18 Drained Triaxial Compressive Test of Modified Cam Clay Material

18.1 Problem Description

The Modified Cam Clay (MCC) constitutive relationship is one of the earliest critical state models for realistically describing the behaviour of soft soils. As a result it is one of the most widely applied stress-strain relationship in the non-linear finite element modeling of practical geotechnical problems. The state at a point in an MCC soil is characterized by three parameters: effective mean stress p', deviatoric (shear stress) q, and specific volume v.

Due to the complexity of the MCC model, very few MCC problems have closed-form solutions, which can be used to verify the accuracy, stability and convergence of MCC finite element algorithms. One of the problems with an analytical solution is the consolidated-drained triaxial test on a MCC sample. In this test, the sample is first consolidated under a hydrostatic pressure, and then sheared by applying additional axial load (see Figure 18-1). The drainage condition is such that there is no build up of excess pore water pressures (i.e. excess pore pressures are allowed to fully dissipate).

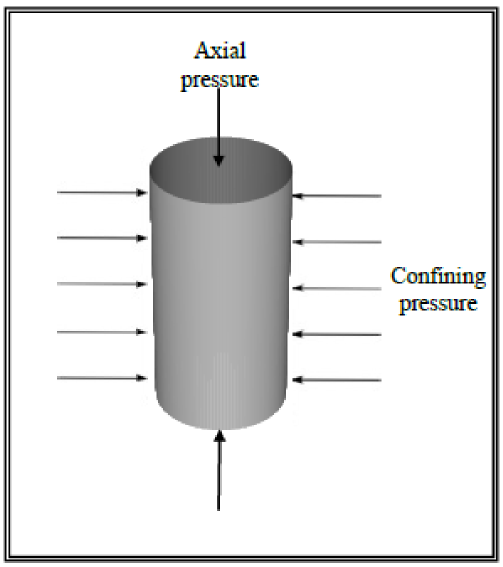

Figure 18-1: Triaxial compressive test of cylindrical soil sample

In ***RS3***, the MCC constitutive model is integrated implicitly over a finite strain increment using the approach presented by Borja [1]. Major advantages of this approach are its accuracy, robustness, and efficiency. The performance of this algorithm in ***RS3*** will be tested in three examples of drained triaxial test. The first test on a normally consolidated clay sample involves only post-yield (elasto-plastic) loading; a behavior that is associated with hardening of the material. The second test is on a lightly over consolidated clay sample where the initial behavior is elastic and it is followed by a transition to elasto-plastic response. The last example demonstrates the behavior of a highly over consolidated clay sample that includes an initial elastic behavior followed by failure and a softening branch in its stress path. The stress paths, initial and final yield surfaces of these tests are shown in Figure 18-2 to Figure 18-4

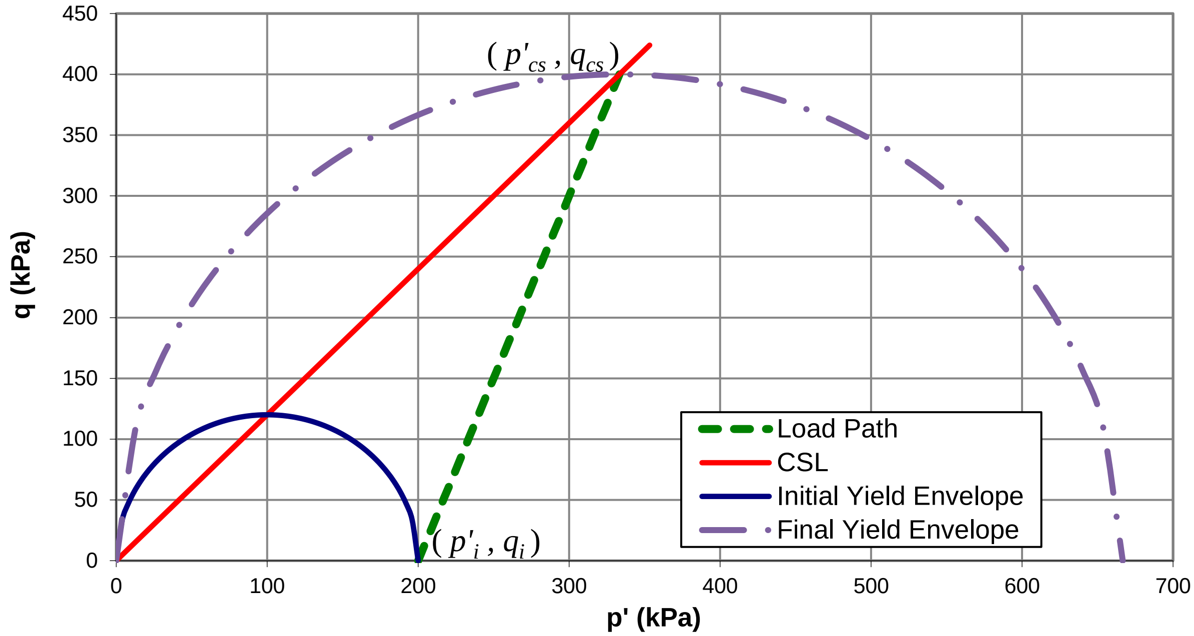

Figure 18-2 : Example 1, drained triaxial compressive test on a normally consolidated clay sample, stress path, initial and final yield surfaces in *p'*-*q* space

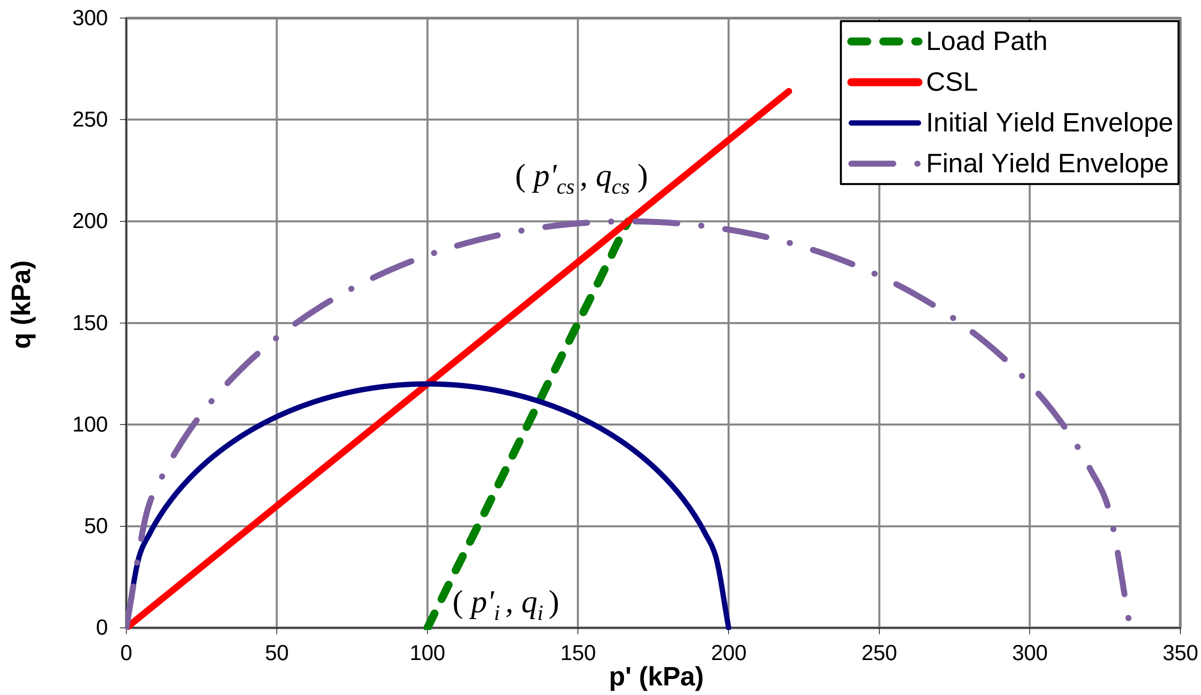

Figure 18-3 : Example 2, drained triaxial compressive test on a lightly over consolidated caly sample(OCR=2), stress pth, initial and fimnaly yield surfaces in *p'*-*q'* space

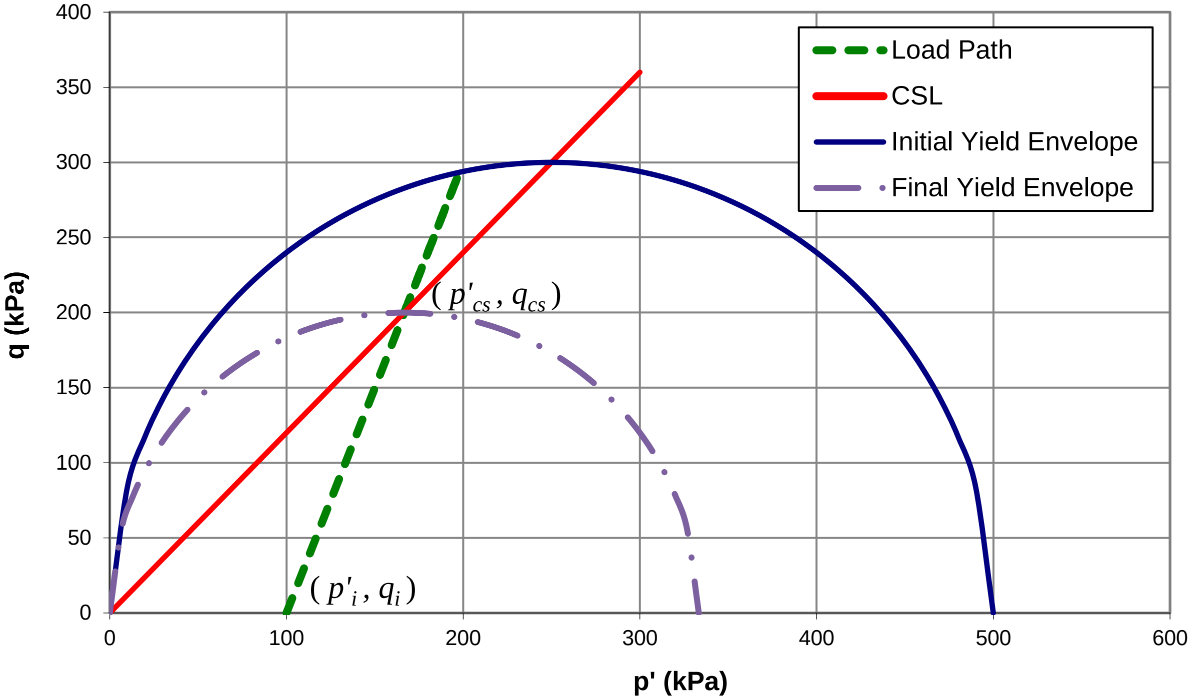

Figure 18-4: Example 3, drained triaxial compressive test on a highly over consolidated clay sample (OCR=5), stress pth, initial and final yield surfaces in *p'=q'* space

For each triaxial test, two plots will be generated to compare the performance of the MCC implementation in ***RS3*** in relation to the drained triaxial test benchmark solution. The first plot examines the relationship between deviatoric (shear stress), q, and axial strain, ε_a, of the test sample, while the second compares volumetric strains, ε_v, to axial strains.

