rocscience

RocPlane

Factor of Safety Calculations – Planar Failures

Theory Manual

Table of Contents

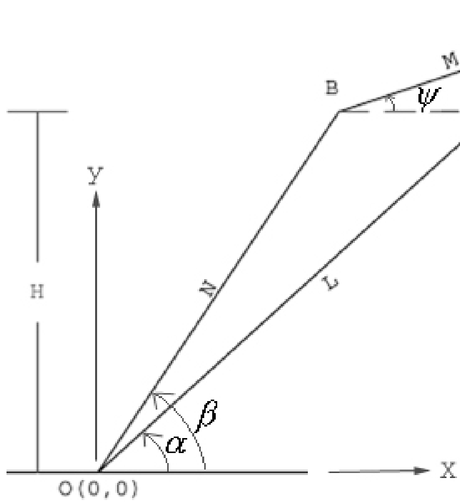
Introduction

This paper documents the calculations used in *RocPlane* to determine the factor of safety for planar failures formed in slopes. This involves the following series of steps:

1. Determine the plane geometry using trigonometry
2. Determine all of the individual forces acting on the failure plane, and then calculate the resultant active and passive force vectors for the failure plane
3. Determine the normal forces on each wedge
4. Compute the resisting forces due to joint shear strength
5. Calculate the safety factor

1. Failure Plane Geometry

1.1. No Tension Crack



Known Parameters:

H	is the slope height
β	is the slope dip
α	is the failure plane dip
ψ	is the upper bench dip
O	is the origin (0,0)
γ	is the rock unit weight

Unknown Parameters:

B	is the intersection point, slope & bench
C	is the intersection point, failure plane & bench
N	is the slope length, origin to B
M	is the bench length, B to C
L	is the failure plane length, origin to C
A	is the wedge area
W	is the wedge weight

1.1.1. Flow Chart

1. Solve for N (eq. (1))
2. Solve for B (eq. (2))
3. Solve for L (eq. (6))
4. Solve for M (eq. (7))
5. Solve for C (eq. (8))
6. Solve for A (eq. (9))
7. Solve for W (eq. (10))

1.1.2. Points and Lengths Calculation

$$N = \frac{H}{\sin\beta} \tag{1}$$

$$B = \{N\cos\beta\,, H\} = \{H\cot\beta\,, H\} \tag{2}$$

To solve for distances L & M, use vector addition:

$$\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$$

$$\begin{Bmatrix} H\cot\beta \\ H \end{Bmatrix} + \begin{Bmatrix} M\cos\psi \\ M\sin\psi \end{Bmatrix} = \begin{Bmatrix} L\cos\alpha \\ L\sin\alpha \end{Bmatrix}$$

This gives two equations:

$$H\cot\beta + M\cos\psi = L\cos\alpha \tag{3}$$

$$H + M\sin\psi = L\sin\alpha \tag{4}$$

From equations (4):

$$M = \frac{L\sin\alpha - H}{\sin\psi} \tag{5}$$

Substituting (5) into (3):

$$H\cot\beta + (L\sin\alpha - H)\cot\psi = L\cos\alpha$$

$$H(\cot\beta - \cot\psi) = L(\cos\alpha - \sin\alpha\cot\psi)$$

$$L = \frac{H(1 - \cot\beta\tan\psi)}{\sin\alpha - \cos\alpha\tan\psi} \tag{6}$$

From equation (3):

$$M = \frac{L\cos\alpha - H\cot\beta}{\cos\psi} \tag{7}$$

To calculate L and M, use equations (6) & (7). Do not use equation (5) because ψ = 0 is common & M is irresolvable using (5).

