rocscience

SWedge

Safety Factor Calculations – Pentahedral Wedges

Theory Manual

Table of Contents

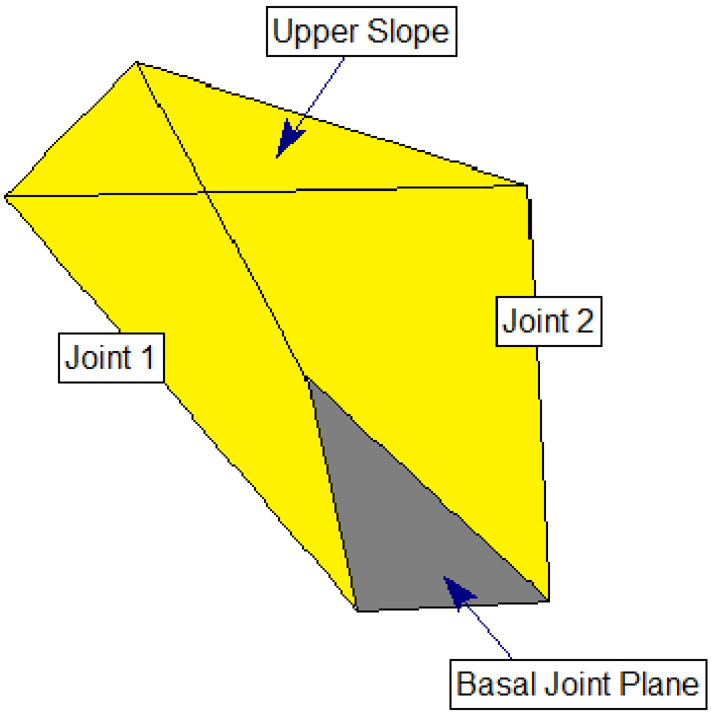
Introduction

This paper documents the calculations used in *SWedge* to determine the safety factor of **pentahedral** wedges formed in rock slopes. This involves the following series of steps:

1. Determine the wedge geometry using block theory (Goodman and Shi, 1985)
2. Determine all of the individual forces acting on a wedge, and then calculate the resultant active and passive force vectors for the wedge
3. Determine the sliding direction of the wedge
4. Determine the normal forces on each wedge plane
5. Compute the resisting forces due to joint shear strength
6. Calculate the safety factor

1. Wedge Geometry – Basal Joint

The orientations of 2 distinct joint planes **and one basal joint** must always be defined for an *SWedge "basal joint"* analysis. To get accurate results, the basal joint must be the one which is closer to the orientation of the ground. Using block theory, *SWedge* determines if a finite wedge can be formed by the intersection of the two joint planes, the basal joint, the slope and the upper face. An optional tension crack can also be included in the analysis.



The method used for determining the wedges is described in the text by Goodman and Shi, "Block Theory and Its Application to Rock Engineering" (1985).

After it is determined that a block can form, all the appropriate spatial orientations of the joint planes are computed to form possible wedges. In general, up to two finite blocks can be formed. The blocks are checked for removability from the slope or both the slope and upper face. Unless the joint planes are parallel, one of the two blocks are eliminated through this process. If, at the end of the block formation process, two valid wedges remain, the wedge with the greatest volume is selected.

The wedges that are formed are pentahedral (5-sided), although tetrahedral (4-sided) wedges may also form if the joints do not intersect the upper face. If a tension crack is included in the analysis, the pentahedrons and tetrahedrons will be truncated by the tension crack plane and hexahedra and pentahedra will result respectively. The wedges are maximized after truncation.

When the wedge coordinates have been determined, the geometrical properties of a wedge can be calculated, including:

- Wedge volume
- Wedge face areas
- Normal vectors for each wedge plane

2. Wedge Forces

All forces on the wedge can be classified as either Active or Passive. In general, Active forces represent driving forces in the safety factor calculation, whereas Passive forces represent resisting forces.

The individual force vectors are computed for each quantity (e.g. wedge weight, bolt force, water force, etc.), and then the resultant Active and Passive force vectors are determined by a vector summation of the individual forces.

