Bolt support systems in *Phase*^{2}
Rocscience Inc, 2001

1. Introduction

Rock bolts are the most commonly used measure for supporting excavations in rock masses. Bolt models are used in various numerical methods such as the Finite Element Method (FEM) (Goodman et al., 1968), the Boundary Element Method (BEM) (Crotty & Wardle, 1985) and the Discrete Element Method (Cundall, 1971).

Phase^2 is a FEM that is used in geotechnical and mining engineering as a tool for designing and the analyzing tunnels, surface excavations and mining excavations (*Phase*^{2}, 1999). Rock support systems are used in civil and mining situations to prevent unraveling of the rock mass in the immediate vicinity of excavations.

This document outlines the four bolt support theories implemented in *Phase*^{2}. Rock bolts in *Phase*^{2} pass through the elements in a mesh and are modeled as one-dimensional elements.

2. Modelling of rock bolts

Rock bolts in *Phase*[2] are divided into four types. End-anchored and fully bonded rock bolts are based on axial deformation. Plain strand considers more complicated model developed in Queen's University, Canada. Swellex bolts considers the shear resistance of the relative movements between bolts and the rock mass.

2.1 End-anchored rock bolt

The end-anchored rock bolt is represented by a one-dimensional deformable element (Figure 1).

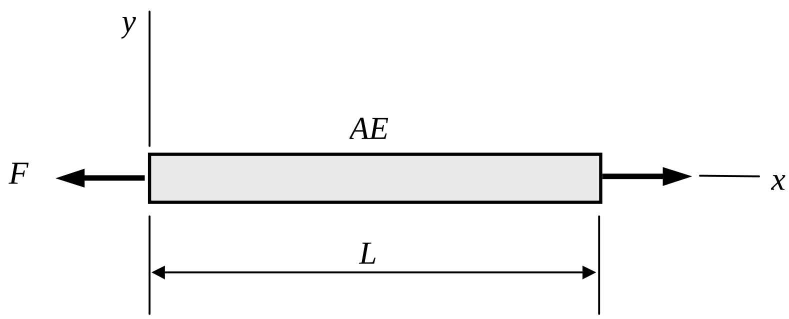

Figure 1. End-anchored bolt model

Each bolt behaves as a single element. Interaction with the finite element mesh is through the endpoints only. The axial force, *F* is calculated from the axial displacement by

$$F = K_b \Delta u \qquad (1)$$

where K_b is the bolt stiffness which equals to EA/L , Δu denotes the relative displacement between the two anchorage points which is $\Delta u = u_1 - u_2$. Failure of an end-anchored rock bolt occurs due to tensile yielding of the bolt material. Therefore, bolt failure is controlled by the yield strength (F_{yield}). An end-anchored bolt has no residual capacity after failure, the entire bolt is considered to have failed.

2.2 Fully-bonded rock bolt

Fully bonded bolts are divided into 'bolt elements' according to where the bolts cross the finite elements. These bolt elements act independently of each other. Bolt elements do not influence each other directly, but only indirectly through their effect on the rock mass.

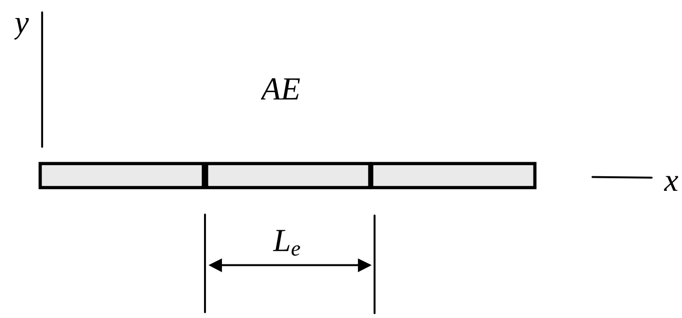

Figure 2. Fully bonded bolt model

The axial force along the bolt determined from the elongation of the bolt element. If the length of a bolt element L_e, is increased by Δu_e then the induced force in the bolt

$$F_e = \frac{AE}{L_e}\Delta u_e \tag{2}$$

If the axial force exceeds the yield strength (F_{yield}) of the bolt material then the bolt force is set to F_{res} (Figure 3)

