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ORIGINAL PAPER

Stability Analysis of Rock Slopes Against Block-Flexure Toppling Failure

Mehdi Amini · Abbas Majdi · Mohammad Amin Veshadi

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Abstract Block-flexure is the most common type of toppling failure in rock slopes. In this case, some rock blocks fail due to tensile bending stresses and some overturn under their own weights. In this paper, first, a literature review of toppling failures is summarized. Then, a theoretical model is proposed for rock slopes with a potential for block-flexure toppling instability. Next, a new analytical approach is presented for the stability analysis of such slopes. Finally, a special computer code is developed for a quick stability assessment of the failures based on the proposed method. This code receives the rock slope parameters from the user as the input data and predicts its stability, along with the corresponding factor of safety against the failure, as the output. In addition, two case studies are used for practical verification of the proposed approach and the corresponding computer code as well.

Keywords Rock slope · Block-flexure toppling · Computer code · Case study

List of Symbols

Parameters

δ	Angle of rock mass stratification with respect to the horizon
α	Angle of total failure plane with respect to the horizon
θ	Angle of face slope with respect to the horizon
H	Slope height
t	Thickness of rock columns
n	Number of rock columns, numbered from bottom to top
P	Inter-column normal force acting at the common boundary of rock columns
Q	Inter-column shear force acting at the common boundary of rock columns
x	Point of application of the inter-column normal force (P)
h	Length of the rock column
W	Weight of the rock column
N	Normal force acting at the base of the rock column
S	Shear force acting at the base of the rock column
e	Point of application of the base normal force (N)
M	Bending moment
φ_1	Frictional angle at the common boundary of rock columns
φ_2	Frictional angle at the base of rock columns
φ_3	Frictional angle of intact rock samples
c	Cohesive strength of intact rock samples
σ_t	Uniaxial tensile strength of rock columns
σ_t^y	Tensile stress of rock columns

Indexes

f	Flexure toppling failure
b	Block toppling failure
sh	Shearing failure
t	Toppling failure
s	Sliding failure

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1 Introduction

Toppling failure is a serious and frequent instability in natural and manmade rock slopes. From the mechanism point of view, the main toppling failures are classified as flexural, blocky, and block-flexure (Goodman and Bray 1976). If a rock mass is composed of a set of parallel discontinuities, dipping steeply against the face slope, then it will act like some rock columns that are placed on top of each other. In such a case, rock columns are under tensile and compressive bending stresses due to their own weights. If the maximum tensile stress in each rock column exceeds its tensile strength, then it fails and topples. Such instability is categorized as flexural toppling failure (Fig. 1a). If one cross-joint set is added to the aforementioned rock mass (Fig. 1b), then the system cannot bear the tensile bending stress and, therefore, the columns may overturn due to their own weights. This type of failure is considered as a typical blocky toppling failure. In real case histories, the aforementioned perfect cases are rarely encountered and toppling failures are mostly of block-flexure nature (Fig. 1c). The latter instability is a combination of blocky and flexural toppling failure modes. Many research papers are available on flexural and blocky toppling failures, and some methods have also been given for their stability analyses, which will be briefly outlined in Sect. 2 of this paper. However, no suitable analytical solution has been presented for the assessment of block-flexure toppling failure as yet. In this paper, the mechanism of such a failure has been studied and a new analytical approach is proposed for its analysis.

2 Literature Review

Müller (1968) was the first who mentioned the overturning of natural rock blocks after studying the instabilities near the famous Vaiont dam lake. In 1971, based on theoretical and experimental modeling, Ashby (1971) presented a simple criterion for the evaluation of such failures. He was the one who, for the first time, proposed the term "toppling" for such instabilities. From 1970 to 1976, some scattered researches on known cases (numerical and experimental modeling of such failures) were published (Erguvanli and Goodman 1970; Cundall 1971; de Freitas and Watters 1973; Hoffmann 1974; Bukovansky et al. 1976). Goodman and Bray (1976) classified toppling failures as "main" and "secondary", based on case study observations and experimental modeling. In main toppling failures (flexural, blocky, and block-flexure), the rock mass weight is the most important factor affecting instability, whereas in secondary ones, other external factors cause the failure. Although some other classifications have been proposed for toppling failures (Cruden 1989), the one stated above has received a better acceptance by rock mechanics researchers. Goodman and Bray presented a step-by-step analytical method for the analysis of the blocky toppling failure in their abovementioned paper. This approach has been presented many times for the analysis of

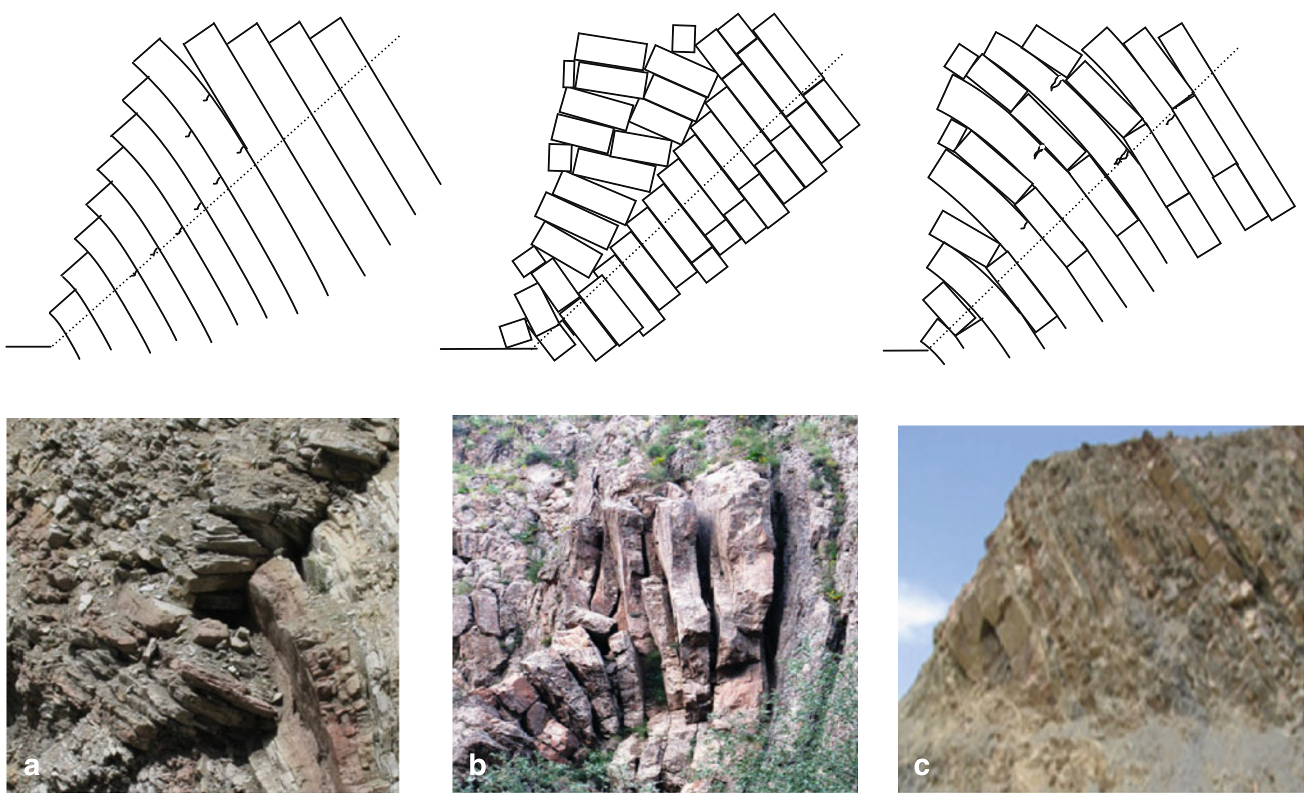


Fig. 1 Possible main toppling failures in rock slopes. **a** Flexural. **b** Blocky. **c** Block-flexure