Five material parameters are required to specify the behaviour of the MCC sample. These are:

1. λ – the slope of the normal compression (virgin consolidation) line and critical state line (CSL) in $v - \ln p'$ space
2. κ – the slope of a swelling (loading-unloading) line in $v - \ln p'$ space
3. *M* – the slope of the CSL in $q - p'$ space
4. $\begin{cases} N - \text{the specific volume of the normal compression line at unit pressure} \\ or \\ \Gamma\ - \text{the specific volume of the CSL at unit pressure} \end{cases}$
5. $\begin{cases} \mu - \text{Poisson's ratio} \\ or \\ G - \text{shear modulus} \end{cases}$

As can be seen from the description of input parameters, the MCC formulation requires specification of either a constant shear modulus G or a constant Poisson's ratio μ, but not both. The verification example will examine the performance of ***RS3*** using both of these options.

The initial state of consolidation of the MCC soil is specified in terms of a pre-consolidation pressure, p_o. (***RS3*** also allows users to specify the initial state of consolidation through the over-consolidation ratio.)

For the test, the following material properties and conditions are assumed:

Table 18-1 : Model parameteres

Parameter	***Value***
N	1.788
M	1.2
λ	0.066
κ	0.0077
G (for the case of constant elasticity)	20000 kPa
μ (for the case of variable elasticity)	0.3
Initial State of the Normally Consolidated Clay	
Preconsolidation pressure, p_o	200 kPa
Initial mean volumetric stress, p'	200 kPa
Initial shear stress, q	0 kPa
Initial State of the Lightly Over Consolidated Clay	
Preconsolidation pressure, p_o	200 kPa
Initial mean volumetric stress,'p	100 kPa
Initial shear stress, q	0 kPa
Initial State of Highly Over Consolidated Clay	
Preconsolidation pressure, p_o	500 kPa
Initial mean volumetric stress, p'	100 kPa
Initial shear stress, q	0 kPa

18.2 Analytical Solution

The analytical solution presented here is adopted from an article by Peric [2]. The solution distinguishes between the volumetric, (p', ε_v), and the deviatoric, (q, ε_q), behaviour of the material.

18.2.1 The volumetric behaviour

Decomposing to its elastic and plastic parts, the rate of the volumetric strain can be obtained from its nonlinear elastic behavior and the hardening rule.

$$\dot{\varepsilon}_v^e = k\dot{p}' = \left(\frac{\kappa}{v_n}\right)\frac{\dot{p}'}{p'} \qquad \dot{\varepsilon}_v^p = \left(\frac{\lambda - \kappa}{v_n}\right)\frac{\dot{p}_0}{(p_0)_n}$$

Considering a general rate of stress, using the definition of the yield surface, the rate of plastic volumetric strain can be rewritten as

$$\dot{\varepsilon}_v^p = \left(\frac{\lambda - \kappa}{v_n}\right)\left(\frac{\dot{p}'}{p'} + \frac{2\eta\dot{\eta}}{M^2 + \eta^2}\right) \quad , \quad \eta = \frac{q}{p'} \quad , \quad \dot{\eta} = \frac{\dot{q}}{\dot{p}'}$$

Integrating the above equation over a finite time increment (step n to step $n+1$), assuming that the change in specific volume is insignificant, results in the following incremental equation

$$\Delta\varepsilon_v^e = \frac{1}{v_n}\ln\left(\frac{p'_{n+1}}{p'_n}\right)^{\kappa} \qquad \Delta\varepsilon_v^p = \frac{1}{v_n}\ln\left(\left(\frac{p'_{n+1}}{p'_n}\right)\left(\frac{M^2 + \eta_{n+1}^2}{M^2 + \eta_n^2}\right)\right)^{\lambda-\kappa}$$

Thus, the total increment of volumetric strain is

$$\Delta\varepsilon_v = \frac{1}{v_n}\ln\left(\left(\frac{p'_{n+1}}{p'_n}\right)^{\lambda}\left(\frac{M^2 + \eta_{n+1}^2}{M^2 + \eta_n^2}\right)^{\lambda-\kappa}\right)$$

Considering a straight stress path in $(p' - q)$ space, with a slope of $(\Delta q/\Delta p') = k$, the above equation can be rewritten as

$$\Delta\varepsilon_v = \frac{1}{v_n}\ln\left(\left(\frac{k - \eta_n}{k - \eta_{n+1}}\right)^{\lambda}\left(\frac{M^2 + \eta_{n+1}^2}{M^2 + \eta_n^2}\right)^{\lambda-\kappa}\right)$$

Note that in the case of a drained triaxial test $k = 3$.

The change in the specific volume can also be calculated from the

$$\Delta v = \frac{\Delta\varepsilon_v}{v_n}$$

18.2.2 The deviatoric behavior

According to the flow rule the rate of plastic strains are calculated as

$$\dot{\varepsilon}_v^p = \dot{\lambda}\frac{\partial F}{\partial p'} \qquad \dot{\varepsilon}_q^p = \dot{\lambda}\frac{\partial F}{\partial q}$$

So the relation between the rate of volumetric strain and the deviatoric one is

$$\dot{\varepsilon}_q^p\frac{\partial F}{\partial p'} = \dot{\varepsilon}_v^p\frac{\partial F}{\partial q}$$

Thus the rate of deviatoric plastic strain is

$$\dot{\varepsilon}_q^p = \frac{2\eta}{M^2 - \eta^2}\dot{\varepsilon}_v^p = \left(\frac{\lambda - \kappa}{v_n}\right)\left(\frac{2\eta}{M^2 - \eta^2}\right)\left(\frac{\dot{p}'}{p'} + \frac{2\eta\dot{\eta}}{M^2 + \eta^2}\right)$$

Once again by considering a straight stress path, with a slope of $(\Delta q/\Delta p') = k$, the plastic deviatoric strain can be calculated as

$$\dot{\varepsilon}_q^p = \left(\frac{\lambda - \kappa}{v_n}\right)\left(\frac{2\eta}{\left(M^2 - \eta^2\right)(k - \eta)} + \frac{4\eta^2}{M^4 + \eta^4}\right)\eta^{\cdot}$$

The elastic portion of the deviatoric strain can be calculated from Hooke's law:

$$\dot{\varepsilon}_q^p = \frac{\dot{q}}{3G}$$

In case the model uses a constant Poisson's ratio, the shear modulus should be calculated as

$$G = \alpha K = \frac{\alpha v_n p'}{\kappa} \quad , \qquad \alpha = \frac{3(1 - 2\mu)}{2(1 + \mu)}$$

Integrating the rate of deviatoric strain over a finite time increment (step *n* to step *n* +1), results in the following incremental equation for the plastic and elastic portion of deviatoric strain

$$\Delta\varepsilon_q^p = \frac{1}{v_n}\ln\left[\left(\frac{M - \eta_{n+1}}{M - \eta_n}\right)^{\frac{(\lambda-\kappa)k}{M(M+k)}}\left(\frac{M + \eta_{n+1}}{M + \eta_n}\right)^{\frac{(\lambda-\kappa)k}{M(M+k)}}\left(\frac{M + \eta_{n+1}}{M + \eta_n}\right)^{\frac{2(\lambda-\kappa)k}{k^2-M^2}}\right] - \frac{2(\lambda - \kappa)}{M v_n}\left[\arctan\left(\frac{\eta_{n+1}}{M}\right) - \arctan\left(\frac{\eta_n}{M}\right)\right]$$

In the case of constant shear modulus the elastic part of the increment of deviatoric strain is

$$\Delta\varepsilon_q^e = \frac{q_{n+1} - q_n}{3G}$$

Otherwise,

$$\Delta\varepsilon_q^e = \frac{1}{v_n}\ln\left(\frac{k - \eta_{n+1}}{k - \eta_n}\right)^{-\frac{\kappa k}{3\alpha}}$$

The volumetric and shear strains calculated in a triaxial test can be related to the axial and radial strains, ε_a and ε_r, respectively, of the test sample. The relationships are as follows

$$\varepsilon_a = \frac{1}{3}\varepsilon_v + \varepsilon_q \quad , \qquad \varepsilon_r = \frac{1}{3}\varepsilon_v - \frac{1}{2}\varepsilon_q$$

The formulations presented above have been implemented in Excel spreadsheets included with this document.

18.3 *RS3* Model

The drained compressive triaxial tests of the MCC sample were modeled in ***RS3*** using 4-noded tetrahedral elements. The deviatoric stress is generated in the sample in two different ways using load-control and displacement-control processes. In the load–control method the axial load is increased in a number of stages that match the load steps used in the analytical solution. In the displacement-control simulations axial displacement is imposed on the sample, once again in a number of stages that match the displacement history of analytical solutions. The boundary conditions and an example of the applied loads used are shown on Figure 18-5 and Figure 18-6 for load-control and displacement-control simulations.

As mentioned before, for each example the stage factors for the axial loads or axial deformation were calculated (from the attached spreadsheet) such that the resulting effective mean and deviatoric stresses conformed to the selected triaxial loading path. In the first test, which starts with stresses on the initial yield envelope, the load path (shown on Figure 18-2) was applied in 32 stages. In Examples 2 and 3 (Figure 18-3 and Figure 18-4), the load path was applied in 35 stages.