2.1. Active Force Vector

The resultant Active force vector is comprised of the following components:

$$\mathrm{A} = \mathrm{W} + \mathrm{C} + \mathrm{X} + \mathrm{U} + \mathrm{E} + \mathrm{B_a}$$

Where:

A is the resultant active force vector

W is the wedge weight vector

C is the external force vector

X is the active pressure force vector

U is the water force vector

E is the seismic force vector

$\mathrm{B_a}$ is the active bolt force vector

2.1.1. Wedge Weight Vector

The wedge weight is usually the primary driving force in the analysis.

$$\mathrm{W} = (\gamma_r V) \cdot \hat{g}$$

Where:

W is the wedge weight vector

γ_r is the unit weight of rock

V is the wedge volume

$\hat{g}$ is the gravity direction

2.1.2. External Force Vector

User-defined external loads can be added on the slope face or upper face.

$$\mathbf{C} = \sum_{i=1}^{n} \mathbf{c_i}$$

Where:

C is the total external force vector

$\mathrm{c_i}$ are the individual external force vectors

2.1.3. Pressure Force (Active) Vector

Pressure force is applied with the **Pressure** option in the Support menu and can be defined as either active or passive.

$$\mathrm{X} = p_s a_s \hat{n}_s + p_u a_u \hat{n}_u$$

Where:

X is the resultant active pressure force vector

p_s is the pressure magnitude on the slope face

a_s is the area of the wedge intersection on the slope face

$\hat{n}_s$ is the unit direction corresponding to the trend/plunge of the slope pressure

p_u is the pressure magnitude on the upper face

a_u is the area of the wedge intersection on the upper face

$\hat{n}_u$ is the unit direction corresponding to the trend/plunge of the upper face pressure

2.1.4. Water Force Vector

In *SWedge*, water pressure can be applied as Joint Water Pressure (acting on the internal joints) and/or Ponded Water Pressure (acting on the slopes).

There are two different methods for defining the existence of water pressure on the joint planes – Constant or Gravitational. When ponded water is applied to the slope and/or upper slope, Gravitation water pressure is used.

Constant Pressure on Each Joint

$$\mathbf{U} = \sum_{i=1}^{3} u_i a_i \hat{n}_i$$

Where:

$\mathbf{U}$ is the resultant water force vector

u_i is the water pressure on the i^{th} joint face

a_i is the area of the i^{th} joint face

$\hat{n}_i$ is the inward (into wedge) normal of the i^{th} joint face

Gravitational Pressure on Each Joint

In *SWedge* it is assumed that the joint water table is a planar surface which is oriented parallel with the upper face. The line of intersection of the water table and the slope surface is a line which is parallel with the crest of the slope. The elevation of the water table surface varies between the toe of the wedge on the slope surface (0 percent filled) and the elevation of the upper face (100 percent filled). *SWedge* allows definition of the elevation by specifying a percent filled variable between 0 and 100.

The water pressure is assumed to be zero along all edges of the wedge that lie on the slope or upper face. In the case of no tension crack, the maximum pressure lies at a point midway along the line of intersection of the two joint planes. This midpoint is half-way between the toe of the wedge and the point at which the line of intersection daylights on the upper face. The magnitude of the pressure is determined based on the vertical distance from the midpoint to where the line of intersection intersects the upper face. The maximum water pressure has a value of $(1/2)\gamma_w H_w$. H_w is the vertical distance between the wedge toe and the point at which the line of intersection daylights on the upper face. The actual pressure distribution on each face is a tetrahedron with volume $Ab/3$ where A is the face area of the slip plane and b is the maximum water pressure. In the case of a tension crack, the maximum pressure is at the base of the tension crack. The magnitude of the pressure is based on the distance from the base of the tension crack to the upper face.