such failures in the form of design charts and computer codes (Hoek and Bray 1977; Zanbak 1983; Choquet and Tanon 1985; Keith Turner and Schuster 1996). After 1986, based on Goodman and Bray's classification, a lot of research was carried out on blocky and flexural toppling failures (Wyllie 1980; Aydan et al. 1989; Aydan and Kawamoto 1992; Adhikary et al. 1997; Bobet 1999; Sageseta et al. 2001; Adhikary and Guo 2002; Amini 2009; Amini el al. 2009; Aydan and Amini 2009; Brideau and Stead 2009; Majdi and Amini 2011). As mentioned earlier in this paper, the research on toppling failures concentrates mostly on blocky and flexural types. But, most failures occurring in nature are of block-flexure mode, for which no suitable analytical method has yet been proposed. It is, therefore, of utmost importance to study the mechanism of such failures and to analyze them rationally. Effort has been made to study them in this paper.

3 Mechanism of Block-Flexure Toppling Failure

Rock is a natural mass and its discontinuities are generally irregular and discontinuous; so, pure toppling failures (flexural and blocky) are rare and most of such failures occurring in nature are of the block-flexure type. Although the real behavior of a rock mass with a potential for block-flexure toppling failure is complicated, it is possible to simplify the problem and propose a theoretical model to achieve an appropriate analytical solution for the failure. In this paper, the following assumptions have been made to simplify the analysis of block-flexure toppling failure:

- Of two adjacent blocks, one has the potential for blocky failure and the other may fail flexurally. In other words, two blocks of the same failure potential do not stand together.
- Blocky and columnar toppling failures are similar.
- All blocks in a rock mass with potential for block-flexure toppling failures have a similar factor of safety equal to that of the whole slope against the failure.
- The total failure plane of the instability lies 10°–20° above the normal line of main discontinuities.

As stated earlier in this paper, in block-flexure toppling failure, some blocks fail due to tensile bending stress and some separate from the cross-joint surface and then all of them topple together. Considering the above assumptions, Fig. 2 has been suggested as a theoretical model to assess the failure. Here, rock columns with natural cross-joints at their pivots having the potential for toppling, sliding, or toppling–sliding exert a special force on their adjacent rock column. The latter rock column that has been subjected to the special force has a potential foe flexural toppling failure and carries a tensile stress at its pivot. This rock column, in turn, exerts a force on its adjacent one that has a potential for blocky toppling failure. If the maximum resultant tensile stress produced at the pivot of the column is greater than the tensile strength of the rock, the rock column cracks and the slope becomes unstable. Therefore, block-flexure toppling failure is a combination of block toppling failure, flexural toppling failure, and the sliding of the rock blocks. The most important parameter in a toppling failure is the point where the inter-column forces act. Some researchers suggested the following assumptions in regards to the determination of the point of action of the inter-column forces (Goodman and Bray 1976; Aydan et al. 1989; Aydan and Kawamoto 1992):

- If block n has the potential for pure block toppling failure, then $x_{n-1} = h_n$.
- If block n has the potential for pure sliding failure, then $x_{n-1} = \frac{h_n}{2}$.
- If block n has the potential for pure flexural toppling failure, then $x_{n-1} = (0.75 - 1)h_n$.

The most critical issue with the limit equilibrium analysis of toppling failures is how to assign the inclination of the total failure plane above which blocks are subjected to overturning. Base friction experiments, carried out by Aydan and Kawamoto (1992), show that the total failure plane of a flexural toppling is perpendicular to the discontinuities. Hence, the angle between the total failure plane and the plane normal to the discontinuities is zero. However, Adhikary et al. (1997), using the centrifuge physical modeling, show that this angle is around 10° above the plane normal to the discontinuities. Also, the new physical model tests carried out by Aydan and Amini (2009) have shown that this angle is in the range of 0–15° for active flexural toppling failures in static and dynamic forms.

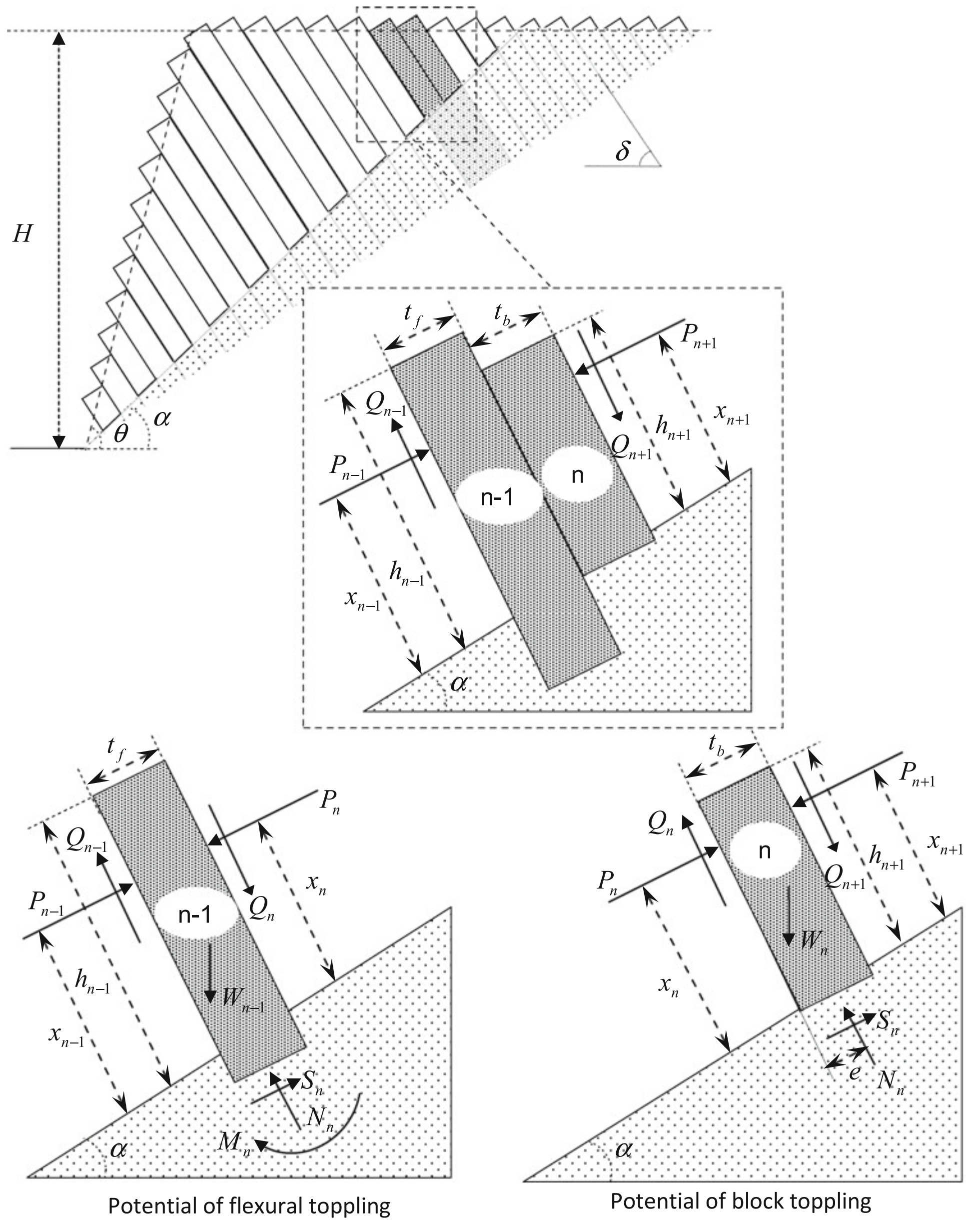
4 Analysis of Block-Flexure Toppling Failure

A theoretical model suitable to be applied to a rock slope having the potential for block-flexure toppling failure is shown in Fig. 2. To study this slope, the following two cases are considered:

- Case 1: one block with a potential for blocky toppling failure, situated between two blocks having potential for flexural toppling failures (Fig. 3a).
- Case 2: one block with a potential for flexural toppling failure, situated between two blocks having potential for blocky toppling failures (Fig. 3b).