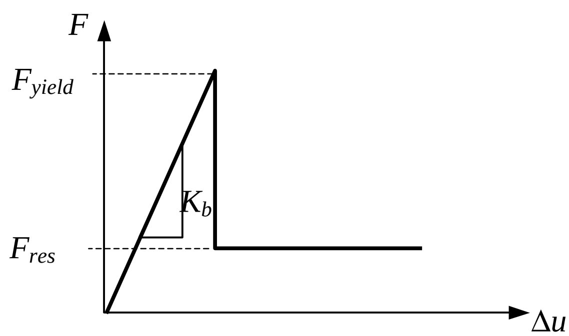

Figure 3. Rock bolt failure criteria

2.3 Plain-strand cable bolt

The Plain-strand cable bolt model is developed at Queen's University in Kingston, Canada. The entire bolt behaves as a single element. Although for purposes of the algorithm, the bolt is discretized according to the intersections with the finite elements, the behaviour of each segment of the bolt has a direct effect on adjacent segments. This is in contrast to the fully bonded cable model, where bolt elements on the same bolt act independently of each other. The stiffness of the grout, and the strength and stiffness of the bolt/grout interface is taken into account. The failure mechanism of the bolt is by tensile rupture of the cable.

Failure of the cable/grout interface also occurs, but it is not a failure mechanism as such, since this interface is always assumed to be in a plastic state as the rock moves. The amount of relative slip at this interface, and the stiffness of the interface, determines how much shear force is generated at the cable.

For information about the development of this model, see the following references (Moosavi, 1997, Moosavi et al. 1996, Hyett et. al. 1996 Hyett et. al. 1995).

2.4 Shear bolts (Swellex/Split sets) – new in version 5.0

The equilibrium equation of a fully grouted rock bolt, Figure 1, may be written as (Farmer, 1975 and Hyett et al., 1996)

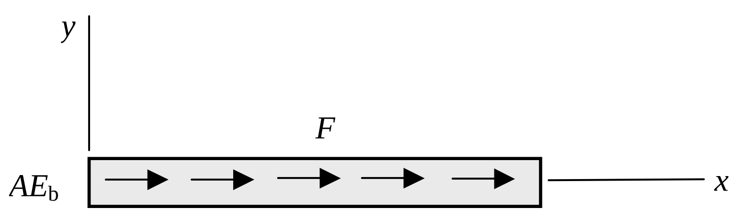

Figure 4 Shear bolt model

$$AE_b \frac{d^2 u_x}{dx^2} + F_s \quad = \quad 0 \tag{3}$$

where F_s is the shear force per unit length and A is the cross-sectional area of the bolt and E_b is the modulus of elasticity for the bolt. The shear force is assumed to be linear function of the relative movement between the rock and the bolt and presented as

$$F_s = k(u_r - u_x) \tag{4}$$

Usually, k is the shear stiffness of the bolt-grout interface measured directly in laboratory pull-out tests. Substitute equation (4) in (3), then the weak form can be expressed as:

$$\delta\Pi = \int (AE_b \frac{d^2u_x}{dx^2} - ku_x + ku_r)\,\delta u\,dx \tag{5}$$

$$\begin{aligned} &= \int \left\{ AE_b \left[\frac{d}{dx}\left(\frac{du_x}{dx}\delta u\right) - \frac{du_x}{dx}\frac{d\delta u}{dx} \right] - (ku_x - ku_r)\delta u \right\} dx \\ &= AE_b \delta u \frac{du_x}{dx}\Big|_0^L \quad - \int \left(AE_b \frac{du_x}{dx}\frac{d\delta u}{dx} + ku_x \delta u \right) dx \quad + \int (ku_r \delta u) dx \end{aligned} \tag{6}$$