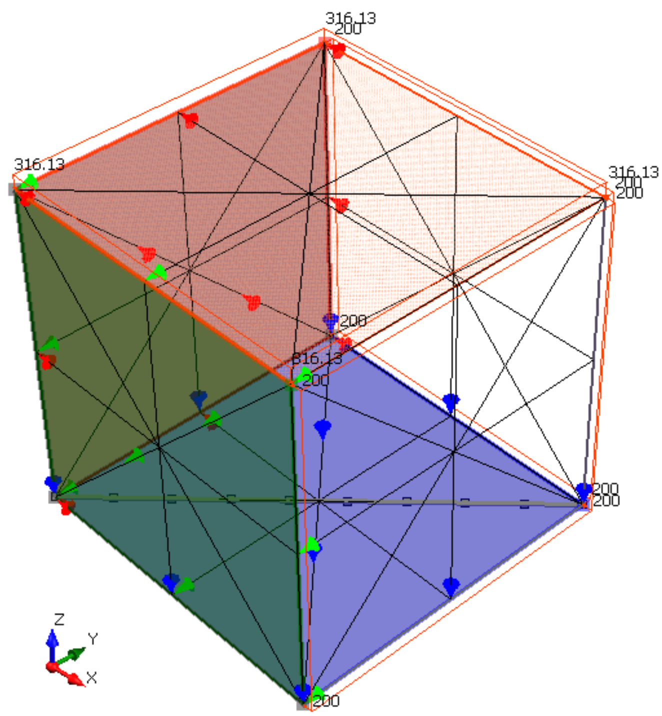

Figure 18-5: Boundary conditions and loads for axisymmetric *RS3* analysis; load-control control simulation

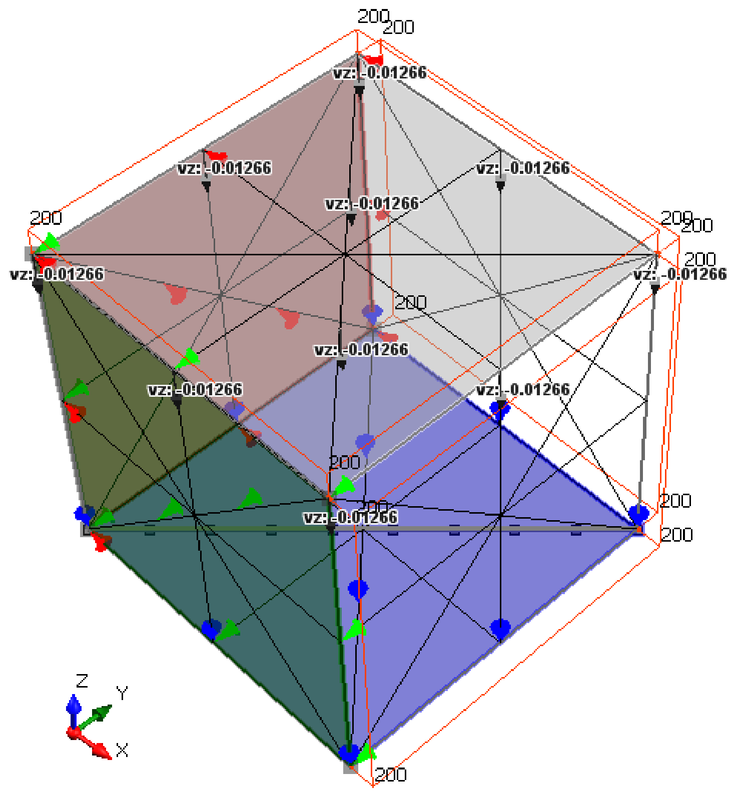

Figure 18-6: Boundary conditions and loads for axisymmetric *RS3* analysis; displacement-control control simulation

18.4 Results

Table 18-2 and Table 18-3 present the variation of the deviatoric stress, axial and volumetric strains calculated from the analytical solution and from ***RS3*** for the first triaxial test example (Example 1). Table 18-2 represents the case of constant shear modulus, while in Table 18-3 the Poisson's ratio is constant**.**

Figure 18-7 and Figure 18-8 show the plots of $\varepsilon_a - q$ and $\varepsilon_a - \varepsilon_v$ obtained from the analytical and numerical solutions for the case of constant shear modulus. Figure 18-9 and Figure 18-10 show the same results but for the case of constant Poisson's ratio.

Accordingly, the results for the second and third examples are summarized in Table 18-4 to Table 18-6 and Figure 18-11 to Figure 18-16.

For all the cases analyzed below, there is a good agreement between the analytical results and the numerical results obtained from ***RS3.***

Table 18-2: Example 1, Triaxial test on a normally consolidated clay sample; results for case of constant shear modulus

	RS3 Load-Control			*RS3* Displacement-Control			Analytical Solution		
No.	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**
1	0.00	0	0	0.00	0.00E+00	0.00E+00	0	0	0
2	12.90	0.00063	0.00109	13.21	0.00065	0.00112	12.90	0.00062	0.00109
3	25.81	0.00144	0.00236	26.27	0.00147	0.00241	25.81	0.00142	0.00236
4	38.71	0.00243	0.00380	39.18	0.00247	0.00385	38.71	0.00239	0.00379
5	51.61	0.00362	0.00536	52.01	0.00366	0.00541	51.61	0.00357	0.00535
6	64.52	0.00501	0.00703	64.83	0.00505	0.00707	64.52	0.00494	0.00702
7	77.42	0.00661	0.00879	77.61	0.00664	0.00881	77.42	0.00653	0.00877
8	90.32	0.00842	0.01061	90.41	0.00844	0.01062	90.32	0.00832	0.01059
9	103.23	0.01045	0.01248	103.22	0.01045	0.01248	103.23	0.01032	0.01246
10	116.13	0.01269	0.01439	115.95	0.01266	0.01436	116.13	0.01254	0.01436
11	129.03	0.01514	0.01632	128.72	0.01509	0.01627	129.03	0.01498	0.01628
12	141.94	0.01783	0.01825	141.50	0.01774	0.01819	141.94	0.01763	0.01822
13	154.84	0.02074	0.02020	154.29	0.02062	0.02011	154.84	0.02052	0.02016
14	167.74	0.02389	0.02213	167.05	0.02373	0.02203	167.74	0.02365	0.02209
15	180.65	0.02730	0.02406	179.85	0.02710	0.02394	180.65	0.02702	0.02402
16	193.55	0.03098	0.02597	192.60	0.03071	0.02583	193.55	0.03067	0.02593
17	206.45	0.03494	0.02786	205.39	0.03462	0.02771	206.45	0.03460	0.02782
18	219.35	0.03923	0.02973	218.18	0.03884	0.02956	219.35	0.03884	0.02968
19	232.26	0.04387	0.03158	230.97	0.04341	0.03140	232.26	0.04344	0.03153
20	245.16	0.04891	0.03339	243.80	0.04837	0.03320	245.16	0.04843	0.03334
21	258.06	0.05440	0.03518	256.64	0.05378	0.03499	258.06	0.05387	0.03513
22	270.97	0.06044	0.03694	269.45	0.05971	0.03674	270.97	0.05984	0.03689
23	283.87	0.06711	0.03867	282.31	0.06627	0.03847	283.87	0.06644	0.03862
24	296.77	0.07457	0.04037	295.20	0.07360	0.04017	296.77	0.07380	0.04032
25	309.68	0.08301	0.04204	308.11	0.08188	0.04184	309.68	0.08212	0.04199
26	322.58	0.09274	0.04368	320.99	0.09141	0.04348	322.58	0.09170	0.04363
27	335.48	0.10422	0.04529	333.91	0.10263	0.04510	335.48	0.10297	0.04524
28	348.39	0.11825	0.04687	346.89	0.11630	0.04669	348.39	0.11669	0.04682
29	361.29	0.13634	0.04842	359.95	0.13383	0.04826	361.29	0.13426	0.04838
30	374.19	0.16195	0.04994	373.29	0.15836	0.04982	374.19	0.15887	0.04990
31	387.09	0.20652	0.05144	386.97	0.20007	0.05140	387.10	0.20061	0.05139

Table 18-3: Example 1, Triaxial test on a normally consolidated clay sample; results for case of constant Poisson's ratio

	RS3 Load-Control			*RS3* Displacement-Control			Analytical Solution		
No.	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv
1	0.00	0.00000	0.00000	0.00	0.00000	0.00000	0.00	0.00000	0
2	12.90	0.00066	0.00109	12.66	0.00065	0.00107	12.90	0.00065	0.0010885
3	25.81	0.00149	0.00236	25.44	0.00147	0.00232	25.81	0.00147	0.0023609
4	38.71	0.00251	0.00380	38.24	0.00247	0.00374	38.71	0.00247	0.0037905
5	51.61	0.00372	0.00536	51.04	0.00366	0.00529	51.61	0.00367	0.005351
6	64.52	0.00512	0.00703	63.87	0.00505	0.00694	64.52	0.00506	0.0070185
7	77.42	0.00673	0.00879	76.69	0.00664	0.00869	77.42	0.00665	0.0087712
8	90.32	0.00855	0.01061	89.55	0.00844	0.01050	90.32	0.00845	0.0105899
9	103.23	0.01058	0.01248	102.43	0.01045	0.01236	103.23	0.01045	0.0124576
10	116.13	0.01281	0.01439	115.24	0.01266	0.01426	116.13	0.01267	0.0143598
11	129.03	0.01527	0.01632	128.09	0.01509	0.01617	129.03	0.01510	0.0162842
12	141.94	0.01794	0.01825	140.95	0.01774	0.01811	141.94	0.01775	0.0182202
13	154.84	0.02085	0.02020	153.83	0.02062	0.02005	154.84	0.02063	0.020159
14	167.74	0.02398	0.02213	166.69	0.02373	0.02198	167.74	0.02374	0.0220935
15	180.65	0.02737	0.02406	179.58	0.02710	0.02390	180.65	0.02710	0.0240175
16	193.55	0.03103	0.02597	192.41	0.03071	0.02581	193.55	0.03072	0.0259263
17	206.45	0.03497	0.02786	205.29	0.03462	0.02769	206.45	0.03463	0.0278161
18	219.35	0.03923	0.02973	218.15	0.03884	0.02956	219.35	0.03885	0.0296835
19	232.26	0.04384	0.03158	231.04	0.04341	0.03140	232.26	0.04342	0.0315264
20	245.16	0.04885	0.03339	243.93	0.04837	0.03322	245.16	0.04838	0.0333428
21	258.06	0.05432	0.03518	256.82	0.05378	0.03501	258.06	0.05379	0.0351314
22	270.97	0.06031	0.03694	269.71	0.05971	0.03677	270.97	0.05972	0.0368911
23	283.87	0.06695	0.03867	282.62	0.06627	0.03851	283.87	0.06628	0.0386214
24	296.77	0.07436	0.04037	295.55	0.07360	0.04021	296.77	0.07360	0.0403218
25	309.68	0.08277	0.04204	308.44	0.08188	0.04188	309.68	0.08188	0.0419923
26	322.58	0.09245	0.04368	321.34	0.09141	0.04353	322.58	0.09141	0.0436327
27	335.48	0.10388	0.04529	334.27	0.10263	0.04514	335.48	0.10264	0.0452433
28	348.39	0.11787	0.04687	347.23	0.11630	0.04673	348.39	0.11631	0.0468244
29	361.29	0.13591	0.04842	360.26	0.13383	0.04829	361.29	0.13383	0.0483764
30	374.19	0.16146	0.04994	373.34	0.15836	0.04984	374.19	0.15838	0.0498996
31	387.09	0.20590	0.05144	386.61	0.20007	0.05138	387.10	0.20007	0.0513945