Analyzing the above two cases and comparing the results with the proposed theoretical model (Fig. 2), it might be inferred that all existing blocks and rock columns

Fig. 2 Block-flexure toppling failure mechanism



could be analyzed and evaluated by means of one of the two abovementioned cases. In this paper, these cases have been studied and the general governing solutions are proposed for their analyses.

- Case 1: one block with a potential for blocky toppling failure situated between two blocks having potential for flexural toppling failures (Fig. 3a).

According to Fig. 3a, since block n can have the potential for toppling, sliding, and toppling–sliding or being stable, the analysis should be divided into the following four categories:

a. Block n has the potential for blocky failure but is stable against sliding. With these assumptions, the following relations can exist (Fig. 4):

$$\begin{cases} e = 0 \\ x_{n-1} = h_n \\ Q_{n-1} = P_{n-1}\tan\varphi_1 \\ Q_n = P_n\tan\varphi_1 \\ S_n < N_n\tan\varphi_2 \end{cases} \tag{1}$$

As shown in Fig. 4, the force P_n is exerted from the rock column with a potential for flexural toppling failure on block n, but its point of action is unknown. Aydan and Kawamoto (1992) suggested the following range for the point of action of the force based on experimental modeling of flexural toppling failure:

$$x_n = \lambda_1 h_{n+1}, \lambda_1 = (0.75 - 1)$$

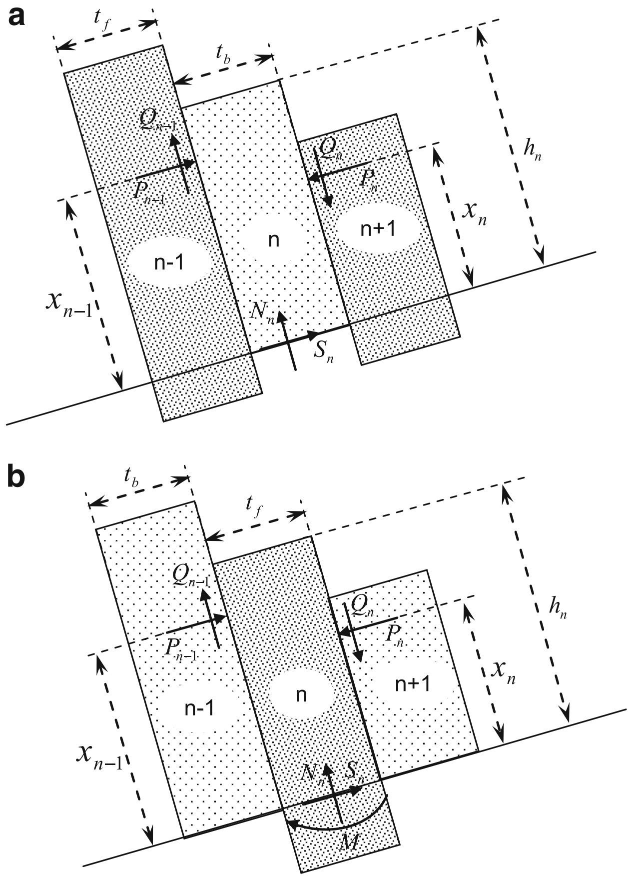


Fig. 3 Two separate parts of a rock slope having the potential for block-flexure toppling failure

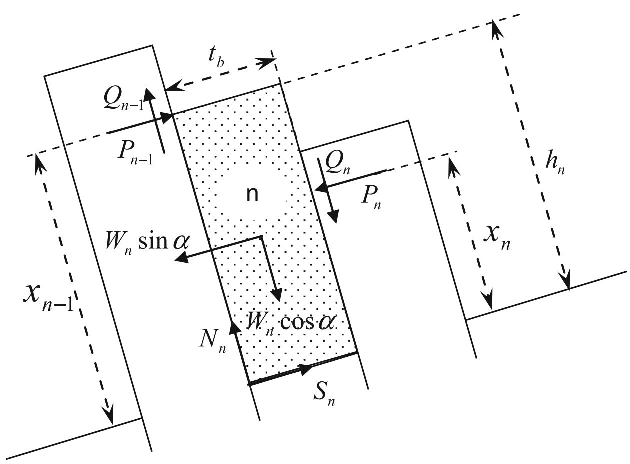


Fig. 4 Analysis of three blocks having the potential for flexural and blocky toppling failures

The best correlation between theoretical and experimental results can be achieved when $\lambda_1 = 1$ is used. On this basis, and using the equation of moment equilibrium with respect

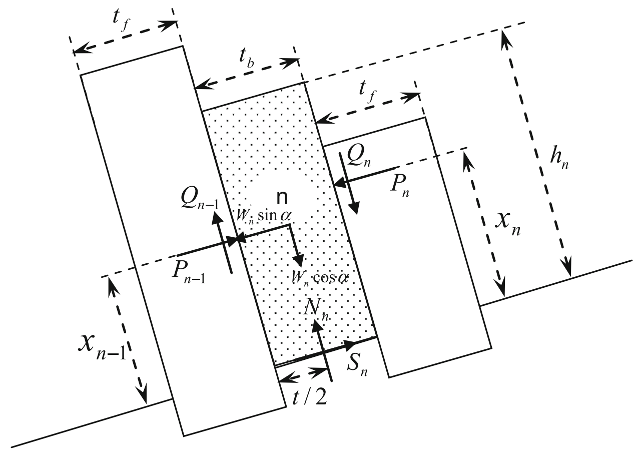


Fig. 5 Analysis of three blocks having the potential for flexural toppling and sliding failures

to point A, the magnitude of the force P_{n-1} can be computed as follows:

$$P_{n-1,t} = \frac{P_n[\lambda_2 h_{n+1} - \tan\varphi_1 \cdot t_b/F_S] + 0.5 w_n[\sin\alpha \cdot h_n - \cos\alpha \cdot t_b]}{h_n} \tag{2}$$

b. Block n has sliding potential but is stable against blocky toppling failure. In this case, the following conditions exist (Fig. 5):

$$\begin{cases} e = \frac{t_b}{2} \\ x_{n-1} = \frac{h_n}{2} \\ S_n = N_n \tan\varphi_2 \\ x_n = \lambda_2 h_{n+1}, \lambda_2 = (0.75 - 1) \end{cases} \tag{3}$$

It is also assumed that:

$$\begin{cases} Q_n \cong P_n \cdot \tan\varphi_1 \\ Q_{n-1} \cong P_{n-1} \cdot \tan\varphi_1 \end{cases} \tag{4}$$