$$C = \{L\cos\alpha\,, L\sin\alpha\} \tag{8}$$

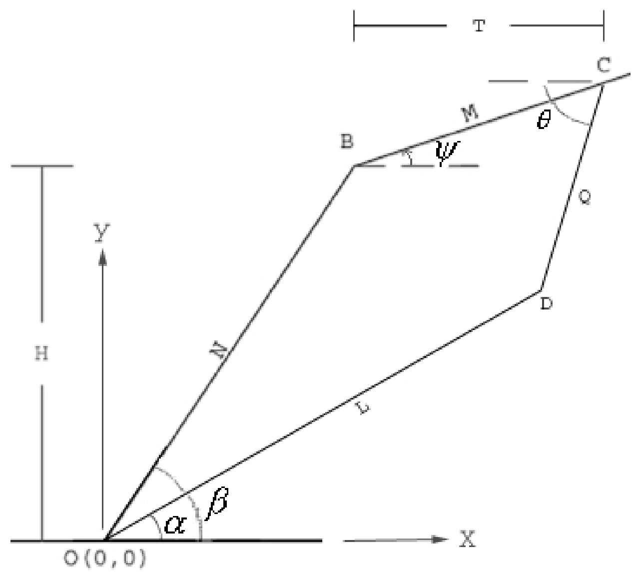
1.1.3. Area Calculation

$$A = \frac{1}{2}\|B \times C\| \tag{9}$$
$$A = \frac{1}{2}\left\|B_x C_y - B_y C_x\right\|$$

1.1.4. Weight Calculation

$$W = A \cdot \gamma \tag{10}$$

1.2. Tension Crack



Known Parameters:		Unknown Parameters:	
H	is the slope height	B	is the slope/bench intersection point
β	slope dip	C	is the tension crack/bench intersection point
α	failure plane dip	D	is the failure plane/tension crack intersection point
ψ	upper bench dip	N	is the slope length, O to B
T	tension crack distance	M	is the bench length, B to C
θ	tension crack dip	L	is the failure plane length, O to D
O	origin (0,0)	Q	is the tension crack length, D to C
γ	rock unit weight	A	is the wedge area
		W	is the wedge weight

1.2.1. Flow Chart

1. Solve for N (eq. (1))
2. Solve for B (eq. (2))
3. Solve for C (eq. (11))
4. Solve for M (eq. (12))
5. Solve for Q (eq. (17))
6. Solve for L (eq. (15))
7. Solve for D (eq. (14))
8. Solve for A (eq. (18))
9. Solve for W (eq. (10))

1.2.2. Points and Lengths Calculation

As in the no tension crack case:

$$N = \frac{H}{\sin\beta}$$

$$B = \{H\cot\beta\,, H\}$$

Now,

$$C = B + \{T, T\tan\psi\} \tag{11}$$

$$M = \frac{T}{\cos\psi} \tag{12}$$

Let's solve for D,Q,L:

$$D = C - \{Q\cos\theta\,, Q\sin\theta\} \tag{13}$$

$$D = \{L\cos\alpha\,, L\sin\alpha\} \tag{14}$$

Equate equations (13) & (14):

$$\{C_x, C_y\} - \{Q\cos\theta\,, Q\sin\theta\} = \{L\cos\alpha\,, L\sin\alpha\}$$

or

$$L = \frac{C_x - Q\cos\theta}{\cos\alpha} \tag{15}$$

and

$$L = \frac{C_y - Q\sin\theta}{\sin\alpha} \tag{16}$$

Equate equations (15) & (16) and solve for Q:

$$Q = \frac{C_y \cot\alpha - C_x}{\sin\theta \cot\alpha - \cos\theta} \tag{17}$$

1.2.3. Area Calculation

$$A = \frac{1}{2}\|B \times D\| + \frac{1}{2}\|(D - B) \times (C - B)\| \tag{18}$$

$$A = \frac{1}{2}\left\|B_x D_y - B_y D_x\right\| + \frac{1}{2}\left\|(D_x - B_x)(C_y - B_y) - (D_y - B_y)(C_x - B_x)\right\|$$

1.2.4. Weight Calculation

$$W = A \cdot \gamma$$

2. Failure Plane Forces

2.1. Water Forces

In *RocPlane*, water pressure can be applied as Plane Water Pressure (acting on the failure plane and tension crack) and/or Ponded Water Pressure (acting on the slopes).

2.1.1. Ponded Water Force

In *RocPlane* it is assumed that the ponded water surface is a horizontal planar surface at some specified depth above the base of the slope. *RocPlane* allows definition of a ponded water depth greater than or equal to zero.