Let is consider the generic element, Figure 5. The displacements u are to be linear in axial coordinate s (Cook, 1981). The displacement field equals to u_1 at one end and u_2 at the other. Then, the displacement at any point along the element can be given as

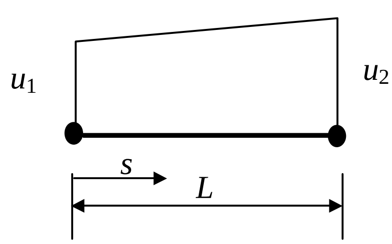

Figure 5 Linear displacement variation

$$u = \frac{L-s}{L}u_1 + \frac{s}{L}u_2 \qquad \text{or} \quad u = \lfloor N \rfloor \{d\} \tag{7}$$

where $\lfloor N \rfloor = \left\lfloor \frac{L-s}{L} \quad \frac{s}{L} \right\rfloor$ and $\{d\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

for the two displacement fields, equation 7 can be written as

$$u=\begin{Bmatrix}u_x\\u_r\end{Bmatrix}=\begin{bmatrix}N_1 & N_2 & 0 & 0\\0 & 0 & N_1 & N_2\end{bmatrix}\begin{Bmatrix}u_{x1}\\u_{x2}\\u_{r1}\\u_{r2}\end{Bmatrix} \tag{8}$$

Then equation (4) can be written as

$$-\int\left(AE_b\frac{du_x}{dx}\frac{d\delta u}{dx}+ku_x\delta u\right)dx+\int(ku_r\delta u)dx=-\quad\lfloor u_{x1}\quad u_{x2}\quad u_{r1}\quad u_{r2}\rfloor\begin{bmatrix}K_b & 0\\0 & -K_r\end{bmatrix}\delta\begin{Bmatrix}u_{x1}\\u_{x2}\\u_{r1}\\u_{r2}\end{Bmatrix} \tag{9}$$

and let us introduce the notation $\lfloor B\rfloor=\lfloor N_{,x}\rfloor$

then $u_{,x}=\dfrac{du}{dx}=\lfloor B\rfloor\{d\}=\left\lfloor-\dfrac{1}{L}\quad\dfrac{1}{L}\right\rfloor\begin{Bmatrix}u_1\\u_2\end{Bmatrix}$ (10)

Hence,

$$[K_b]=\int_0^L\left\{AE_b\begin{bmatrix}N_{1,x}N_{1,x} & N_{1,x}N_{2,x}\\N_{2,x}N_{1,x} & N_{2,x}N_{2,x}\end{bmatrix}+k\begin{bmatrix}N_1N_1 & N_1N_2\\N_2N_1 & N_2N_2\end{bmatrix}\right\}dx \tag{11}$$

$$[K_b]=\frac{AE_b}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}+k\int_0^L\begin{bmatrix}\left(1-\frac{x}{L}\right)^2 & \left(1-\frac{x}{L}\right)\frac{x}{L}\\\left(1-\frac{x}{L}\right)\frac{x}{L} & \left(\frac{x}{L}\right)^2\end{bmatrix}dx \tag{12}$$

$$[K_b]=\frac{AE_b}{L}\begin{bmatrix}1 & -1\\-1 & 1\end{bmatrix}+\frac{kL}{3}\begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix} \tag{13}$$

and

$$[K_r]=k\begin{bmatrix}N_1N_1 & N_1N_2\\N_2N_1 & N_2N_2\end{bmatrix}=\frac{kL}{3}\begin{bmatrix}1 & 0.5\\0.5 & 1\end{bmatrix} \tag{14}$$

Equations (13) and (14) are used to assemble the stiffness for bolts. *Phase*^{2} uses bolts that are not necessarily connected to the element vertices, mapping procedure carried out to transfer the effect to the element vertices. This procedure is done for each bolt segment by mapping the stiffness by the shape function depends on the intersected side of the elements.