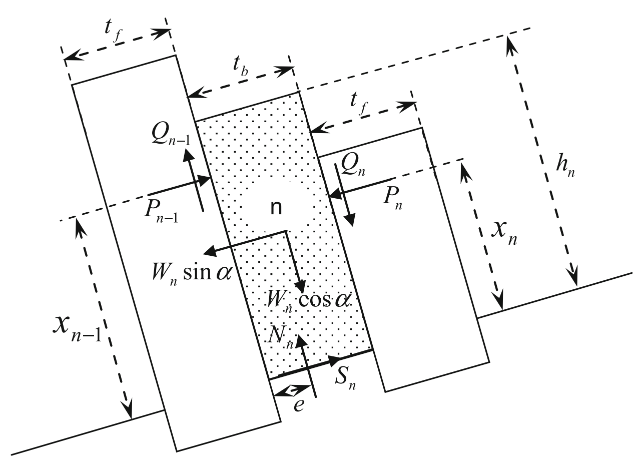


Fig. 6 Analysis of three blocks having the potential for flexural toppling and blocky toppling–sliding failures

Under such conditions, the magnitude of P_{n-1} can be computed, using limit equilibrium equations as follows:

$$P_{n-1,s} = P_n + \frac{w_n(\sin\alpha - \cos\alpha\tan\varphi_2/F_S)}{1 - \tan\varphi_1\tan\varphi_2/F_S} \tag{5}$$

c. Block n has the potential for blocky toppling and sliding failures (Fig. 6). Sagaseta (1986) analyzed a single column with potential for toppling–sliding failure on the basis of the dynamic equation of equilibrium. The mechanism and conditions of a single column and rock slopes having the potential for combined sliding and toppling failures were investigated and a complete solution for the analysis of such a case was presented by Aydan et al. (1989). On the basis of this approach, the failure may be classified into the following cases:

- Transition from sliding mode to combined sliding and toppling mode; in this case, the following conditions exist:

$$\begin{cases} S_n = N_n\tan\varphi_2 \\ Q_n = P_n\tan\varphi_1 \\ Q_{n-1} = P_{n-1}\tan\varphi_1 \\ a_x^i \geq 0 \\ a_y^i = a_\theta^i = 0 \end{cases}$$

Under the aforesaid conditions and on the basis of the dynamic equation of equilibrium, the magnitude of inter-column forces in such a case can be computed as follows (Aydan et al. 1989):

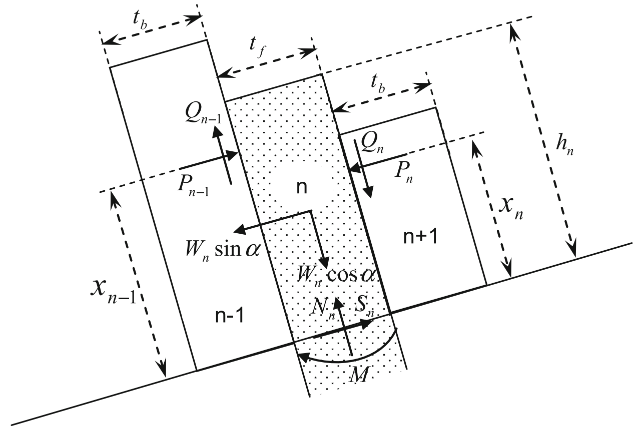


Fig. 7 Analysis of three blocks having potential for block-flexure toppling failures

such a case can be computed as follows (Aydan et al. 1989):

$$\begin{aligned} P_{n-1,t,s} = P_i[&(4t_b^2 - 2h_i^2 - 6h_it_b\tan\varphi_2) \\ &+ \tan\varphi_1(\tan\varphi_1(2t_b^2 - 4h_i^2)/F_S + 6h_it_b)/F_S]K^{-1} \\ &+ W_i[\sin\alpha(4t_b^2 + h_i^2 - 3h_it_b\tan\varphi_2) \\ &- \cos\alpha(\tan\varphi_2(t_b^2 + 4h_i^2) - 3h_it_b)]K^{-1} \end{aligned} \tag{6.2}$$

where $K = [(4(t_b^2 + h_i^2) - 6h_ih_{i-1}) - \tan\varphi_2(6h_{i-1}t_b + 4\tan\varphi_1 (t_b^2 + h_i^2)/F_S)]$

d. Block n is stable against blocky toppling and sliding modes. Therefore, $P_{n-1} = 0$ (Fig. 7).

$$P_{n-1,s,t} = \frac{P_i[0.5h_i(1 + \tan\varphi_1\tan\varphi_2/F_S) - t_b\tan\varphi_1] + 0.5W_i\cos\alpha(h_i\tan\varphi_2/F_S - t_b)}{h_{i-1} - 0.5h_i(1 - \tan\varphi_1\tan\varphi_2/F_S)} \tag{6.1}$$

- Transition from toppling mode to combined sliding and toppling mode; in this case, the following conditions exist:

$$\begin{cases} S_n = N_n\tan\varphi_2 \\ Q_n = P_n\tan\varphi_1 \\ Q_{n-1} = P_{n-1}\tan\varphi_1 \\ a_\theta^i \geq 0 \\ a_x^i = 0.5a_\theta^i h_i \\ a_y^i = -0.5a_\theta^i t_b \end{cases}$$

Similarly, on the basis of the dynamic equation of equilibrium, the magnitude of inter-column forces in

After investigating all the above cases, the final value of P_{n-1} may be determined as follows:

$$P_{n-1} = \mathrm{Max}(P_{n-1,t}, P_{n-1,s}, P_{n-1,s,t}, P_{n-1,t,s}, 0)$$

- Case 2: one block with a potential for flexural toppling failure situated between two blocks having potential for blocky toppling failures (Fig. 3b).

Since block n can have the potential for flexural toppling or shearing, the case may be studied under the following two categories:

a. Block n has the potential for a flexural toppling, then:

$$\begin{cases} Q_n = P_n \cdot \tan\varphi_1 \\ Q_{n-1} = P_{n-1} \cdot \tan\varphi_1 \\ e = \frac{t_f}{2} \\ x_{n-1} = \lambda_1 h_n, \lambda_1 = (0.75 - 1) \end{cases} \quad (7)$$

Writing the limit equilibrium equations for this block, the magnitudes of the moment and the normal force at the base of the block can be determined as follows:

$$\Sigma F_N = 0 \Rightarrow N = w_n \cos\alpha + Q_n - Q_{n-1} \cong w_n \cos\alpha \quad (8.1)$$

$$\Sigma M = 0 \Rightarrow M = w_n \sin\alpha \frac{h_n}{2} + P_n x_n - Q_n \frac{t_f}{2} - Q_{n-1} . \frac{t_f}{2} - P_{n-1} x_{n-1} \quad (8.2)$$

Fig. 8 Rock slope with a potential for block-flexure toppling failure facing Chalus Road

On the other hand, the maximum tensile stress at the base of this block can be computed as follows:

$$\sigma_t^{y=t/2} = \frac{0.5M \cdot t}{I} - \frac{N}{t} \quad (9.1)$$

Under limit equilibrium conditions, the maximum tensile stress is equal to the tensile strength of the rock block; hence, Eq. 9.1, considering the factor of safety, can be rewritten in the following form:

$$M = \frac{2I}{t}\left(\frac{\sigma_t}{F_S} + \frac{N}{t}\right) \quad (9.2)$$

Substituting M and N from Eqs. 8.1 and 8.2 into Eq. 9.2, the value of P_{n-1} can be determined as follows:

$$P_{n-1} = \frac{P_n\left(x_n - \frac{1}{2}\tan\varphi_1 \cdot t\right) + w_n \sin\alpha \cdot \frac{h_n}{2} - \frac{2I}{t}\left(\frac{\sigma_t}{F_S} + \frac{w_n \cos\alpha}{t}\right)}{x_{n-1} + \frac{1}{2}\tan\varphi_1 \cdot t} \quad (10)$$

Therefore, P_n is the force that the block $n + 1$ exerts on block n. Since the latter block has the potential for toppling, sliding, toppling–sliding, or being stable, the force P_{n-1} can be categorized into the following four groups:

- If block $n + 1$ has the potential for toppling failure but is stable against sliding, then $x_n = h_{n+1}$ and P_{n-1} is designated as $P_{n-1,t}$.
- If block $n + 1$ has sliding potential but is stable against toppling, then $x_n = 0.5h_{n+1}$ and P_{n-1} is designated as $p_{n-1,s}$.
- If block $n + 1$ has the potential for toppling and sliding failures, then $x_n = (0.5 - 1)\, h_{n+1}$ and P_{n-1} is designated as $P_{n-1,s,t}$.
- If block $n + 1$ is stable against toppling and sliding failures, then $P_{n-1} = 0$.