No Tension Crack:

$$\mathbf{U} = \frac{1}{6}\sum_{i=1}^{2} p^3 \gamma_w H_w a_i \hat{n}_i$$

Where:

$\mathbf{U}$ is the resultant water pressure force vector

p is the proportion filled = percent filled / 100

γ_w is the unit weight of water

H_w is the vertical height of the line of intersection of joint 1 and joint 2

a_i is the area of the i^{th} joint face

$\hat{n}_i$ is the inward (into wedge) normal of the i^{th} joint face

With Tension Crack:

$$\mathbf{U} = \frac{1}{3}\sum_{i=1}^{3} p^3 \gamma_w H_w a_i \hat{n}_i$$

Where:

$\mathbf{U}$ is the resultant water pressure force vector

p is the proportion filled = percent filled / 100

γ_w is the unit weight of water

H_w is the vertical height of the line of intersection of joint 1 and joint 2

a_i is the area of the i^{th} joint face

$\hat{n}_i$ is the inward (into wedge) normal of the i^{th} joint face

Note: The above applies to cases where either no ponded water exists or ponded water exists, but Slope Surface Type is set to Impervious. The joint water pressure is computed independent of the ponded water surface.

In *SWedge*, when both ponded water and joint water exists and the Slope Surface Type is set to Pervious, the water table is defined by a combination of water surface planes consisting of the joint water surfaces and the ponded water surface.

Where the elevations of the wetted joint extents are below the ponded water elevation, water pressure magnitudes are computed based on the vertical distance from the ponded water elevation. Where the elevations of the wetted joint extents are above the ponded water elevation, water pressure magnitudes are computed based on the vertical distance from the joint water surfaces.

Gravitational Pressure on Each Slope

In *SWedge* it is assumed that the ponded water surface is a horizontal planar surface at some specified depth above the base of the slope. *SWedge* allows definition of a ponded water depth greater than or equal to zero.

The water pressure is assumed to be zero along the ponded water surface. The magnitude of the pressure is determined based on the vertical distance from the ponded surface. The maximum water pressure has a value of $\gamma_w H_w$. In this case, H_w is the vertical distance between the wedge toe and the ponded water surface. The pressure magnitude at each vertex of the slope face is computed then averaged.

$$P_{ij} = \gamma_w \left(H_w - d_{ij} \right)$$

Where:

P_{ij} is the water pressure magnitude at the j^{th} vertex on the i^{th} slope face

γ_w is the unit weight of water

H_w is the vertical height of ponded water surface above the wedge toe

d_{ij} is the vertical height of the j^{th} vertex on the i^{th} slope face above the wedge toe

$$\mathbf{U} = \sum_{i=1}^{2} \bar{P}_i a_i \hat{n}_i$$

Where:

U is the resultant ponded water force vector

$\bar{P}_i$ is the average pressure magnitude over the i^{th} slope face

a_i is the area of the i^{th} joint face

$\hat{n}_i$ is the inward (into wedge) normal of the i^{th} slope face

2.1.5. Seismic Force Vector

This determines the seismic force vector if the **Seismic** option is applied. The trend and plunge of the direction of the seismic force is first converted to a vector in Cartesian coordinates.

In the case of the basal joint, the force can be applied in the direction of maximum downward gradient of the basal joint (i.e. down-slope on the basal plane) since it is the most likely direction of sliding and the factor of safety is minimized if seismic force is applied in the sliding direction.

Seismic force can also be applied in the direction of the projection of the maximum downward gradient of the basal joint on the horizontal plane.

$$\mathbf{E} = (k\gamma_r V) \cdot \hat{e}$$

Where:

E is the seismic force vector

k is the seismic coefficient

γ_r is the unit weight of rock

V is the wedge volume

$\hat{e}$ is the direction of seismic force

2.1.6. Active Bolt Force Vector

SWedge contains a very simple bolt model. The bolt force is taken as the capacity of the bolt with a direction defined by the user (trend/plunge). The force is assumed to go through the centroid of the wedge. The user must determine the capacity based on the tensile strength of the bolt, bond strength, and plate capacity. The efficiency of the bolt depending on its orientation is not accounted for.

$$\mathbf{B_a} = \sum_{i=1}^{n} c_i \hat{e}_i$$

Where:

$\mathrm{B_a}$ is the active bolt force vector

c_i is the capacity of the i^{th} bolt

$\hat{e}_i$ is the unit direction vector of the i^{th} bolt

2.1.7. Passive Force Vector

The resultant Passive Force Vector is the sum of the bolt, shotcrete and pressure (passive) support force vectors.

$$\mathrm{P} = \mathrm{H} + \mathrm{Y} + \mathrm{B_p}$$

Where:

P is the resultant passive force vector

H is the shotcrete shear resistance force vector

Y is the passive pressure force vector

$\mathrm{B_p}$ is the resultant passive bolt force vector

2.1.8. Pressure Force (Passive) Vector

Pressure force is applied with the **Pressure** option in the Support menu and can be defined as either active or passive.

$$\mathbf{Y} = p_s a_s \hat{n}_s + p_u a_u \hat{n}_u$$

Where:

Y is the resultant passive pressure force vector

p_s is the pressure magnitude on the slope face

a_s is the area of the wedge intersection on the slope face

$\hat{n}_s$ is the unit direction corresponding to the trend/plunge of the slope pressure

p_u is the pressure magnitude on the upper face

a_u is the area of the wedge intersection on the upper face

$\hat{n}_u$ is the unit direction corresponding to the trend/plunge of the upper face pressure

2.1.9. Shotcrete Shear Resistance

The shotcrete support model in *SWedge* assumes that the shotcrete fails by punching shear. Basically, the wedge shears through the shotcrete on the slope face. The shear zones are along the line of intersections of joint 1 and joint 2 on the slope face. To account for adhesion failure coupled with bending failure, factor the shear strength accordingly.

$$\mathbf{Y} = (L_1 + L_2) t \tau_s \hat{n}_s$$

Where:

$\mathbf{Y}$ is the shotcrete shear resistance force vector

L_1 is the trace length of joint 1 on slope face

L_2 is the trace length of joint 2 on slope face

t is the shotcrete thickness

τ_s is the shotcrete shear strength

$\hat{n}_s$ is the slope face unit normal pointing into slope

2.1.10.Passive Bolt Force Vector

SWedge contains a very simple bolt model. The bolt force is taken as the capacity of the bolt with a direction defined by the user (trend/plunge). The force is assumed to go through the centroid of the wedge. The user must determine the capacity based on the tensile strength of the bolt, bond strength, and plate capacity. The efficiency of the bolt depending on its orientation is not accounted for.

$$\mathbf{B_p} = \sum_{i=1}^{n} c_i \hat{e}_i$$

Where:

B_p is the passive bolt force vector

c_i is the capacity of the i^{th} bolt

$\hat{e}_i$ is the unit direction vector of the i^{th} bolt

3. Sliding Direction

Next, the sliding direction of the wedge must be determined. The sliding (deformation) direction is computed by considering active forces only ($\mathbf{A}$ vector). Passive forces ($\mathbf{P}$ vector) *DO NOT* influence sliding direction.

The calculation algorithm is based on the method presented in chapter 9 of "Block Theory and its application to rock engineering", by Goodman and Shi (1985).

In *SWedge*, with pentahedral wedges, there are 6 possible directions ($\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_3, \hat{s}_{12}, \hat{s}_{13}, \hat{s}_{23}$). These represent the modes of: lifting ($\hat{s}_0$), sliding on a single joint plane ($\hat{s}_1, \hat{s}_2, \hat{s}_3$), or sliding along the line of intersection of two joint planes ($\hat{s}_{12}, \hat{s}_{13}, \hat{s}_{23}$).

Calculation of the sliding direction is a two-step process:

1. Compute all possible sliding directions; and
2. Determine which one of the possible sliding directions is the actual valid direction.