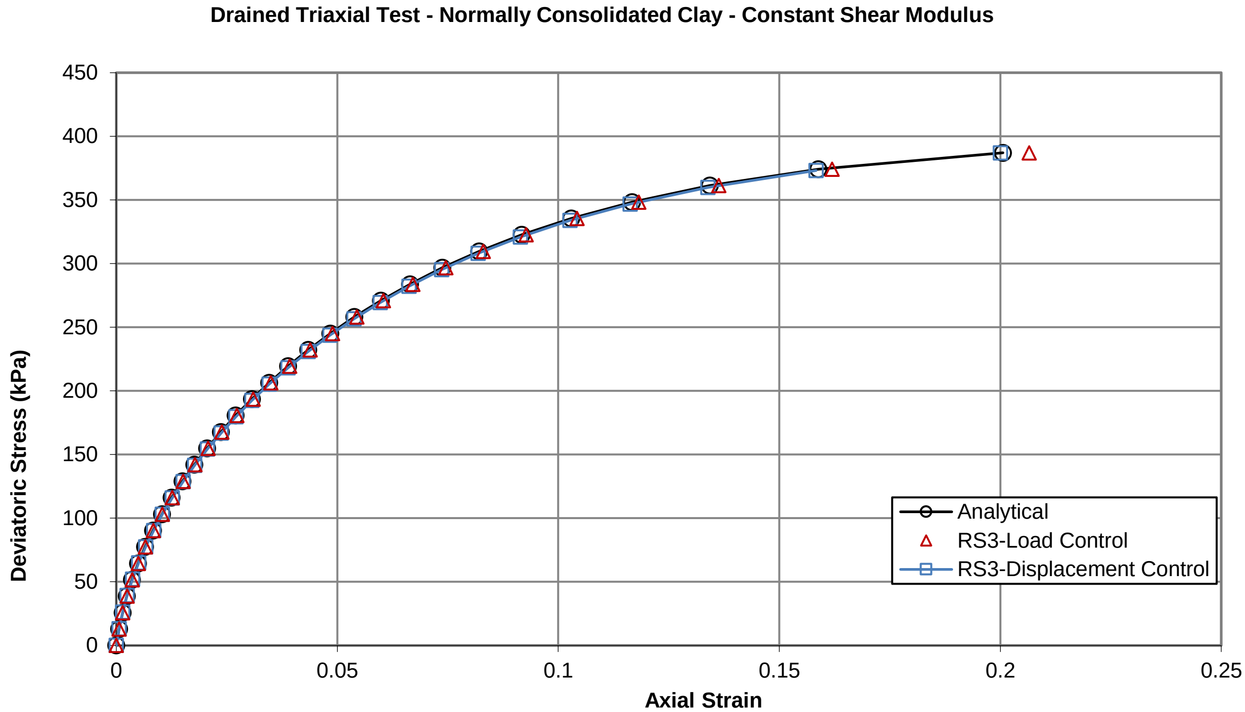

Figure 18-7: Variation of deviatoric stress with axial strain for Example 1 case of constant shear modulus

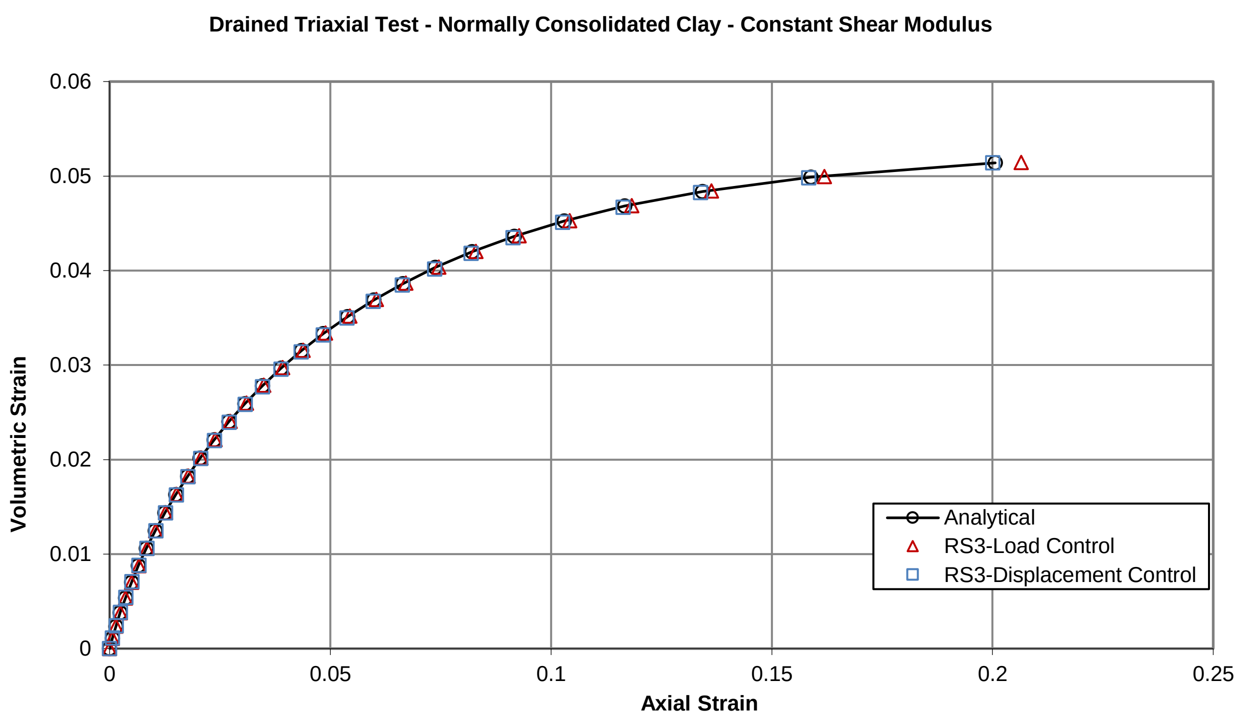

Figure 18-8: Variation of volumetric strain with axial strain for Example 1 case of constant shear modulus

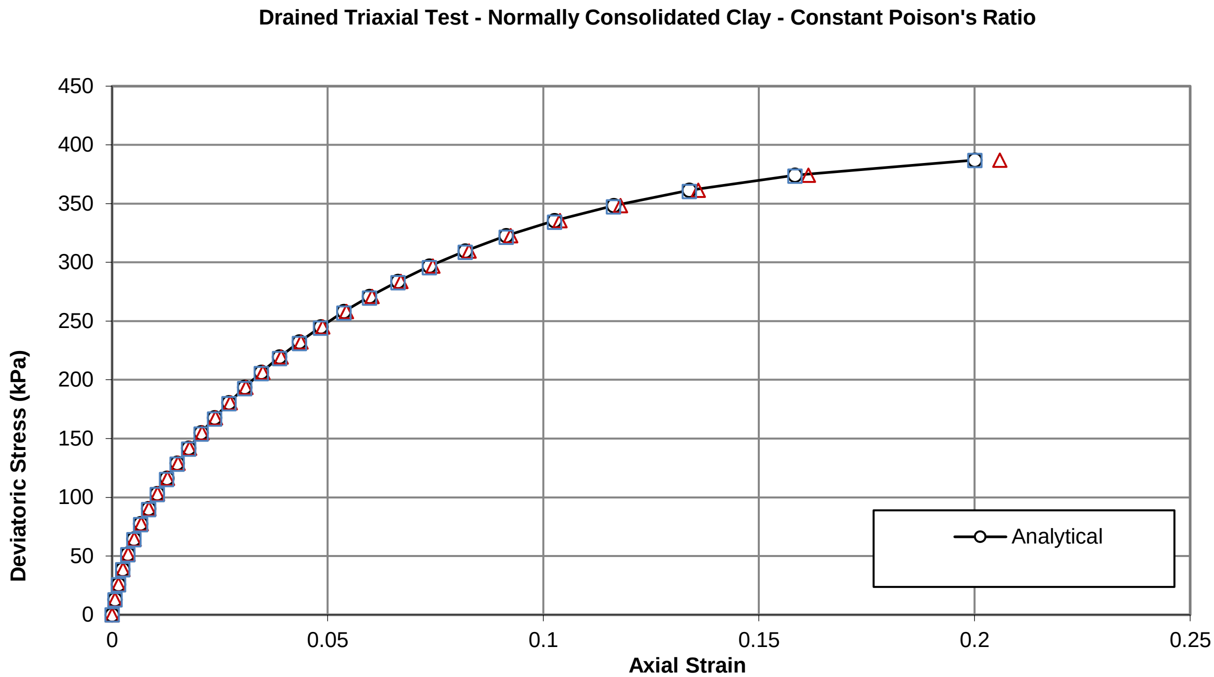

Figure 18-9: Variation of deviatoric stress with axial strain for Example 1 case of constant Poisson's ratio