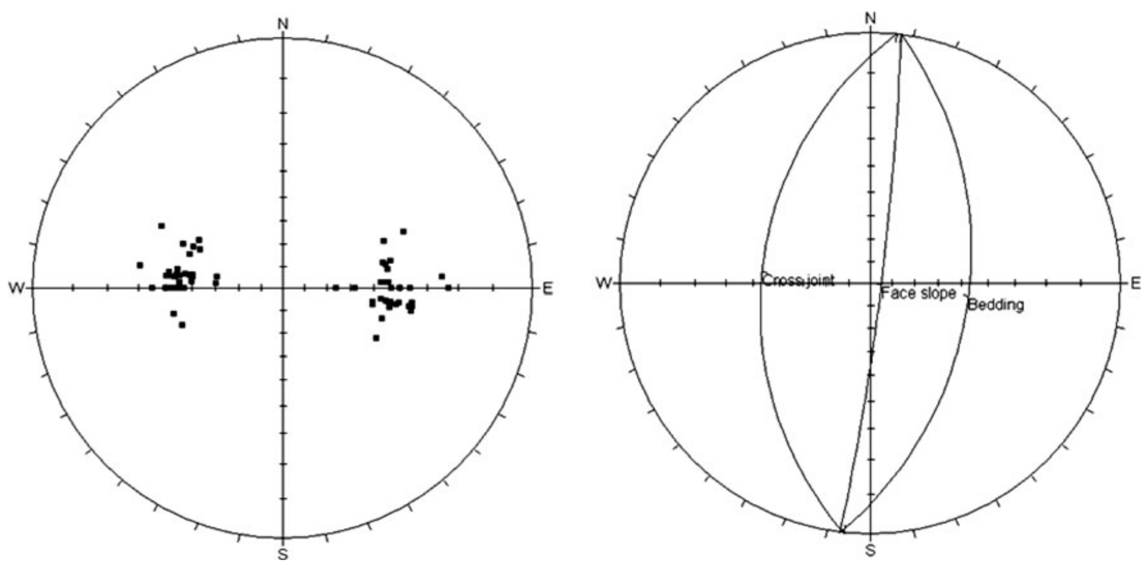


Fig. 9 Stereonet diagrams of discontinuities and face slope of the Chalus Road rock mass

b. If block n has shearing potential, then:

$$\begin{cases} e = \frac{t_f}{2} \\ S_n = N_n \tan\varphi_3 + ct_f \end{cases} \tag{11}$$

It is also assumed that:

$$\begin{cases} Q_n \cong P_n \cdot \tan\varphi_1 \\ Q_{n-1} \cong P_{n-1} \cdot \tan\varphi_1 \end{cases} \tag{12}$$

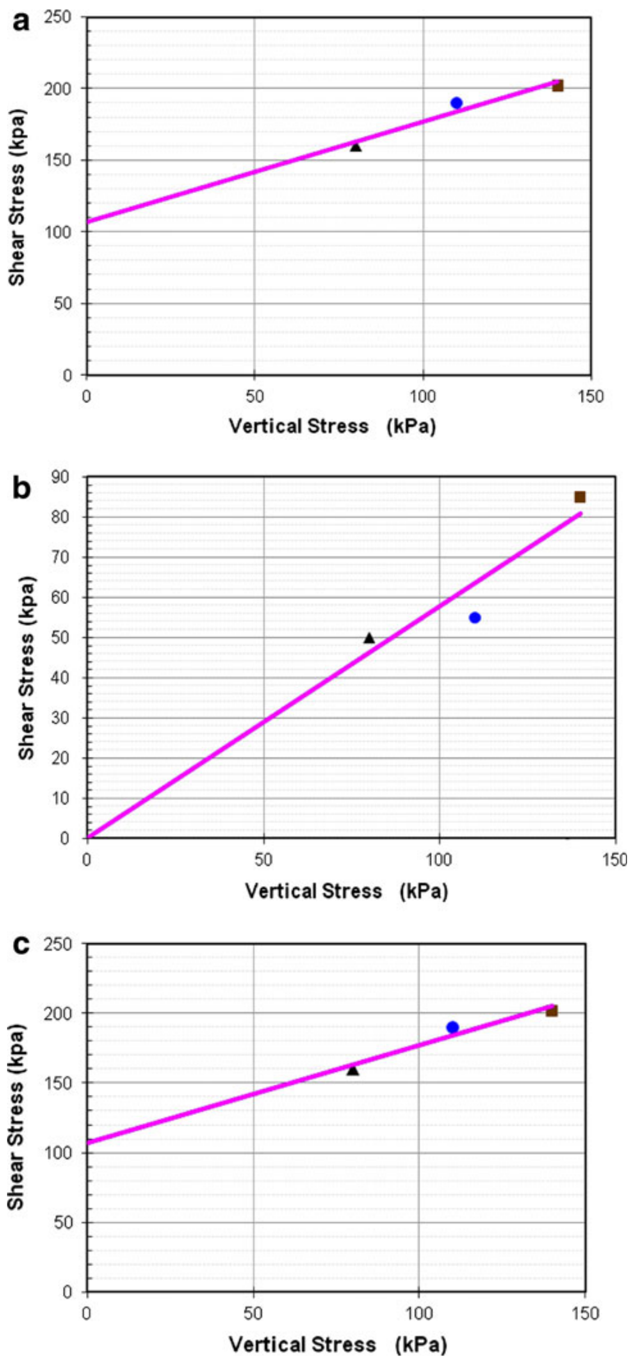


Fig. 10 Direct shear test results of the samples taken from the rock mass facing Chalus Road. **a** Sandstone joints. **b** Shale. **c** Intact rock

With regards to Eqs. 11 and 12 along with taking the limit equilibrium equations into account, the resulting expression is used to determine the inter-column force P_{n-1}:

$$P_{n-1,sh} = P_n + \frac{w_n(\sin\alpha - \cos\alpha\tan\varphi_3/F_S) - ct_f/F_S}{1 - \tan\varphi_1\tan\varphi_3/F_S} \tag{13}$$

It must be borne in mind that the real magnitude of P_{n-1} is computed using the following relation:

$$P_{n-1} = \mathrm{Max}(P_{n-1,t}, P_{n-1,s}, P_{n-1,t,s}, P_{n-1,sh}, 0)$$

Under limit equilibrium conditions, $F_s = 1$, the inter-column forces can be computed for all rock columns by using the above relations. Knowing the sign of P_0 (the assumed force required for the stability of block 1), the stability of the rock slope against block-flexure toppling failure can be evaluated as follows:

1. If $P_0 > 0$, then the slope is unstable.
2. If $P_0 < 0$, then the slope is stable.
3. If $P_0 = 0$, then the slope is in the limit equilibrium condition.

To determine the factor of safety of the slope against block-flexural toppling failure, P_0 is assumed to be 0 and then F_s can be computed by trial and error.