3.1. Step 1: Compute List of 4 Possible Sliding Directions

3.1.1. Lifting

$$\hat{s}_0 = \hat{a} = \frac{\mathrm{A}}{\|\mathrm{A}\|}$$

Where:

$\hat{s}_0$ is the falling or lifting direction

$\hat{a}$ is the unit direction of the resultant active force

A is the active force vector

3.1.2. Sliding on a Single Face i

$$\hat{s}_i = \frac{(\hat{n} \times \mathbf{A}) \times \hat{n}_i}{\|(\hat{n}_i \times \mathbf{A}) \times \hat{n}_i\|}$$

Where:

$\hat{s}_i$ is the sliding direction on joint i

$\hat{n}_i$ is the normal to joint face i directed into wedge

A is the active force vector

3.1.3. Sliding on Two Faces i and j

$$\hat{s}_{ij} = \frac{\hat{n}_i \times \hat{n}_j}{\|\hat{n}_i \times \hat{n}_j\|} \operatorname{sign}\left((\hat{n}_i \times \hat{n}_j) \cdot \mathbf{A}\right)$$

Where:

$\hat{s}_{ij}$ is the sliding direction on joint i and j (along line of intersection)

$\hat{n}_i$ is the normal to joint face i directed into wedge

$\hat{n}_j$ is the normal to joint face j directed into wedge

A is the active force vector

3.2. Step 2: Compute Which of the Possible Sliding Directions is Valid

For a pentahedral wedge, the following 8 cases are tested. Whichever satisfies the given inequalities is the sliding direction of the wedge. If none of these tests satisfies the given inequalities, the wedge is unconditionally stable.

3.2.1. Lifting Wedge

$$\mathrm{A} \cdot \hat{n}_1 > 0$$
$$\mathrm{A} \cdot \hat{n}_2 > 0$$
$$\mathrm{A} \cdot \hat{n}_3 > 0$$
$$\mathrm{A} \cdot W < 0$$

Where:

A is the active force vector

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

W is the weight vector

3.2.2. Sliding on Joint 1

$$\mathbf{A} \cdot \hat{n}_1 \leq 0$$
$$\hat{s}_1 \cdot \hat{n}_2 > 0$$
$$\hat{s}_1 \cdot \hat{n}_3 > 0$$

Where:

A is the active force vector

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

$\hat{s}_1$ is the sliding direction on joint 1

3.2.3. Sliding on Joint 2

$$\mathrm{A} \cdot \hat{n}_2 \leq 0$$
$$\hat{s}_2 \cdot \hat{n}_1 > 0$$

$$\hat{s}_2 \cdot \hat{n}_3 > 0$$

Where:

$\mathbf{A}$ is the active force vector

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

$\hat{s}_2$ is the sliding direction on joint 2

3.2.4. Sliding on Basal Joint (Joint 3)

$$\mathrm{A} \cdot \hat{n}_3 \leq 0$$
$$\hat{s}_3 \cdot \hat{n}_1 > 0$$
$$\hat{s}_3 \cdot \hat{n}_2 > 0$$

Where:

$\mathbf{A}$ is the active force vector

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

$\hat{s}_3$ is the sliding direction on basal joint (joint 3)

3.2.5. Sliding on the Intersection of Joint 1 and Joint 2

$$\hat{s}_{12} \cdot \hat{n}_3 > 0$$
$$\hat{s}_1 \cdot \hat{n}_2 \leq 0$$
$$\hat{s}_2 \cdot \hat{n}_1 \leq 0$$

Where:

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

$\hat{s}_{12}$ is the sliding direction on the line of intersection of joints 1 and 2

$\hat{s}_1$ is the sliding direction on joint 1

$\hat{s}_2$ is the sliding direction on joint 2

3.2.6. Sliding on the Intersection of Joint 1 and Basal Joint

$$\hat{s}_{13} \cdot \hat{n}_2 > 0$$
$$\hat{s}_1 \cdot \hat{n}_3 \leq 0$$
$$\hat{s}_3 \cdot \hat{n}_1 \leq 0$$

Where:

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

$\hat{s}_{13}$ is the sliding direction on the line of intersection of joints 1 and 3

$\hat{s}_1$ is the sliding direction on joint 1

$\hat{s}_3$ is the sliding direction on joint 3

3.2.7. Sliding on the Intersection of Joint 2 and Basal Joint

$$\hat{s}_{23} \cdot \hat{n}_1 > 0$$
$$\hat{s}_2 \cdot \hat{n}_3 \leq 0$$
$$\hat{s}_3 \cdot \hat{n}_2 \leq 0$$

Where:

$\hat{n}_i$ is the inward (into the wedge) normal of joint i

$\hat{s}_{23}$ is the sliding direction on the line of intersection of joints 2 and 3

$\hat{s}_2$ is the sliding direction on joint 2

$\hat{s}_3$ is the sliding direction on joint 3

3.2.8. Slumping on the Intersection of Joint 1 and Joint 2

This is a rotational mode of failure which can occur when the mode of sliding is along the basal failure plane. In the case where the active force (line of thrust), which is assumed to go through the centroid of the wedge, lies outside the planes of failure, a rotation of the wedge can occur resulting in slumping of the wedge (See below figure). Since Goodman and Shi (1985) does not account for rotational modes of failure, special consideration must be given to this mechanism. If this mechanism is not accounted for, the block will be computed as sliding along the basal failure plane and the factor of safety in a number of cases will be greatly overestimated.

In this case, *SWedge* assumes that the block slides down the intersection of Joint 1 and Joint 2 and that the basal plane has no influence on the factor of safety. This is an extremely conservative assumption in a number of cases. Especially those in which the line of thrust through the centroid of the wedge approaches the basal failure plane. However, in cases where the area of the basal plane becomes small, this assumption is more accurate than assuming some stabilizing contribution from the basal plane. It is important to keep in mind that this is a lower bound solution.

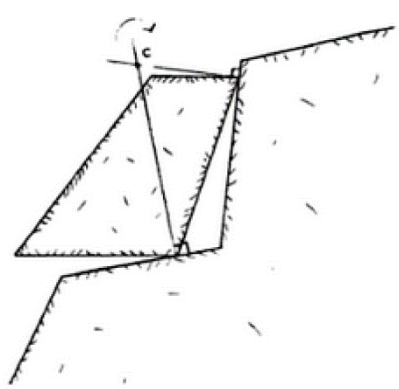


Figure 5 - Block Slumping, with backward rotation about an instantaneous center at C.

(from Goodman 1995, “Block theory and its application”)

4. Normal Force

The calculation of the normal forces on each of the two joint planes for a wedge first requires the calculation of the sliding direction. Once the sliding direction is known, the following equations are used to determine the normal forces given a resultant force vector, $\mathbf{F}$. The force vector, $\mathbf{F}$, is generally either the active or the passive resultant force vector.

4.1. Lifting Wedge

$$N_1 = 0$$
$$N_2 = 0$$
$$N_3 = 0$$

Where:

N_i is the normal force on the i^{th} joint

4.2. Sliding on Joint 1

$$N_1 = -\mathbf{F} \cdot \hat{n}_1$$
$$N_2 = 0$$
$$N_3 = 0$$

Where:

N_i is the normal force on the i^{th} joint

$\mathbf{F}$ is the force vector

$\hat{n}_1$ is the inward (into the wedge) normal of joint 1

4.3. Sliding on Joint 2

$$N_1 = 0$$
$$N_2 = -\mathbf{F} \cdot \hat{n}_2$$
$$N_3 = 0$$

Where:

N_i is the normal force on the i^{th} joint

$\mathbf{F}$ is the force vector

$\hat{n}_2$ is the inward (into the wedge) normal of joint 2

4.4. Sliding on Basal Joint (Joint 3)

$$N_1 = 0$$
$$N_2 = 0$$

$$N_3 = -\mathbf{F} \cdot \hat{n}_3$$

Where:

N_i is the normal force on the i^{th} joint

$\mathbf{F}$ is the force vector

$\hat{n}_3$ is the inward (into the wedge) normal of joint 3

4.5. Sliding on Joints 1 and Joint 2

$$N_1 = -\frac{(\mathbf{F} \times \hat{n}_2) \cdot (\hat{n}_1 \times \hat{n}_2)}{(\hat{n}_1 \times \hat{n}_2) \cdot (\hat{n}_1 \times \hat{n}_2)}$$

$$N_2 = -\frac{(\mathbf{F} \times \hat{n}_1) \cdot (\hat{n}_2 \times \hat{n}_1)}{(\hat{n}_2 \times \hat{n}_1) \cdot (\hat{n}_2 \times \hat{n}_1)}$$

$$N_3 = 0$$

Where:

N_i is the normal force on the i^{th} joint

$\mathbf{F}$ is the force vector

$\hat{n}_1$ is the inward (into the wedge) normal of joint 1

$\hat{n}_2$ is the inward (into the wedge) normal of joint 2

4.6. Sliding on Joint 1 and Basal Joint

$$N_1 = -\frac{(\mathbf{F} \times \hat{n}_3) \cdot (\hat{n}_1 \times \hat{n}_3)}{(\hat{n}_1 \times \hat{n}_3) \cdot (\hat{n}_1 \times \hat{n}_3)}$$

$$N_2 = 0$$

$$N_3 = -\frac{(\mathbf{F} \times \hat{n}_1) \cdot (\hat{n}_3 \times \hat{n}_1)}{(\hat{n}_3 \times \hat{n}_1) \cdot (\hat{n}_3 \times \hat{n}_1)}$$

Where:

N_i is the normal force on the i^{th} joint

$\mathbf{F}$ is the force vector

$\hat{n}_1$ is the inward (into the wedge) normal of joint 1

$\hat{n}_2$ is the inward (into the wedge) normal of joint 2

$\hat{n}_3$ is the inward (into the wedge) normal of basal joint (joint 3)

4.7. Sliding on Joint 2 and Basal Joint

$$N_1 = 0$$

$$N_2 = -\frac{(\mathbf{F} \times \hat{n}_3) \cdot (\hat{n}_2 \times \hat{n}_3)}{(\hat{n}_2 \times \hat{n}_3) \cdot (\hat{n}_2 \times \hat{n}_3)}$$

$$N_3 = -\frac{(\mathbf{F} \times \hat{n}_2) \cdot (\hat{n}_3 \times \hat{n}_2)}{(\hat{n}_3 \times \hat{n}_2) \cdot (\hat{n}_3 \times \hat{n}_2)}$$

Where:

N_i is the normal force on the i^{th} joint

$\mathbf{F}$ is the force vector

$\hat{n}_1$ is the inward (into the wedge) normal of joint 1

$\hat{n}_2$ is the inward (into the wedge) normal of joint 2

$\hat{n}_3$ is the inward (into the wedge) normal of basal joint (joint 3)

5. Shear Strength

There are three joint strength models available in *SWedge*:

1. Mohr-Coulomb
2. Barton-Bandis
3. Power Curve

Shear strength is computed based on the normal stress acting on each joint plane. The normal stress is computed based on the active and passive normal forces computed on the joint planes using the equations in the previous section.

5.1. Compute Normal Stress on Each Joint

First compute the stress on each joint plane based on the normal forces computed in Section 5.

$$\sigma_{n_i} = \frac{N_i}{a_i}$$

Where:

σ_{n_i} is the normal stress on the i^{th} joint

N_i is the normal force on the i^{th} joint

a_i is the area of the i^{th} joint

5.2. Compute Shear Strength of Each Joint

Use the strength criteria defined for the joint, and the normal stress, to compute the shear strength.

5.2.1. Mohr-Coulomb Strength Criterion

$$\tau_i = c_i + \sigma_{n_i} \tan \phi_i$$

Where:

τ_i is the shear strength of the i^{th} joint

c_i is the cohesion of the i^{th} joint

σ_{n_i} is the normal stress on the i^{th} joint

ϕ_i is the friction angle of the i^{th} joint

5.2.2. Barton-Bandis Strength Criterion

$$\tau_i = \sigma_{n_i} \tan\left[JRC_i \log_{10}\left(\frac{JCS_i}{\sigma_{n_i}}\right) + \phi_{r_i}\right]$$

Where:

τ_i	is the shear strength of the i^{th} joint
JRC_i	is the joint roughness coefficient of the i^{th} joint
JCS_i	is the joint compressive strength of the i^{th} joint
σ_{n_i}	is the normal stress on the i^{th} joint
ϕ_{r_i}	is the residual friction angle of the i^{th} joint