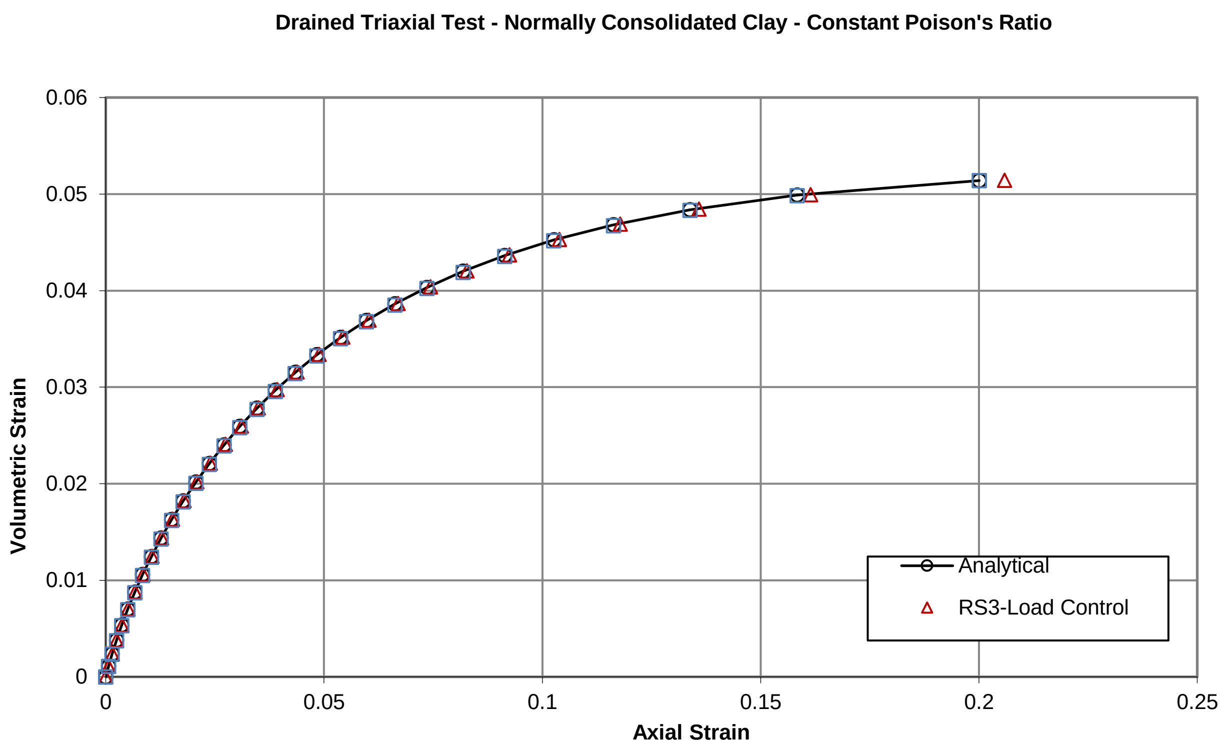

Figure 18-10: Variation of volumetric strain with axial strain for Example 1 case of constant Poisson's ratio

Table 18-4: Example 2, Triaxial test on a lightly over consolidated clay sample; results for case of constant shear modulus

	RS3 Load-Control			*RS3* Displacement-Control			Analytical Solution		
No.	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv
1	0	0	0	0.00	0	0	0	0	0
2	27.85	0.00062	0.00047	29.12	0.00065	0.00049	27.85	0.00062	0.00047
3	55.71	0.00123	0.00091	66.75	0.00147	0.00107	55.71	0.00123	0.00091
4	83.56	0.00183	0.00131	111.49	0.00247	0.00171	83.56	0.00183	0.00131
5	111.42	0.00242	0.00169	113.26	0.00366	0.00220	111.42	0.00242	0.00169
6	114.27	0.00435	0.00248	115.28	0.00505	0.00276	114.27	0.00434	0.00248
7	117.13	0.00636	0.00327	117.53	0.00664	0.00338	117.13	0.00634	0.00326
8	119.99	0.00845	0.00405	119.99	0.00844	0.00405	119.99	0.00842	0.00405
9	122.85	0.01061	0.00483	122.65	0.01045	0.00477	122.85	0.01057	0.00482
10	125.70	0.01287	0.00560	125.46	0.01266	0.00553	125.70	0.01281	0.00559
11	128.56	0.01521	0.00636	128.43	0.01509	0.00633	128.56	0.01514	0.00636
12	131.42	0.01766	0.00713	131.53	0.01774	0.00715	131.42	0.01758	0.00712
13	134.28	0.02021	0.00788	134.74	0.02062	0.00800	134.28	0.02011	0.00787
14	137.13	0.02287	0.00863	138.03	0.02373	0.00886	137.13	0.02276	0.00862
15	139.99	0.02567	0.00937	141.41	0.02710	0.00974	139.99	0.02554	0.00937
16	142.85	0.02860	0.01011	144.83	0.03071	0.01062	142.85	0.02845	0.01010
17	145.71	0.03168	0.01084	148.31	0.03462	0.01151	145.71	0.03152	0.01084
18	148.56	0.03492	0.01157	151.82	0.03884	0.01239	148.56	0.03474	0.01156
19	151.42	0.03836	0.01229	155.35	0.04341	0.01327	151.42	0.03815	0.01228
20	154.28	0.04199	0.01301	158.90	0.04837	0.01415	154.28	0.04176	0.01300
21	157.14	0.04587	0.01371	162.45	0.05378	0.01502	157.14	0.04561	0.01371
22	159.99	0.05000	0.01442	165.99	0.05971	0.01587	159.99	0.04971	0.01441
23	162.85	0.05444	0.01512	169.53	0.06627	0.01672	162.85	0.05411	0.01511
24	165.71	0.05923	0.01581	173.06	0.07360	0.01756	165.71	0.05886	0.01580
25	168.57	0.06443	0.01649	176.57	0.08188	0.01838	168.57	0.06401	0.01649
26	171.42	0.07012	0.01717	180.06	0.09141	0.01919	171.42	0.06965	0.01717
27	174.28	0.07641	0.01785	183.54	0.10263	0.01999	174.28	0.07586	0.01784
28	177.14	0.08343	0.01852	186.95	0.11630	0.02077	177.14	0.08280	0.01851
29	180.00	0.09139	0.01918	190.36	0.13383	0.02153	180.00	0.09064	0.01917
30	182.85	0.10057	0.01984	193.75	0.15836	0.02230	182.85	0.09968	0.01983
31	185.71	0.11144	0.02049	197.35	0.20007	0.02308	185.71	0.11034	0.02049
32	188.57	0.12476	0.02114	197.28	0.20007	0.02308	188.57	0.12336	0.02113
33	191.43	0.14197	0.02178	197.28	0.20007	0.02308	191.43	0.14010	0.02177
34	194.26	0.16717	0.02241	197.28	0.20007	0.02308	194.28	0.16363	0.02241
35	197.10	0.20863	0.02304	197.28	0.20007	0.02308	197.14	0.20374	0.02304

	RS3 Load-Control			*RS3* Displacement-Control			Analytical Solution		
No.	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv
1	0	0	0	0.00	0	0	0	0	0
2	27.85	0.00062	0.00047	29.12	0.00065	0.00049	27.85	0.00062	0.00047

3	55.71	0.00123	0.00091	66.75	0.00147	0.00107	55.71	0.00123	0.00091
4	83.56	0.00183	0.00131	111.49	0.00247	0.00171	83.56	0.00183	0.00131
5	111.42	0.00242	0.00169	113.26	0.00366	0.00220	111.42	0.00242	0.00169
6	114.27	0.00435	0.00248	115.28	0.00505	0.00276	114.27	0.00434	0.00248
7	117.13	0.00636	0.00327	117.53	0.00664	0.00338	117.13	0.00634	0.00326
8	119.99	0.00845	0.00405	119.99	0.00844	0.00405	119.99	0.00842	0.00405
9	122.85	0.01061	0.00483	122.65	0.01045	0.00477	122.85	0.01057	0.00482
10	125.70	0.01287	0.00560	125.46	0.01266	0.00553	125.70	0.01281	0.00559
11	128.56	0.01521	0.00636	128.43	0.01509	0.00633	128.56	0.01514	0.00636
12	131.42	0.01766	0.00713	131.53	0.01774	0.00715	131.42	0.01758	0.00712
13	134.28	0.02021	0.00788	134.74	0.02062	0.00800	134.28	0.02011	0.00787
14	137.13	0.02287	0.00863	138.03	0.02373	0.00886	137.13	0.02276	0.00862
15	139.99	0.02567	0.00937	141.41	0.02710	0.00974	139.99	0.02554	0.00937
16	142.85	0.02860	0.01011	144.83	0.03071	0.01062	142.85	0.02845	0.01010
17	145.71	0.03168	0.01084	148.31	0.03462	0.01151	145.71	0.03152	0.01084
18	148.56	0.03492	0.01157	151.82	0.03884	0.01239	148.56	0.03474	0.01156
19	151.42	0.03836	0.01229	155.35	0.04341	0.01327	151.42	0.03815	0.01228
20	154.28	0.04199	0.01301	158.90	0.04837	0.01415	154.28	0.04176	0.01300
21	157.14	0.04587	0.01371	162.45	0.05378	0.01502	157.14	0.04561	0.01371
22	159.99	0.05000	0.01442	165.99	0.05971	0.01587	159.99	0.04971	0.01441
23	162.85	0.05444	0.01512	169.53	0.06627	0.01672	162.85	0.05411	0.01511
24	165.71	0.05923	0.01581	173.06	0.07360	0.01756	165.71	0.05886	0.01580
25	168.57	0.06443	0.01649	176.57	0.08188	0.01838	168.57	0.06401	0.01649
26	171.42	0.07012	0.01717	180.06	0.09141	0.01919	171.42	0.06965	0.01717
27	174.28	0.07641	0.01785	183.54	0.10263	0.01999	174.28	0.07586	0.01784
28	177.14	0.08343	0.01852	186.95	0.11630	0.02077	177.14	0.08280	0.01851
29	180.00	0.09139	0.01918	190.36	0.13383	0.02153	180.00	0.09064	0.01917
30	182.85	0.10057	0.01984	193.75	0.15836	0.02230	182.85	0.09968	0.01983
31	185.71	0.11144	0.02049	197.35	0.20007	0.02308	185.71	0.11034	0.02049