As it can be seen, the above approach needs a lot of calculations, which is time consuming if the calculations are carried out manually. Thus, based on the method

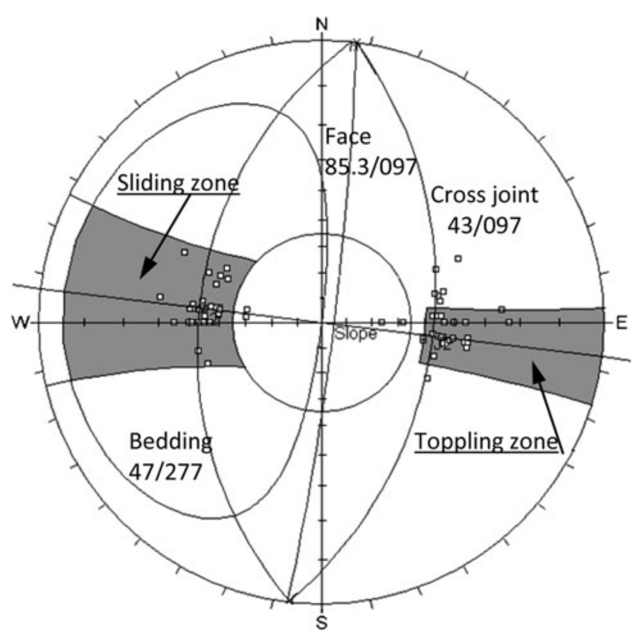


Fig. 11 Kinematic stability analysis of the rock slope facing Chalus Road

proposed in this paper, a FORTRAN computer program was developed to simplify the stability analysis of rock slopes against block-flexure toppling failure. This program receives the rock slope parameters from the users and calculates the magnitude of the inter-column forces and the corresponding factor of safety.

5 Case Studies

To verify the results of the proposed analytical method, two real case studies were selected and have been analyzed using the corresponding computer code. The first case is the rock slope facing Tehran-Chalus Road near the Amir-Kabir Dam Lake and the second is the Galandrood mine slope, which is located in the north of Iran. The first case has always been stable against toppling failure and no local or total failures have ever been reported. The second case, although stable and there were no signs of total or even local failure, presently, there is an obvious indication of local failure. The two real cases, as practical examples, have been analyzed by using the theoretical method proposed in this paper with the aid of corresponding computer codes. The results were compared with the in situ observations.

Table 1 Results of the stability analysis of the rock slope facing Chalus Road

```
                   Computer program title: "BFTOP"
                       Coded by: MEHDI AMINI
 Purpose:  Analysis of rock slopes against block-flexure toppling failure

 ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                            Input section
 ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

     Project name:GALOOS       Date:12/07/2011        Analyzed by:M.AMINI

    Geometrical and geomechanical parameters of the slope and the rock mass
    ------------------------------------------------------------------------
        Inclination of the face slope:                  85.30  Degree
        Inclination of the layers:                      47.00  Degree
        Inclination of the total failure plane:         53.00  Degree
        Inclination of the slope top surface:           29.00  Degree
        Height of the slope:                            20.45       m
        Average thickness of the layers:                 2.31       m
        Friction angle at the column-column contacts:   30.00  Degree
        Friction angle at the base-column contacts:     35.00  Degree
        Cohesive strength of the base-column contacts:   0.10     MPa
        Friction angle at the intact rock column:       45.00  Degree
        Cohesive strength of the intact rock column:     1.11     MPa
        Unit weight of the rock columns:                26.50  KN/m^3
        Tensile strength of the rock columns:            5.50     MPa

 +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                            Computation section
 +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

                      Specifications of rock columns
 -------------------------------------------------------------------------------
  N   h(m)   h/t  Pbt(MN)  Ps(MN)   Pbs,t(MN) Pbt,s(MN) Pft(MN) Psh(MN) FAILURE
                                                                          MODES
 -------------------------------------------------------------------------------
 16  0.82  0.35  -0.035   -0.373   -0.053   -0.060
 15  1.80  0.78                                      -1.964   -40.350
 14  2.79  1.21   0.007   -0.339   -0.029    0.011
 13  3.77  1.63                                      -1.047   -41.339    S
 12  4.75  2.06   0.047   -0.305   -0.010    0.025
 11  5.74  2.48                                      -0.641   -42.233    T
 10  6.72  2.91   0.089   -0.270   -0.051    0.063
  9  7.70  3.33                                      -0.388   -43.122    A
  8  8.69  3.76   0.130   -0.236   -0.094    0.102
  7  9.67  4.19                                      -0.200   -44.016    B
  6  9.31  4.03   0.143   -0.225   -0.145    0.155
  5  7.62  3.30                                      -0.299   -43.018    L
  4  5.92  2.56   0.072   -0.284   -0.011    0.128
  3  4.23  1.83                                      -0.801   -41.436    E
  2  2.53  1.10   0.001   -0.334   -0.029    0.062
  1  0.84  0.36                                      -3.169   -39.893
 ------------------------------------------------------------------------------
 N=column number,  h:height of rock column,  Pb:inter-column force due to block
 toppling, Ps:inter-column force due to sliding, Ps,t:inter-column force due to
 sliding & toppling,Pt,s:inter-column force due to toppling & sliding, Pf:inter-
 column force due toflexural toppling, Psh:inter-column force due to shearing

 ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                               Output section
 ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

                      Stability assessment of the rock slope
                      --------------------------------------
1.PO= -3168.3 KN, hence the slope is UNSTABLE against block-flexure toppling failure

2.Safety factor of the slope against block-flexure toppling failure is:  2.6037
```

5.1 Case Study 1 (the Rock Slope Facing Tehran-Chalus Road)

The rock slope of Tehran-Chalus Road illustrated in Fig. 8 has been used as one of the practical examples. The rock mass consists of thick sandstone with thin inter-bedded shale layers. The data about the geometry of the slope and the rock mass discontinuities were gathered through site investigations and have been analyzed by using DIPS software. The results are shown in Fig. 9. As can be seen from Fig. 9, there is one dominating bedding plane plus one cross-joint set (two sets of discontinuities altogether) in the rock mass. Persistence of the bedding plane is such that it can be seen throughout the total rock slope regularly, but persistence of the cross-joint set is, approximately, equal to the thickness of sandstone layers and does not cover the whole rock mass continuously. To obtain the geomechanical properties of the rock mass, some block samples were taken from the sandstone and shale layers. The samples were transferred to the laboratory for the required testing. The shear test results obtained from the samples are shown in Fig. 10. In this slope, the inter-bedded shale layers do not show much effect on toppling failure due to their high flexibility and low thickness. However, they show a significant reduction of the friction between the sandstone and shale layers' contacts, which simplifies the sliding of the layers over each other. Therefore, the shale samples were used to determine the shear strength parameters of the bedding plane contacts, whereas the sandstone samples were used to determine the shear strength parameters of the cross-joints' surfaces, density, and tensile and compressive strengths of the rock columns. The result of the kinematic analysis of this slope is shown in Fig. 11. As can be seen, the poles of the rock mass cross-joints are in the sliding plane zone and those of the rock mass beddings are in the toppling instability zone. Since there are no continuous cross-joints in the rock mass, the slope only has the potential for block-flexure toppling failure. Then, stability