3. Numerical bolt implementation in *Phase*^{2}

Bolts in *Phase*^{2} are modeled as single elements. In *Phase*^{2} programs, a bolt is discretized according to the manner in which it intersects with finite elements in a mesh. For plain cable bolts and shear bolts, the behaviour of each segment of a bolt has a direct influence on adjacent segments. For fully bonded bolts, a segment indirectly influences adjacent segments through its effect on the rock mass.

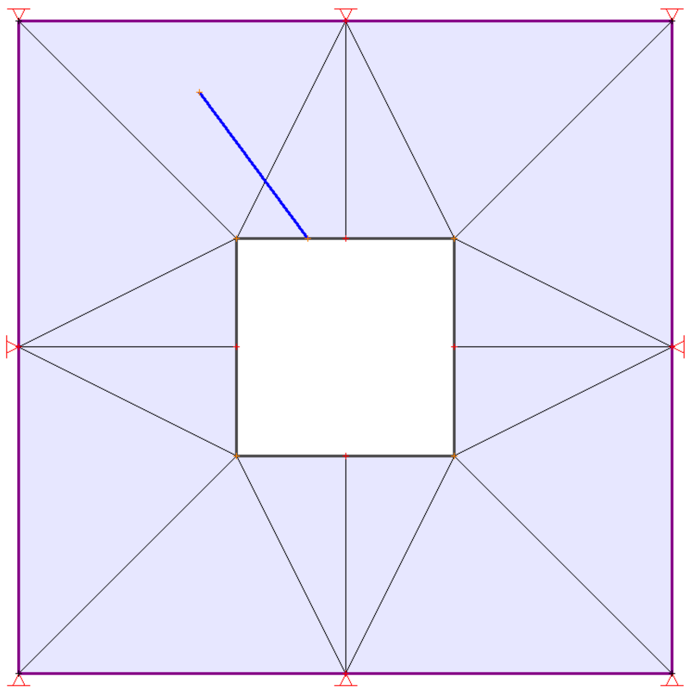

Figure 6 *Phase*^{2} model

In general, there is a common procedure for handling bolt models in the compute engine of *Phase*^{2}. This procedures is outlined as follows:

- Each bolt is defined by a start and an end point (Figure 6).
- A stiffness is calculated for a bolt depending on its model. The bolt stiffness is then transferred to element nodes according to the following approach
  - Transform the global coordinates of each bolt into the local coordinate systems of elements.

- Map the shape function of the transformed coordinate points as follows
    - Find the shape function for each end point based on its local coordinate system.
    - Define a shape matrix that stores the shape function of the bolt with respect to the elements in the mesh (as shown for elements 14 and 17 in Figure 7).
- Transfer the bolt stiffness of each bolt segment to new stiffness based on mapping the bolts' stiffness values to the nodes of the intersecting elements (as shown in nodes 8,9 16 and 17 in Figure 7).

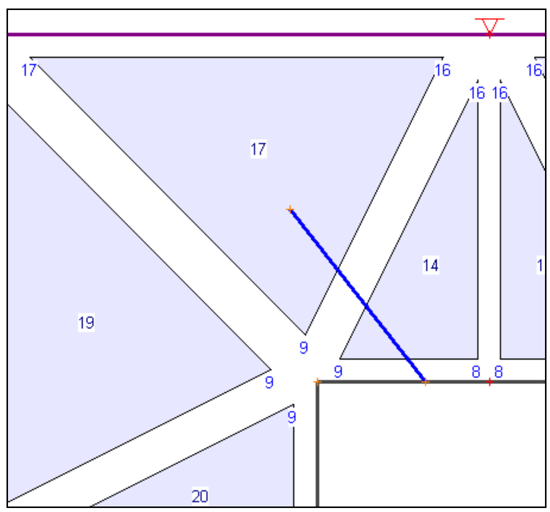

Figure 7. Enlarged view of the bolt region

Based on the outlined procedure, the global stiffness system is assembled. In the case of yielded bolts, an iterative process is used to update bolt stresses.

3. Reference

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