32	188.57	0.12476	0.02114	197.28	0.20007	0.02308	188.57	0.12336	0.02113
33	191.43	0.14197	0.02178	197.28	0.20007	0.02308	191.43	0.14010	0.02177
34	194.26	0.16717	0.02241	197.28	0.20007	0.02308	194.28	0.16363	0.02241
35	197.10	0.20863	0.02304	197.28	0.20007	0.02308	197.14	0.20374	0.02304

Table 18-5: Example 2, Triaxial test on a lightly over consolidated clay sample; results for case of constant Poisson's ratio

	RS3 Load-Control			*RS3* Displacement-Control			Analytical Solution		
No.	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**
1	0	0	0	0.00	0	0	0	0	0
2	27.86	0.00118	0.00047	28.24	0.00120	0.00048	27.85	0.00118	0.00047
3	55.71	0.00227	0.00091	56.44	0.00230	0.00092	55.71	0.00227	0.00091
4	83.56	0.00328	0.00131	84.14	0.00330	0.00132	83.56	0.00328	0.00131
5	111.42	0.00422	0.00169	110.87	0.00420	0.00168	111.42	0.00422	0.00169
6	114.27	0.00619	0.00248	114.29	0.00620	0.00249	114.27	0.00617	0.00248
7	117.13	0.00823	0.00327	117.10	0.00820	0.00326	117.13	0.00820	0.00326
8	119.99	0.01034	0.00405	119.94	0.01030	0.00403	119.99	0.01031	0.00405
9	122.85	0.01254	0.00483	122.80	0.01250	0.00481	122.85	0.01250	0.00482
10	125.70	0.01482	0.00560	125.68	0.01480	0.00559	125.70	0.01477	0.00559
11	128.56	0.01720	0.00636	128.45	0.01710	0.00634	128.56	0.01713	0.00636
12	131.42	0.01967	0.00713	131.34	0.01960	0.00711	131.42	0.01959	0.00712
13	134.28	0.02225	0.00788	134.23	0.02220	0.00787	134.28	0.02216	0.00787
14	137.13	0.02495	0.00863	136.99	0.02480	0.00859	137.13	0.02484	0.00862
15	139.99	0.02777	0.00937	139.83	0.02760	0.00933	139.99	0.02764	0.00937
16	142.85	0.03073	0.01011	142.74	0.03060	0.01008	142.85	0.03058	0.01010
17	145.71	0.03384	0.01084	145.59	0.03370	0.01081	145.71	0.03367	0.01084
18	148.56	0.03711	0.01157	148.40	0.03690	0.01153	148.56	0.03693	0.01156
19	151.42	0.04057	0.01229	151.30	0.04040	0.01226	151.42	0.04036	0.01228
20	154.28	0.04424	0.01301	154.12	0.04400	0.01296	154.28	0.04400	0.01300
21	157.14	0.04813	0.01371	156.99	0.04790	0.01368	157.14	0.04787	0.01371
22	159.99	0.05229	0.01442	159.83	0.05200	0.01437	159.99	0.05200	0.01441
23	162.85	0.05676	0.01512	162.65	0.05640	0.01507	162.85	0.05643	0.01511
24	165.71	0.06157	0.01581	165.52	0.06120	0.01576	165.71	0.06120	0.01580
25	168.57	0.06680	0.01649	168.39	0.06640	0.01645	168.57	0.06638	0.01649
26	171.42	0.07251	0.01717	171.21	0.07200	0.01712	171.42	0.07204	0.01717
27	174.28	0.07882	0.01785	174.09	0.07830	0.01780	174.28	0.07828	0.01784
28	177.14	0.08587	0.01852	176.93	0.08520	0.01847	177.14	0.08524	0.01851
29	180.00	0.09385	0.01918	179.80	0.09310	0.01914	180.00	0.09310	0.01917
30	182.85	0.10305	0.01984	182.68	0.10220	0.01980	182.85	0.10216	0.01983
31	185.71	0.11394	0.02049	185.52	0.11280	0.02045	185.71	0.11285	0.02049
32	188.57	0.12729	0.02114	188.43	0.12590	0.02110	188.57	0.12589	0.02113
33	191.43	0.14454	0.02178	191.36	0.14260	0.02176	191.43	0.14265	0.02177
34	194.28	0.16907	0.02242	194.31	0.16620	0.02241	194.28	0.16620	0.02241
35	197.11	0.21116	0.02304	197.33	0.20630	0.02308	197.14	0.20632	0.02304

	RS3 Load-Control			*RS3* Displacement-Control			Analytical Solution		
No.	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**	**q (kPa)**	**Axial strain, εa**	**Volumetric strain, εv**
1	0	0	0	0.00	0	0	0	0	0
2	27.86	0.00118	0.00047	28.24	0.00120	0.00048	27.85	0.00118	0.00047

3	55.71	0.00227	0.00091	56.44	0.00230	0.00092	55.71	0.00227	0.00091
4	83.56	0.00328	0.00131	84.14	0.00330	0.00132	83.56	0.00328	0.00131
5	111.42	0.00422	0.00169	110.87	0.00420	0.00168	111.42	0.00422	0.00169
6	114.27	0.00619	0.00248	114.29	0.00620	0.00249	114.27	0.00617	0.00248
7	117.13	0.00823	0.00327	117.10	0.00820	0.00326	117.13	0.00820	0.00326
8	119.99	0.01034	0.00405	119.94	0.01030	0.00403	119.99	0.01031	0.00405
9	122.85	0.01254	0.00483	122.80	0.01250	0.00481	122.85	0.01250	0.00482
10	125.70	0.01482	0.00560	125.68	0.01480	0.00559	125.70	0.01477	0.00559
11	128.56	0.01720	0.00636	128.45	0.01710	0.00634	128.56	0.01713	0.00636
12	131.42	0.01967	0.00713	131.34	0.01960	0.00711	131.42	0.01959	0.00712
13	134.28	0.02225	0.00788	134.23	0.02220	0.00787	134.28	0.02216	0.00787
14	137.13	0.02495	0.00863	136.99	0.02480	0.00859	137.13	0.02484	0.00862
15	139.99	0.02777	0.00937	139.83	0.02760	0.00933	139.99	0.02764	0.00937
16	142.85	0.03073	0.01011	142.74	0.03060	0.01008	142.85	0.03058	0.01010
17	145.71	0.03384	0.01084	145.59	0.03370	0.01081	145.71	0.03367	0.01084
18	148.56	0.03711	0.01157	148.40	0.03690	0.01153	148.56	0.03693	0.01156
19	151.42	0.04057	0.01229	151.30	0.04040	0.01226	151.42	0.04036	0.01228
20	154.28	0.04424	0.01301	154.12	0.04400	0.01296	154.28	0.04400	0.01300
21	157.14	0.04813	0.01371	156.99	0.04790	0.01368	157.14	0.04787	0.01371
22	159.99	0.05229	0.01442	159.83	0.05200	0.01437	159.99	0.05200	0.01441
23	162.85	0.05676	0.01512	162.65	0.05640	0.01507	162.85	0.05643	0.01511
24	165.71	0.06157	0.01581	165.52	0.06120	0.01576	165.71	0.06120	0.01580
25	168.57	0.06680	0.01649	168.39	0.06640	0.01645	168.57	0.06638	0.01649
26	171.42	0.07251	0.01717	171.21	0.07200	0.01712	171.42	0.07204	0.01717
27	174.28	0.07882	0.01785	174.09	0.07830	0.01780	174.28	0.07828	0.01784
28	177.14	0.08587	0.01852	176.93	0.08520	0.01847	177.14	0.08524	0.01851
29	180.00	0.09385	0.01918	179.80	0.09310	0.01914	180.00	0.09310	0.01917
30	182.85	0.10305	0.01984	182.68	0.10220	0.01980	182.85	0.10216	0.01983
31	185.71	0.11394	0.02049	185.52	0.11280	0.02045	185.71	0.11285	0.02049
32	188.57	0.12729	0.02114	188.43	0.12590	0.02110	188.57	0.12589	0.02113
33	191.43	0.14454	0.02178	191.36	0.14260	0.02176	191.43	0.14265	0.02177
34	194.28	0.16907	0.02242	194.31	0.16620	0.02241	194.28	0.16620	0.02241
35	197.11	0.21116	0.02304	197.33	0.20630	0.02308	197.14	0.20632	0.02304

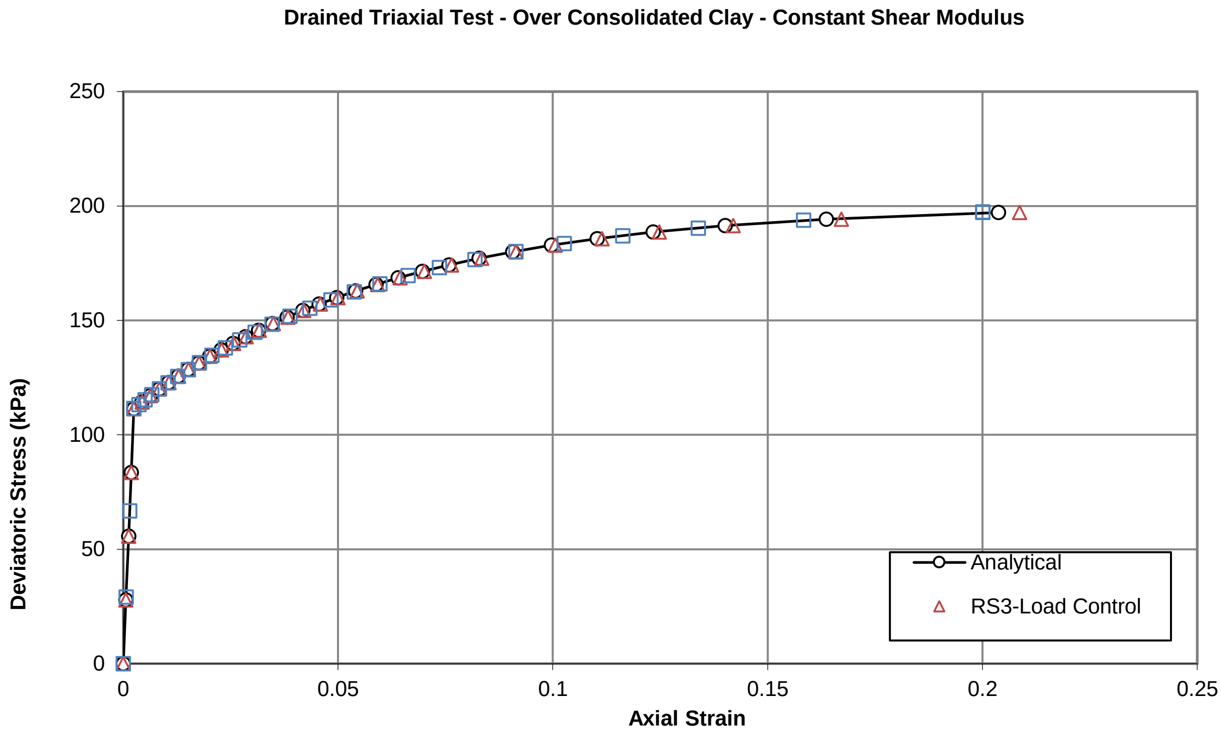

Figure 18-11: Variation of deviatoric stress with axial strain for Example 2 case of constant shear modulus