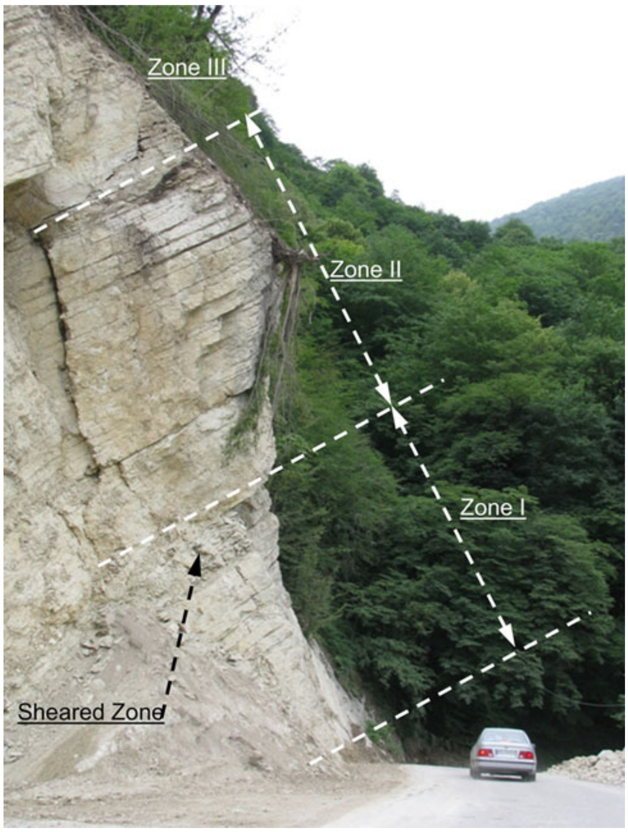


Fig. 12 Rock slope facing Galandrood mine

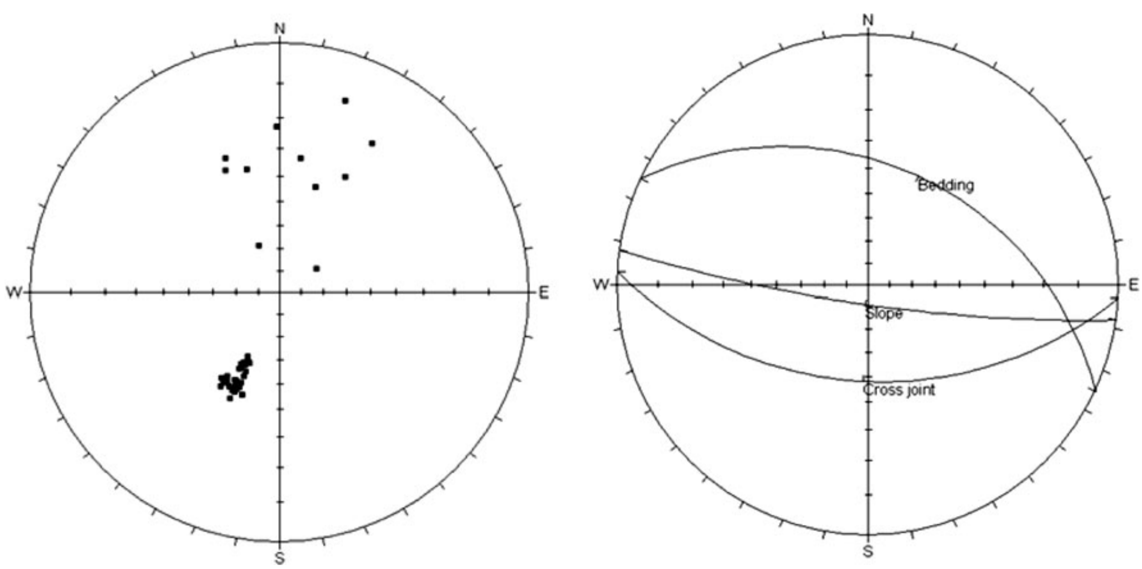


Fig. 13 Stereonet diagrams of discontinuities and face slope of Galandrood mine rock mass

Table 2 Stability analysis results of the Galandrood mine slope

```
                  Computer program title: "BFTOP"
                      Coded by: MEHDI AMINI
Purpose:  Analysis of rock slopes against block-flexure toppling failure

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                            Input section
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

   Project name:GALANDROOD       Date:12/07/2011        Analyzed by:M.AMINI

   Geometrical and geomechanical parameters of the slope and the rock mass
   ----------------------------------------------------------------------
       Inclination of the face slope:                    81.00  Degree
       Inclination of the layers:                        39.00  Degree
       Inclination of the total failure plane:           58.00  Degree
       Inclination of the slope top surface:             32.00  Degree
       Height of the slope:                              16.50       m
       Average thickness of the layers:                   0.30       m
       Friction angle at the column-column contacts:     26.00  Degree
       Friction angle at the base-column contacts:       26.00  Degree
       Cohesive strength of the base-column contacts:     0.01     MPa
       Friction angle at the intact rock column:         45.00  Degree
       Cohesive strength of the intact rock column:       1.11     MPa
       Unit weight of the rock columns:                  27.00  KN/m^3
       Tensile strength of the rock columns:              5.00     MPa

++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
                            Computation section
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

                   Specifications of rock columns
```

N	h(m)	h/t	Pbt(MN)	Ps(MN)	Pbs,t(MN)	Pbt,s(MN)	Pft(MN)	Psh(MN)	FAILURE MODES
95	0.04	0.13	-0.001	-0.004	-0.001	-0.002			
94	0.16	0.53					-0.322	-1.587	S
93	0.30	1.00	0.000	-0.002	-0.001	-0.002			
92	0.44	1.47					-0.145	-1.590	T
91	0.59	1.97	0.001	-0.001	0.000	-0.001			
90	0.73	2.43					-0.091	-1.592	A
89	0.87	2.90	0.002	0.000	0.000	0.000			
88	1.01	3.37					-0.065	-1.593	B
87	1.15	3.83	0.003	0.002	0.001	0.000			
86	1.29	4.30					-0.049	-1.595	L
85	1.43	4.77	0.004	0.003	0.001	0.000			
84	1.57	5.23					-0.038	-1.597	E
83	1.71	5.70	0.005	0.005	0.002	0.001			
82	1.85	6.17					-0.030	-1.599	
81	1.99	6.63	0.005	0.006	0.002	0.001			
80	2.13	7.10					-0.025	-1.600	
79	2.27	7.57	0.006	0.007	0.003	0.002			
78	2.41	8.03					-0.020	-1.601	S
77	2.55	8.50	0.007	0.009	0.003	0.002			
76	2.69	8.97					-0.015	-1.602	T
75	2.83	9.43	0.008	0.010	0.004	0.002			
74	2.97	9.90					-0.011	-1.603	A
73	3.11	10.37	0.009	0.012	0.005	0.003			
72	3.25	10.83					-0.008	-1.605	B
71	3.39	11.30	0.010	0.013	0.005	0.003			
70	3.53	11.77					-0.004	-1.606	L
69	3.67	12.23	0.011	0.014	0.006	0.003			
68	3.81	12.70					-0.001	-1.607	E
67	3.95	13.17	0.012	0.016	0.006	0.004			
66	4.09	13.63					0.002	-1.608	
65	4.23	14.10	0.014	0.018	0.008	0.005			
64	4.37	14.57					0.005	-1.608	
63	4.51	15.03	0.018	0.022	0.012	0.009			
62	4.65	15.50					0.009	-1.607	S
61	4.79	15.97	0.022	0.027	0.016	0.012			
60	4.93	16.43					0.012	-1.605	T
59	5.07	16.90	0.027	0.031	0.020	0.016			
58	5.21	17.37					0.016	-1.604	A
57	5.35	17.83	0.031	0.035	0.024	0.020			
56	5.49	18.30					0.020	-1.602	B
55	5.63	18.77	0.036	0.039	0.028	0.023			
54	5.77	19.23					0.023	-1.601	L
53	5.91	19.70	0.040	0.043	0.032	0.027			
52	6.05	20.17					0.027	-1.599	E