The water pressure is assumed to be zero along the ponded water surface. The magnitude of the pressure is determined based on the vertical distance from the ponded surface. The maximum water pressure has a value of $\gamma_w H_w$. In this case, H_w is the vertical distance between the wedge toe and the ponded water surface. The pressure and force along the slope face is computed as:

Case 1: Slope Face Partially Wetted by Ponded Water ($0 \leq H_w < B_y$)

$$P_1 = \gamma_w H_w$$

$$P_2 = 0$$

$$L_{w_1} = \frac{H_w}{\sin\beta}$$

$$U_x^{ponded} = \frac{P_1 + P_2}{2} L_{w_1} \sin\beta$$

$$U_y^{ponded} = -\frac{P_1 + P_2}{2} L_{w_1} \cos\beta$$

Case 2: Upper Face Partially Wetted by Ponded Water ($B_y \leq H_w < C_y$)

$$P_1 = \gamma_w H_w$$

$$P_2 = \gamma_w \left(H_w - B_y\right)$$

$$P_3 = 0$$

$$L_{w_1} = \frac{B_y}{\sin\beta}$$

$$L_{w_2} = \frac{H_w - B_y}{\sin\psi}$$

$$U_x^{ponded} = \frac{P_1 + P_2}{2} L_{w_1} \sin\beta + \frac{P_2 + P_3}{2} L_{w_2} \sin\psi$$

$$U_y^{ponded} = -\frac{P_1 + P_2}{2} L_{w_1} \cos\beta - \frac{P_2 + P_3}{2} L_{w_2} \cos\psi$$

Case 3: Upper Face Fully Submerged by Ponded Water ($H_w \geq C_y$)

$$P_1 = \gamma_w H_w$$
$$P_2 = \gamma_w (C_y - B_y)$$
$$P_3 = \gamma_w (H_w - C_y)$$

$$L_{w_1} = \frac{B_y}{\sin\beta}$$
$$L_{w_2} = \frac{C_y - B_y}{\sin\psi}$$

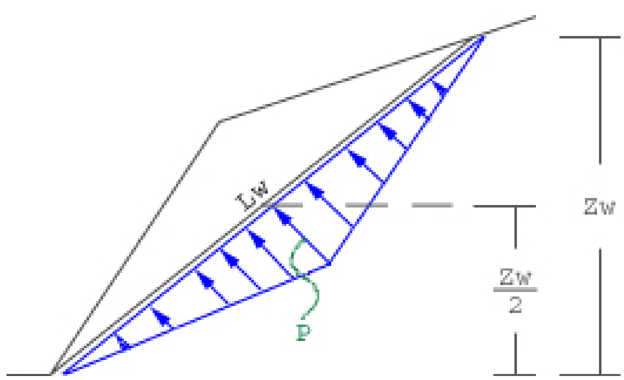
$$U_x^{ponded} = \frac{P_1 + P_2}{2} L_{w_1} \sin\beta + \frac{P_2 + P_3}{2} L_{w_2} \sin\psi$$
$$U_y^{ponded} = -\frac{P_1 + P_2}{2} L_{w_1} \cos\beta - \frac{P_2 + P_3}{2} L_{w_2} \cos\psi$$

Where:

P_1, P_2, P_3	are the ponded water pressures along the slope
γ_w	is the unit weight of ponded water
H_w	is the depth of ponded water above the toe
L_w	is the wetted length
U^{ponded}	is the slope plane ponded water force
B_y	is the vertical coordinate of the intersection point, slope & bench
C_y	is the vertical coordinate of the intersection point, failure plane & bench

2.1.2. Plane Water Force – No Tension Crack

Case 1: Maximum Pressure Mid Height



$$0 \leq Z_w \leq L \sin\alpha$$
$$L_w = \frac{Zw}{sin\,\alpha}$$
$$P = \frac{1}{2} Z_w \gamma_w$$

$$U = \frac{1}{2} P \cdot L_w = \frac{1}{2}\left(\frac{1}{2} Z_w \cdot \gamma_w\right)\left(\frac{Z_w}{\sin\alpha}\right) = \frac{{Z_w}^2 \cdot \gamma_w}{4\,sin\,\alpha} \quad (19)$$

Where:

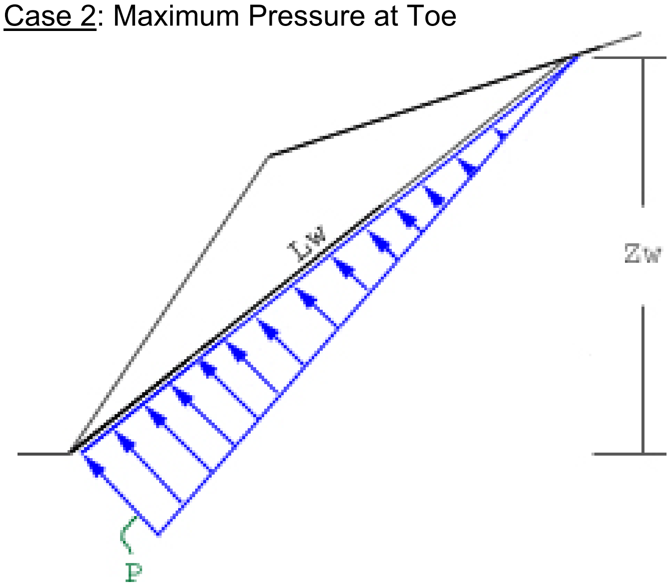
Z_w is the height of water on the failure plane

L_w is the wetted length

P is the maximum water pressure

U is the failure plane water force

Case 2: Maximum Pressure at Toe



$$L_w = \frac{Z_w}{\sin\alpha}$$

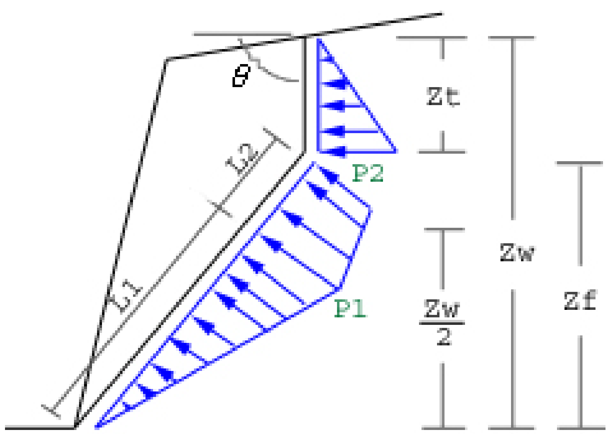
$$P = \gamma Z_w$$

$$U = \frac{1}{2}P \cdot L_w = \frac{1}{2}(\gamma \cdot Z_w)\left(\frac{Z_w}{\sin\alpha}\right) \quad (20)$$

$$U = \frac{{Z_w}^2 \cdot \gamma_w}{2\sin\alpha}$$

2.1.3. Plane Water Force – Tension Crack

Case 1: Maximum Pressure Mid Height



$$Z_t = Z_w - Z_f$$

$$Z_f = D_y = L\sin\alpha$$

Where:

Z_t is the height of water on the tension crack

Z_f is the height of water on the failure plane

L is the failure plane length

U is the failure plane water force

V is the tension crack water force

Type A: If $Z_w \leq Z_f$

$$U = \frac{{Z_w}^2 \cdot \gamma_w}{4 \sin \alpha} \tag{21}$$

$$V = 0$$

Type B: If $Z_w > Z_f$ and $\frac{Z_w}{2} < Z_f$

$$L_2 = L - L_1 \tag{22}$$

$$L_1 = \frac{Z_w}{2 \sin \alpha}$$

$$L_2 = L - L_1$$

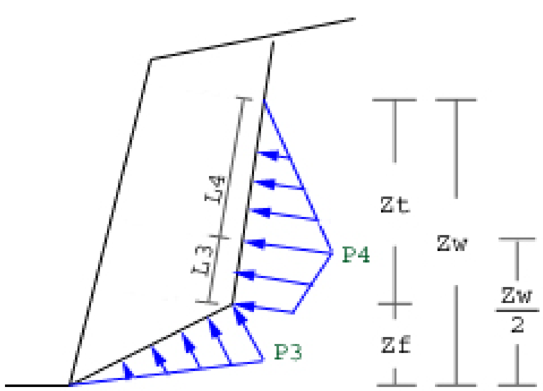
$$P_1 = \frac{1}{2} Z_w \cdot \gamma_w$$