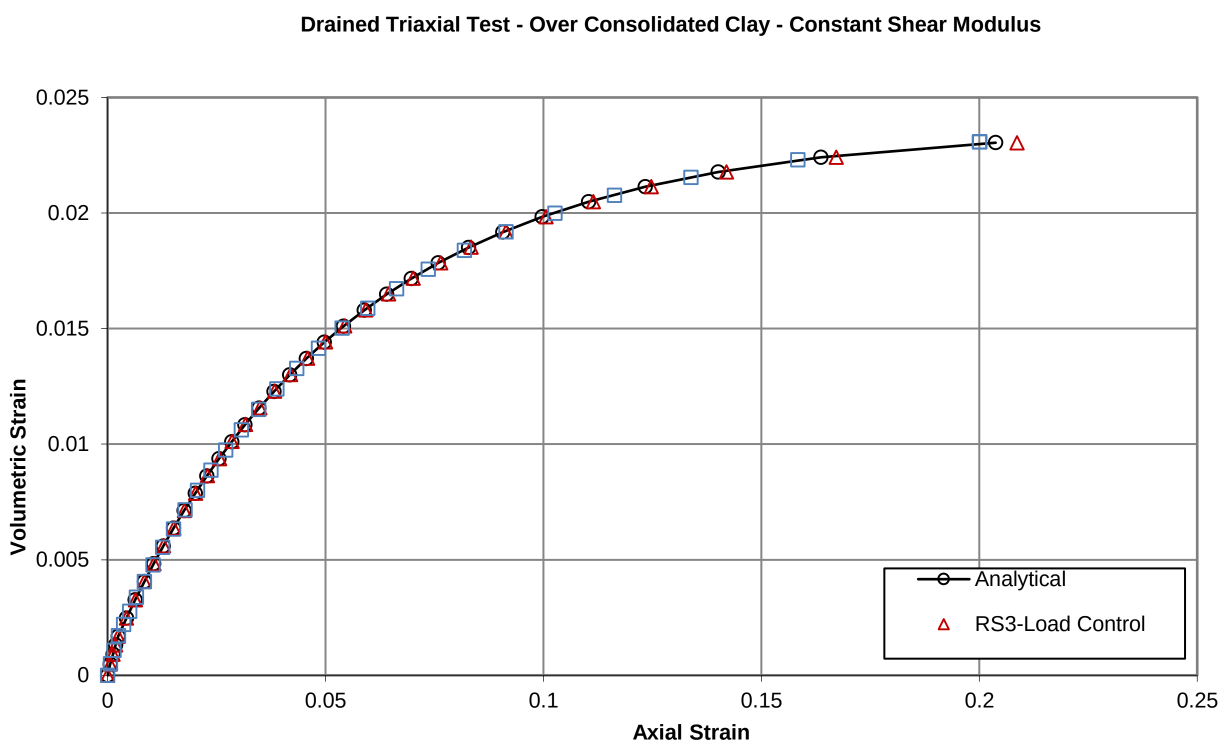

Figure 18-12: Variation of volumetric strain with axial strain for Example 2 case of constant shear modulus

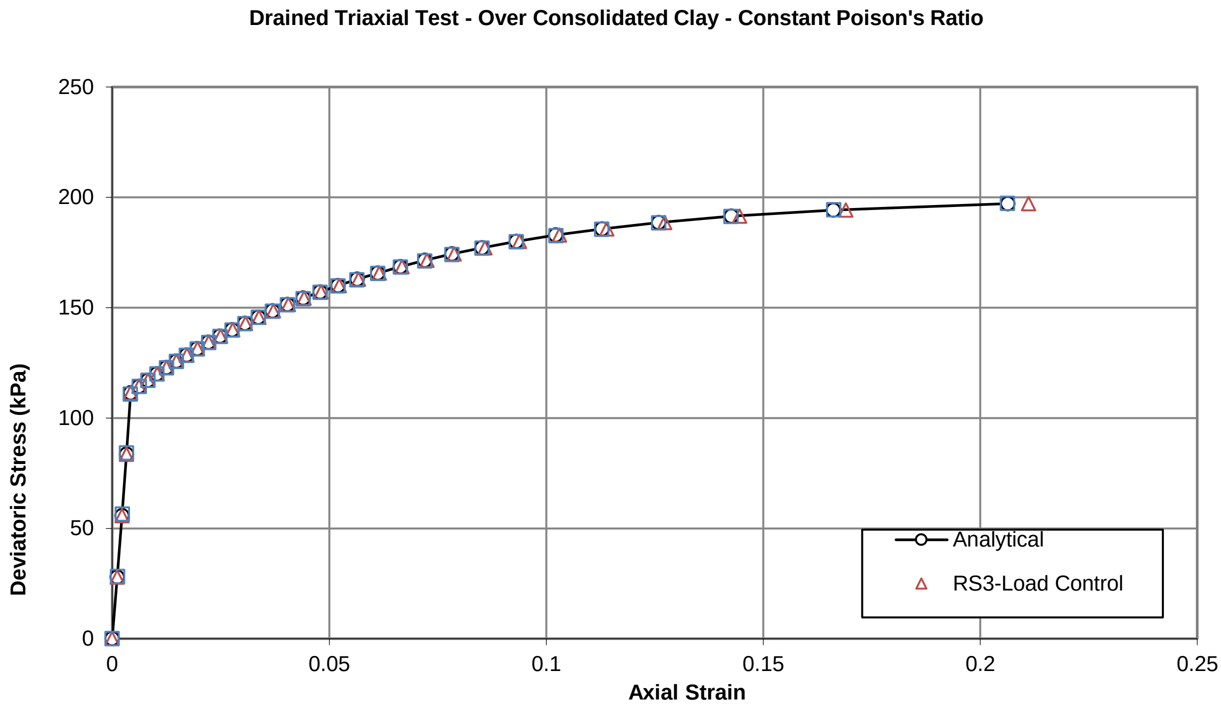

Figure 18-13: Variation of deviatoric stress with axial strain for Example 2 case of constant Poisson's ratio

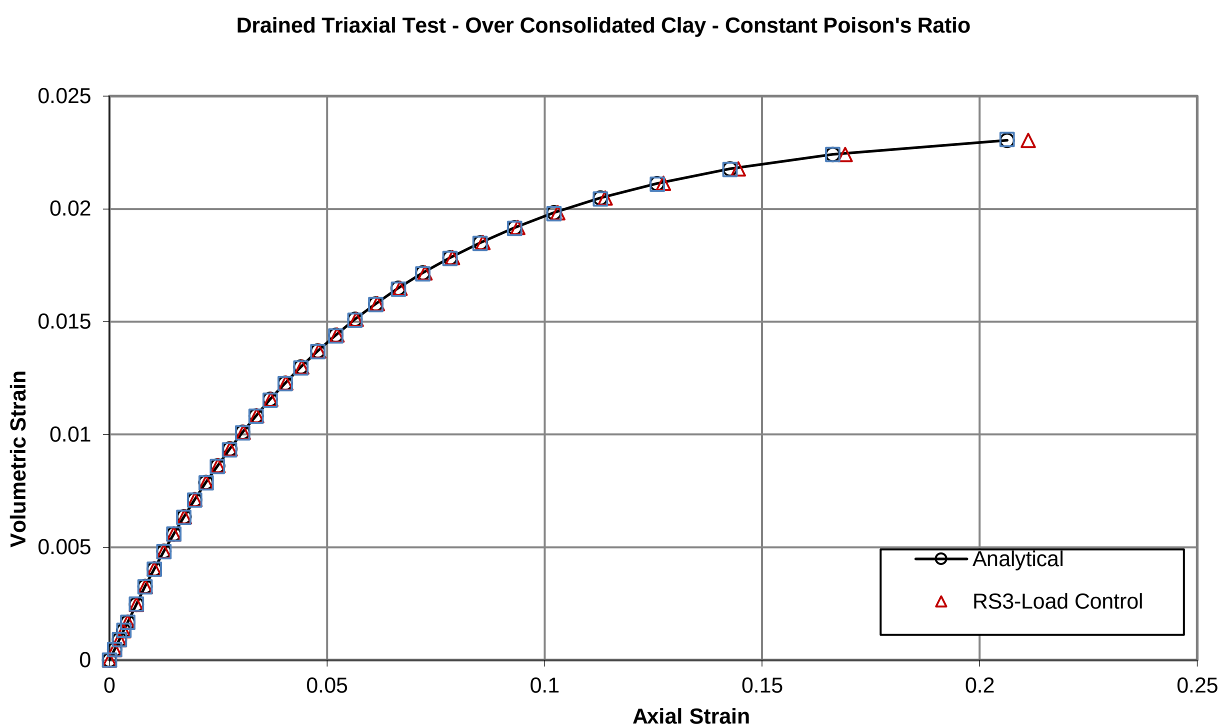

Figure 18-14: Variation of volumetric strain with axial strain for Example 2 case of constant Poisson's ratio

Table 18-6: Example 2, Triaxial test on a highly over consolidated clay sample; results for case of constant Poisson's ratio