Table 2 continued

51	6.19	20.63	0.044	0.047	0.036	0.031			
50	6.33	21.10					0.030	-1.598	
49	6.47	21.57	0.048	0.051	0.039	0.034			
48	6.47	21.57					0.033	-1.595	
47	6.33	21.10	0.052	0.053	0.045	0.040			
46	6.20	20.67					0.033	-1.591	S
45	6.06	20.20	0.051	0.052	0.045	0.040			
44	5.93	19.77					0.031	-1.590	T
43	5.79	19.30	0.049	0.049	0.042	0.038			
42	5.65	18.83					0.028	-1.590	A
41	5.52	18.40	0.045	0.045	0.038	0.034			
40	5.38	17.93					0.025	-1.591	B
39	5.24	17.47	0.041	0.041	0.035	0.031			
38	5.11	17.03					0.021	-1.592	L
37	4.97	16.57	0.036	0.037	0.031	0.027			
36	4.83	16.10					0.018	-1.594	E
35	4.70	15.67	0.032	0.033	0.026	0.023			
34	4.56	15.20					0.014	-1.595	
33	4.43	14.77	0.027	0.029	0.022	0.019			
32	4.29	14.30					0.010	-1.597	
31	4.15	13.83	0.022	0.024	0.017	0.014			
30	4.02	13.40					0.006	-1.599	S
29	3.88	12.93	0.017	0.020	0.013	0.010			
28	3.74	12.47					0.001	-1.601	T
27	3.61	12.03	0.012	0.015	0.008	0.005			
26	3.47	11.57					-0.003	-1.603	A
25	3.33	11.10	0.010	0.013	0.006	0.003			
24	3.20	10.67					-0.007	-1.603	B
23	3.06	10.20	0.009	0.011	0.005	0.003			
22	2.93	9.77					-0.011	-1.602	L
21	2.79	9.30	0.008	0.010	0.005	0.003			
20	2.65	8.83					-0.015	-1.601	E
19	2.52	8.40	0.007	0.009	0.004	0.002			
18	2.38	7.93					-0.019	-1.599	
17	2.24	7.47	0.006	0.007	0.004	0.002			
16	2.11	7.03					-0.025	-1.598	
15	1.97	6.57	0.005	0.006	0.003	0.002			
14	1.83	6.10					-0.031	-1.597	S
13	1.70	5.67	0.005	0.005	0.003	0.001			
12	1.56	5.20					-0.037	-1.596	T
11	1.43	4.77	0.004	0.003	0.002	0.001			
10	1.29	4.30					-0.048	-1.594	A
9	1.15	3.83	0.003	0.002	0.001	0.001			
8	1.02	3.40					-0.063	-1.593	B
7	0.88	2.93	0.002	0.000	0.001	0.000			
6	0.74	2.47					-0.088	-1.591	L
5	0.61	2.03	0.001	-0.001	0.000	0.000			
4	0.47	1.57					-0.136	-1.589	E
3	0.33	1.10	0.000	-0.002	0.000	0.000			
2	0.20	0.67					-0.274	-1.587	
1	0.06	0.20	-0.001	-0.004	-0.001	-0.002			

```
-----------------------------------------------------------------------------
N=column number,  h:height of rock column,  Pb:inter-column force due to block
toppling, Ps:inter-column force due to sliding, Ps,t:inter-column force due to
sliding & toppling,Pt,s:inter-column force due to toppling & sliding, Pf:inter-
column force due toflexural toppling, Psh:inter-column force due to shearing

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                              Output section
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                   Stability assessment of the rock slope
                   --------------------------------------
1.P0= -0.112 KN, hence the slope is STABLE against block-flexure toppling failure

2.SAFETY FACTOR of the slope against block-flexure toppling failure is:  1.3802
```

of the slope against the failure has been studied by means of the proposed analytical method using the corresponding computer code. The results shown in Table 1 indicate that the slope is stable against block-flexure toppling failure with $F_s = 2.60$. It is almost 30 years since the slope was excavated and there has been no signs of any instability. The in situ observation confirms the results obtained by the analytical method proposed in this paper.

5.2 Case Study 2 (the Rock Slope Facing Galandrood Mine)

The rock slope facing Galandrood mine illustrated in Fig. 12 has been used as the other practical example. As can be seen from the photography, an obvious local instability has been exhibited in the rock slope in zone II; however, the rock mass as a whole is stable. The

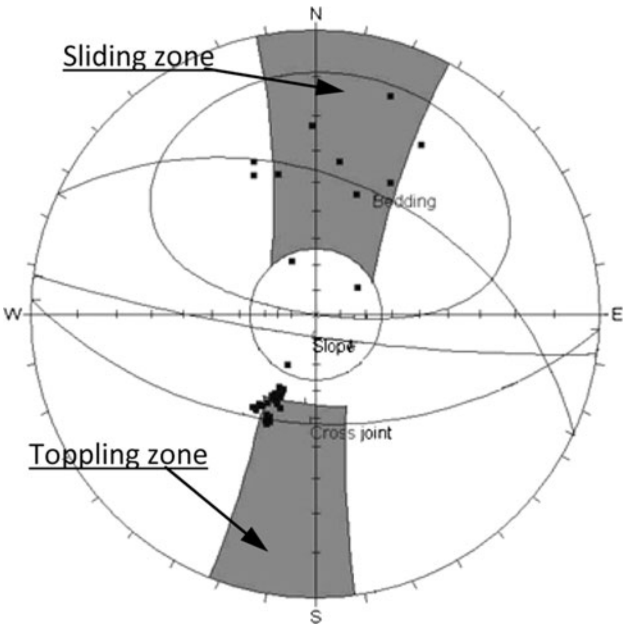

Fig. 14 Kinematic stability analysis of the Galandrood mine slope

geometrical parameters of the rock mass discontinuities and kinematic analyses of this slope are shown in Fig. 13. It seems from these figures that the dominating failure in this slope is flexural toppling, though there are a few cross-joints in the rock mass too. These joints have crossed the rock bedding planes in such a way that some rock columns exhibit potential for blocky toppling failure. The stability of this slope against pure flexural toppling failure was investigated and the results have already been published (Majdi and Amini 2011). The analysis has yielded the overall factor of safety of this slope against flexural toppling failure as 1.18–4.36. In the current paper, this case study has been further investigated and reassessed by using the proposed analytical method against block-flexure toppling failure. The results illustrated in Table 2 confirm that the slope is stable against a total block-flexure toppling failure. The above prediction is comparable with the actual observation as well. It should be borne in mind that some of the existing rock columns in zone II (Fig. 12) as a block have been sheared and show a clockwise rotation downwards movement due to the vertical movement that had taken place in the immediate underlain rock column located at the top of zone I. The aforementioned vertical movement of the latter rock column occurred due to the compressibility of the underneath sheared zone shown in the same figure (Fig. 14).

6 Conclusions

Perfect toppling failures (pure blocky or flexural toppling) are rare in nature because rocks are brittle and their discontinuities are irregular; therefore, most real toppling instabilities are of the block-flexure type. In this paper, block-flexure toppling failure has been studied. The mechanism of such failure has been investigated. Then, an appropriate theoretical model along with an analytical solution has been proposed to analyze and evaluate such instabilities. Since the rock slope stability analysis against the block-flexure toppling failure is a step-by-step procedure, it will be time consuming if the calculations are carried out manually. Thus, based on the method proposed in this paper, a FORTRAN computer program was developed to simplify the stability analysis of rock slopes against the failure. This code receives the rock slope parameters from the user and predicts its stability, along with the corresponding factor of safety against the failure. In addition, two case studies have been used for the practical verification of the proposed approach and the corresponding computer code. The results revealed that the proposed model can be used for the analysis of such instabilities.

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