$$P_2 = \gamma_w \cdot Z_t$$

$$U = \frac{1}{2} P_1 \cdot L_1 + \frac{1}{2}(P_1 + P_2) L_2$$

$$V = \frac{{Z_t}^2 \cdot \gamma_w}{2 \sin \theta}$$

Type C: If $Z_w > Z_f$ and $\frac{Z_w}{2} \geq Z_f$



$$P_3 = \gamma \cdot Z_f \tag{23}$$

$$P_4 = \frac{1}{2} \gamma \cdot Z_w$$

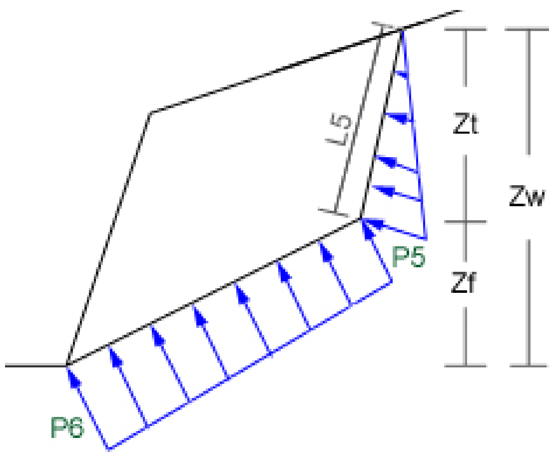
$$L_3 = \frac{\left(\frac{Z_w}{2} - Z_f\right)}{\sin \theta}$$

$$L_4 = \frac{Z_w}{2 \sin \theta}$$

$$U = \frac{1}{2} L \cdot P_3$$

$$V = \frac{1}{2}(P_3 + P_4)L_3 + \frac{1}{2}P_4 L_4$$

Case 2: Maximum Pressure at Toe



$$P_5 = \gamma \cdot Z_t \quad (24)$$

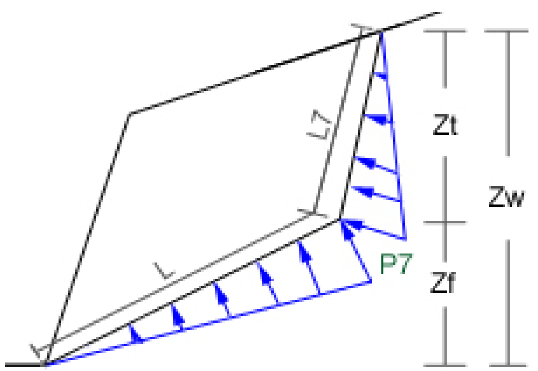
$$P_6 = \gamma \cdot Z_w$$

$$L_5 = \frac{Z_t}{\sin\theta}$$

$$U = \frac{1}{2}(P_5 + P_6)L$$

$$V = \frac{1}{2}P_5 \cdot L_5$$

Case 3: Maximum Pressure at Base of Tension Crack



$$Z_t = Z_w - Z_f$$

$$P_7 = \gamma \cdot Z_t \quad (25)$$

$$L_7 = \frac{Z_t}{\sin\theta}$$

$$U = \frac{1}{2}P_7 \cdot L$$

$$V = \frac{1}{2}P_7 \cdot L_7$$

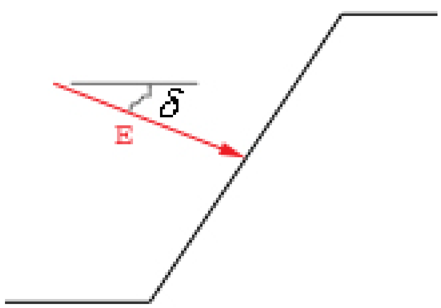
Note: The above applies to cases where either no ponded water exists or ponded water exists, but Slope Surface Type is set to Impervious. The plane water pressure is computed independent of the ponded water surface.

In *RocPlane*, when both ponded water and plane water exists and the Slope Surface Type is set to Pervious, the water table is defined by a combination of water surface planes consisting of the plane water surfaces and the ponded water surface. The plane water surface is defined by a plane parallel to the upper face and a plane coinciding with the slope face.

Where the elevations of the wetted plane extents are below the ponded water elevation, water pressure magnitudes are computed based on the vertical distance from the ponded water elevation. Where the elevations of the wetted joint extents are above the ponded water elevation, water pressure magnitudes are computed based on the vertical distance from the plane water surfaces. The plane water pressure is computed wherever the depth of water does not vary linearly, at:

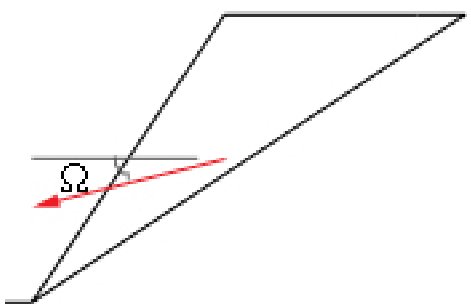
- At the toe of the wedge
- Directly below where the ponded water surface intersects the slope or upper face
- Directly below the slope crest
- At the top of the wedge
- At the base of the tension crack (if applicable)