	RS3 Displacement-Control			Analytical Solution		
No.	q (kPa)	Axial strain, εa	Volumetric strain, εv	q (kPa)	Axial strain, εa	Volumetric strain, εv
1	0.00	0.000000	0.000000	0.00	0.000000	0.000000
2	72.51	0.003000	0.001200	73.35	0.003029	0.001211
3	145.95	0.005500	0.002200	146.69	0.005518	0.002207
4	218.52	0.007600	0.003040	220.04	0.007634	0.003054
5	293.36	0.009500	0.003787	293.39	0.009474	0.003790
6	290.49	0.011100	0.003301	290.37	0.011149	0.003280
7	287.38	0.012900	0.002770	287.36	0.012888	0.002765
8	284.39	0.014700	0.002255	284.35	0.014695	0.002246
9	281.36	0.016600	0.001728	281.34	0.016576	0.001722
10	278.45	0.018500	0.001219	278.32	0.018536	0.001195
11	275.36	0.020600	0.000675	275.31	0.020581	0.000663
12	272.41	0.022700	0.000151	272.30	0.022719	0.000127
13	269.33	0.025000	-0.000401	269.29	0.024957	-0.000413
14	266.39	0.027300	-0.000933	266.27	0.027304	-0.000957
15	263.35	0.029800	-0.001487	263.26	0.029771	-0.001506
16	260.34	0.032400	-0.002038	260.25	0.032369	-0.002060
17	257.39	0.035100	-0.002585	257.24	0.035111	-0.002618
18	254.39	0.038000	-0.003145	254.22	0.038014	-0.003181
19	251.37	0.041100	-0.003713	251.21	0.041095	-0.003748
20	248.35	0.044400	-0.004285	248.20	0.044377	-0.004320
21	245.36	0.047900	-0.004857	245.19	0.047885	-0.004897
22	242.34	0.051700	-0.005441	242.17	0.051650	-0.005479
23	239.40	0.055700	-0.006015	239.16	0.055711	-0.006065
24	236.40	0.060100	-0.006603	236.15	0.060115	-0.006657
25	233.41	0.064900	-0.007196	233.14	0.064921	-0.007253
26	230.40	0.070200	-0.007796	230.12	0.070206	-0.007855
27	227.39	0.076100	-0.008402	227.11	0.076072	-0.008462
28	224.39	0.082700	-0.009011	224.10	0.082655	-0.009074
29	221.44	0.090100	-0.009616	221.09	0.090148	-0.009691
30	218.45	0.098800	-0.010234	218.07	0.098830	-0.010314
31	215.49	0.109100	-0.010853	215.06	0.109138	-0.010942
32	212.49	0.121800	-0.011479	212.05	0.121802	-0.011576
33	209.53	0.138200	-0.012108	209.04	0.138189	-0.012215
34	206.64	0.161400	-0.012732	206.02	0.161369	-0.012859
35	203.85	0.201100	-0.013328	203.01	0.201143	-0.013510

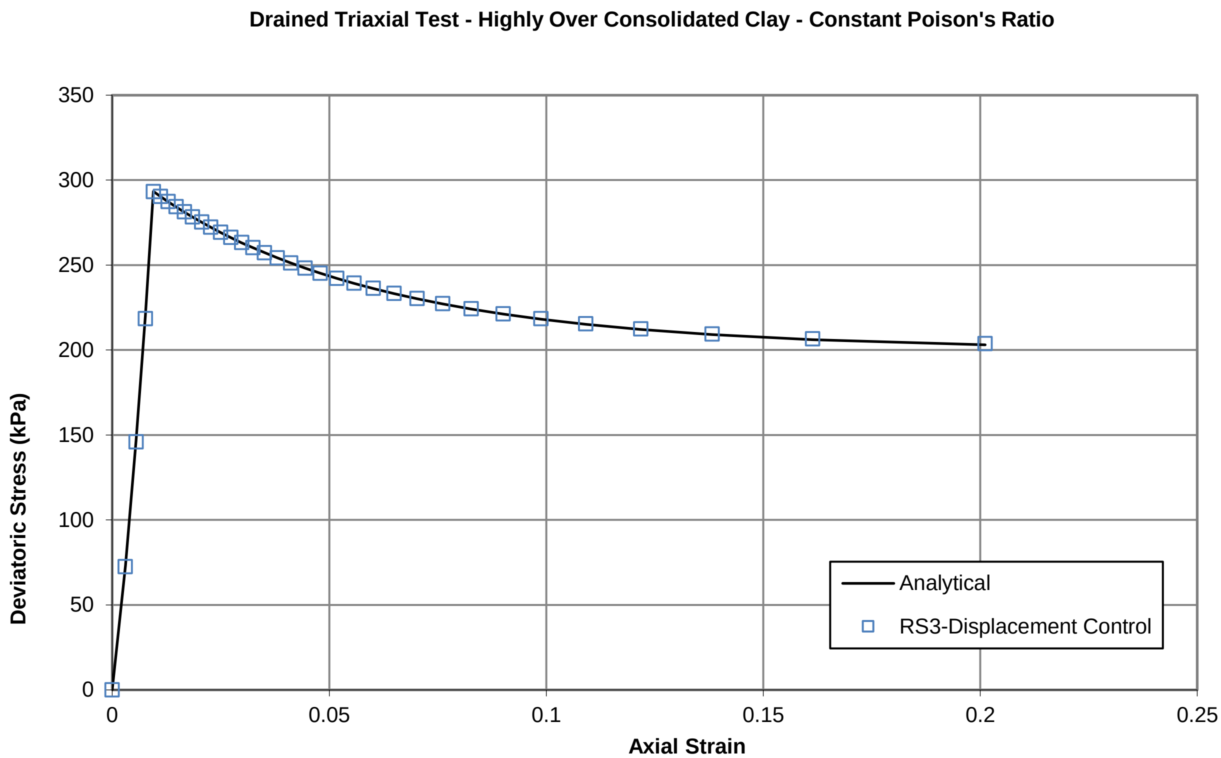

Figure 18-15: Variation of deviatoric stress with axial strain for Example 3 case of constant Poisson's ratio

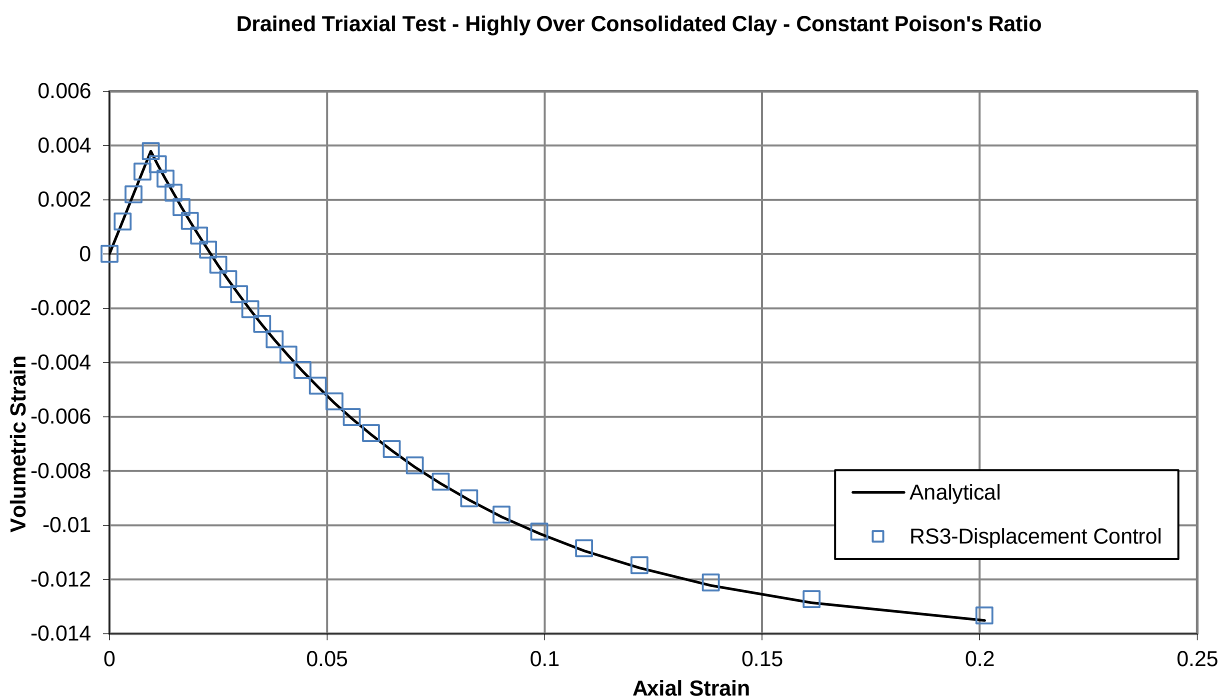

Figure 18-16: Variation of volumetric strain with axial strain for Example 3 case of constant Poisson's ratio

18.5 References

1. R.I. Borja (1991), Cam-Clay plasticity, Part II: Implicit integration of constitutive equation based on a nonlinear elastic stress predictor, Computer Methods in Applied Mechanics and Engineering, 88, 225-240.
2. D. Peric´ (2006), Analytical solutions for a three-invariant Cam clay model subjected to drained loading histories, Int. J. Numer. Anal. Meth. Geomech., 30, 363–387.

18.6 Data Files

The input data files for the drained triaxial compressive testing of Modified Cam Clay samples are:

File name	*Example No.*	*Assumption*	*Simulation type*	*Consolidation*
V018 cnstG NC load.rs3model	1	$G = const.$	Load Control	Normal
V018 cnstv NC load.rs3model	1	$\mu = const.$	Load Control	Normal
V018 cnstG NC displ.rs3model	1	$G = const.$	Displacement Control	Normal
V018 cnstv NC disp.rs3model	1	$\mu = const.$	Displacement Control	Normal
V018 cnstG OC load.rs3model	2	$G = const.$	Load Control	Over
V018 cnstv OC load.rs3model	2	$\mu = const.$	Load Control	Over
V018 cnstG OC disp.rs3model	2	$G = const.$	Displacement Control	Over
V018 cnstv OC disp.rs3model	2	$\mu = const.$	Displacement Control	Over
V018 cnstv HighOC disp.rs3model	3	$\mu = const.$	Displacement Control	Highly Over

These can be found in the *RS3* installation folder. Also included in the installation folder are Microsoft Excel spreadsheet files:

V018 (constant G) – NC Clay.xls
V018 (constant v) – NC Clay.xls
V018 (constant G) – OC Clay.xls
V018 (constant v) – OC Clay.xls
V018 - (constant v) – Highly OC.xls

that implement the closed-form solutions for drained triaxial compressive testing for Modified Cam Clay soils.