5.2.3. Power Curve Strength Criterion

$$\tau_i = c_i + a_i\left(\sigma_{n_i} + d_i\right)^{b_i}$$

Where:

τ_i	is the shear strength of the i^{th} joint
a_i, b_i, c_i, d_i	are the strength parameters of the i^{th} joint
σ_{n_i}	is the normal stress on the i^{th} joint

5.3. Compute Resisting Force due to Shear Strength

Force acts in a direction opposite to the direction of sliding (deformation).

$$J_i = \tau_i a_i \cos\theta_i$$

Where:

J_i	is the magnitude of the resisting force due to the shear strength of the i^{th} joint
τ_i	is the shear strength of the i^{th} joint
a_i	is the area of the i^{th} joint
θ_i	is the angle between the sliding direction and the i^{th} joint

6. Factor of Safety

SWedge computes 3 separate factors of safety:

1. Lifting factor of safety
2. Unsupported factor of safety
3. Supported factor of safety

The reported factor of safety is the maximum of the above three factors of safety. The logic of this is simple; support is assumed to never decrease the factor of safety from the unsupported value. The factor of safety can never be less than if the wedge was lifting with only support to stabilize it.

The equations are based on three joint planes making up a pentahedral wedge.

The limit equilibrium safety factor calculations only consider force equilibrium in the direction of sliding. Moment equilibrium is not considered.

Factor of Safety:

$$FS = \max\left(FS_f, FS_u, FS_s\right)$$

Where:

FS_f is the falling factor of safety

FS_u is the unsupported factor of safety

FS_s is the supported factor of safety

6.1. Factor of Safety Definition

$$factor\ of\ safety = \frac{resisting\ forces\ \ (e.g.\ shear\ or\ tensile\ strength,\ support)}{driving\ forces\ \ (e.g.\ weight,\ seismic,\ water)}$$

6.2. Lifting Factor of Safety

The lifting factor of safety assumes that only passive support acts to resist movement. Basically, the wedge is assumed to be lifting off the failure planes so no influence of the joint planes (shear strength, failure direction) is incorporated. Driving forces are due to the active forces on the wedge as defined in Section 3.1. The lifting direction is calculated from the direction of the active force vector.

$$FS_f = \frac{-\mathrm{P} \cdot \hat{s}_0}{\mathrm{A} \cdot \hat{s}_0}$$

Where:

FS_f is the falling factor of safety

P is the resultant passive force vector (Section 3.2)

A is the resultant active force vector (Section 3.1)

$\hat{s}_0$ is the falling direction (Section 4)

6.3. Unsupported Factor of Safety

The unsupported factor of safety assumes that shear strength acts to resist movement. No passive support force is used.

Driving forces are due to the active forces on the wedge as defined in Section 3.1.

The sliding direction is calculated from the equations in Section 4. The shear strength is calculated based on the normal forces from the active force vector only. Normal forces from the passive force vector are not included.

$$FS_u = \frac{\sum_{i=1}^{2} J_i^u}{\mathrm{A} \cdot \hat{s}}$$

Where:

FS_u is the unsupported factor of safety

J_i^u is the magnitude of the resisting force due to the unsupported shear strength of the i^{th} joint (Section 6.3)

A is the resultant active force vector (Section 3.1)

$\hat{s}$ is the sliding direction (Section 4)

6.4. Supported Factor of Safety

The supported factor of safety assumes that passive support forces and shear strength act to resist movement.

Driving forces are due to the active forces on the wedge as defined in Section 3.1. The sliding direction is calculated from the equations in Section 4. The shear strength is calculated based on the normal force calculated from the active force vector plus the passive force vector.

$$FS_s = \frac{-\mathrm{P} \cdot \hat{s} + \sum_{i=1}^{2} J_i^s}{\mathrm{A} \cdot \hat{s}}$$

Where:

FS_s is the supported factor of safety

J_i^s is the magnitude of the resisting force due to the supported shear strength of the i^{th} joint (Section 6.3)

P is the resultant passive force vector (Section 3.2)

A is the resultant active force vector (Section 3.1)

$\hat{s}$ is the sliding direction (Section 4)