2.2. External Force



$$E_x = E \cdot \cos\delta$$
$$E_y = E \cdot \sin\delta$$

2.3. Seismic Force



$$S = W_y \cdot \alpha_s$$

$$W_y = -W$$

$$S_x = S \cdot \cos\Omega$$

$$S_y = S \cdot \sin\Omega$$

Where:

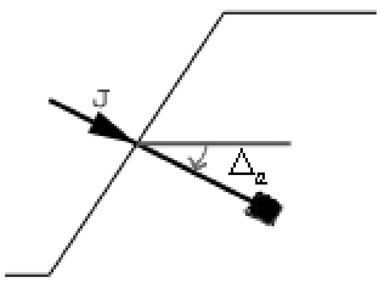
S is the seismic force

α_s is the seismic coefficient

W is the weight of wedge

W_y is the directional weight component

2.4. Bolt Force



Active Bolt Force:

$$J_x = J \cdot \cos \Delta_a$$

$$J_y = -J \cdot \sin \Delta_a$$

Passive Bolt Force:

$$K_x = K \cdot \cos \Delta_p$$

$$K_y = -K \cdot \sin \Delta_p$$

Where:

J is the active bolt force

Δ_a is the active bolt angle

K is the passive bolt force

Δ_p is the passive bolt angle

2.5. Active Water Force (Tension Crack Water Force)

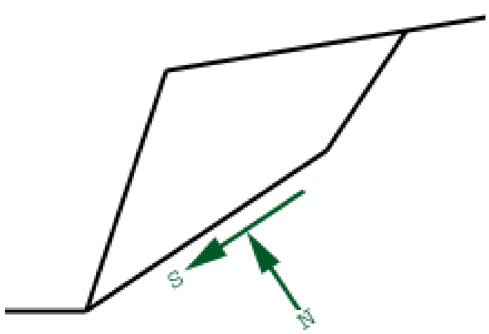
$$V_x = -V \cdot \sin \theta$$

$$V_y = V \cdot \cos \theta$$

Where:

V is the tension crack water force

2.6. Normal Force and Shear Force on Failure Plane



$$W_y = -W$$

Where:

W is the wedge weight

Active Forces Only:

$$\sum F_y \uparrow^{+} \qquad F_y = W_y + E_y + S_y + J_y + V_y + U_y^{ponded} \tag{26}$$

$$F_y = -A \cdot \gamma - E \cdot \sin\delta - S \cdot \sin\Omega - J \cdot \sin\Delta_a + V \cdot \cos\theta + U_y^{ponded}$$

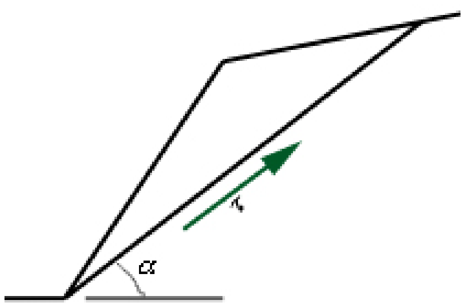
$$\sum F_x \rightarrow^{+} \qquad F_x = E_x + S_x + J_x + V_x + U_x^{ponded} \tag{27}$$

$$F_x = E \cdot cos\,\delta - S \cdot cos\,\Omega + J \cdot cos\,\Delta_a - V \cdot sin\,\theta + U_x^{ponded}$$

$$N = -\left(F_y + K_y\right)\cos\alpha + (F_x + K_x)\sin\alpha - U \tag{28}$$

$$S = -F_y \cdot \sin\alpha - F_x \cdot \cos\alpha \tag{29}$$

2.7. Shear Strength on Failure Plane



Strength Criterion: Mohr Coulomb

$$\tau = c \cdot L + N \cdot \tan\phi + \underbrace{K_x \cdot \cos\alpha + K_y \cdot \sin\alpha}_{passive\ bolt} \tag{30}$$

Where:

c is the cohesion

N is the normal force

ϕ is the friction angle

L is the length of failure surface

3. Factor of Safety

$$\mathrm{FS} = \frac{resisting\ forces}{driving\ forces}$$

$$\mathrm{FS} = \frac{shear\ strength}{shear\ force} = \frac{\tau}{S}$$