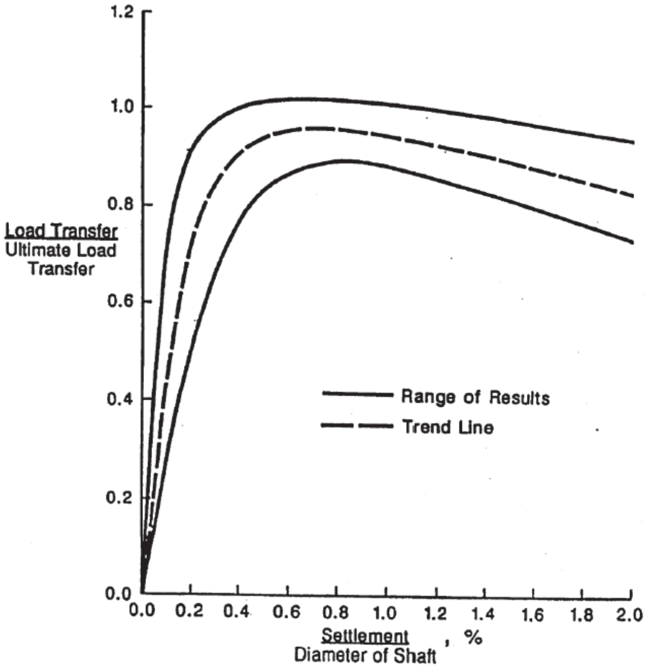

Figure 6-8: Drilled Clay Skin Friction (t-z) Load Transfer Curve

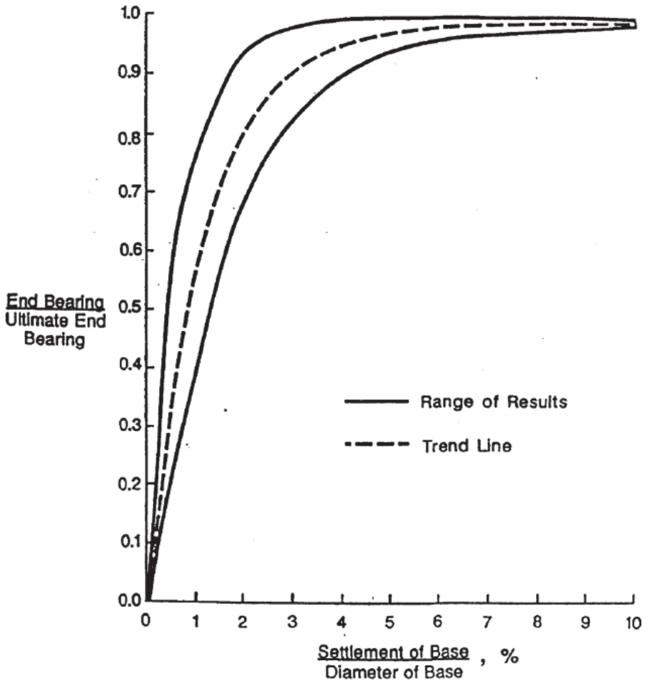

Figure 6-9: Drilled Clay End Bearing (Q-z) Load Transfer Curve

6.7 Drilled Sand

The load-transfer curves presented below were developed by Reese and O’Neill (1988) based on the study of tests of drilled shafts end bearing vs settlement results.

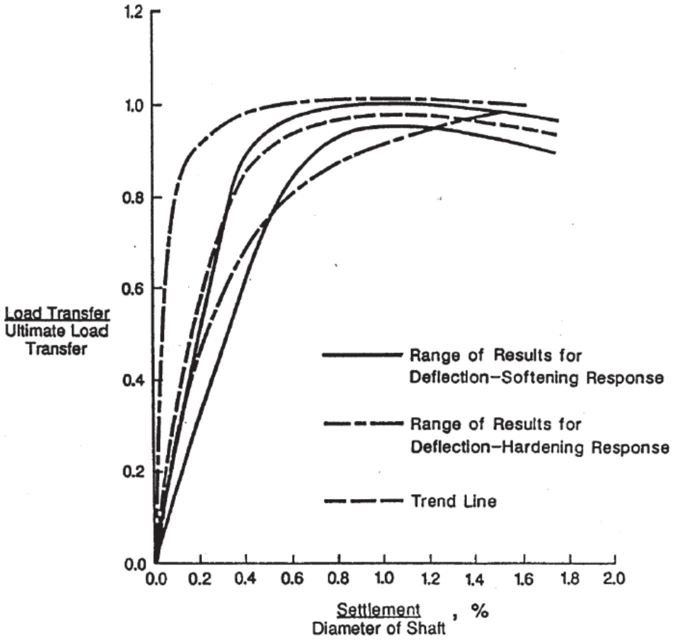

Figure 6-10: Drilled Sand Skin Friction (t-z) Load Transfer Curve

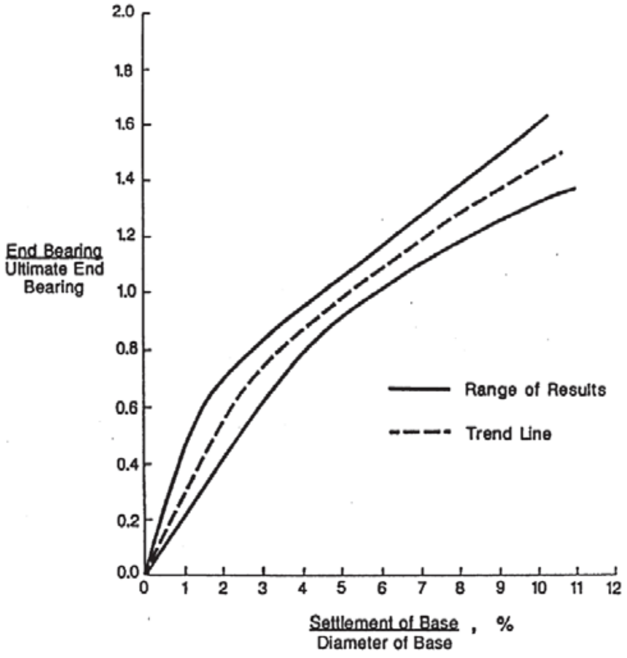

Figure 6-11: Drilled Sand End Bearing (Q-z) Load Transfer Curve

7 References

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