Dynamic Analysis

Introduction

In RS3, you can perform true dynamic analysis of geotechnical structures due to earthquake loading or other dynamically applied loading. This document outlines the background theories and options in RS3 to carry out dynamic analysis.

Rayleigh Damping

RS3 allows the user to introduce Rayleigh damping to the model. With this type of damping, the damping matrix that relates the damping force and velocity of the system is expressed solely in terms of the stiffness and mass matrix of the system. In this way the damping becomes proportional to the mass and stiffness of the system.

$$C = \alpha_M \times M + \beta_K \times K$$

In a multiple degree of freedom model the system has a large number of natural frequencies but due to the nature of the problem a constant level of damping for all frequencies is not possible. Rayleigh damping allows the user to define the damping ratio for two frequencies and the remainder of the frequencies are defined by a curve similar to curve below. Typically the frequencies between the two that were defined have a damping ratio lower than the specified damping ratios, and frequencies outside this range are damped more heavily.
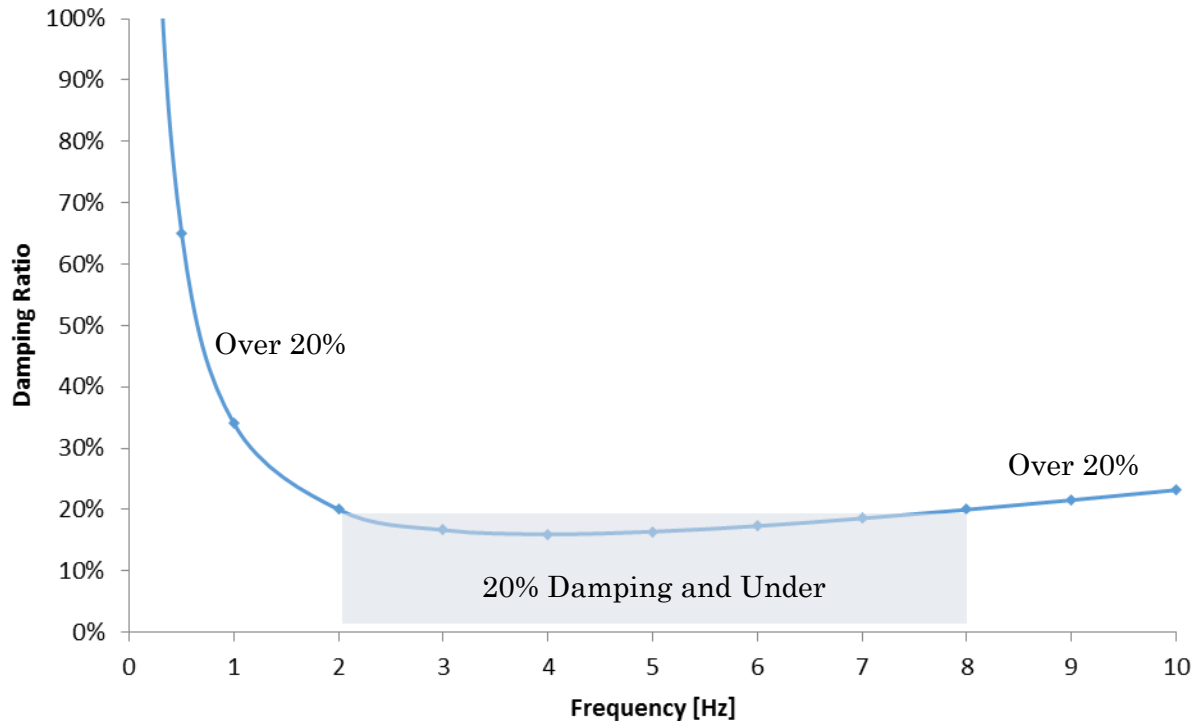
Over 20%

Over 20%

20% Damping and Under

Damping Ratio

Frequency [Hz]

Figure 1 - Damping Ratio Plot, 20% damping at 2 and 8 Hz

The user therefore may specify the two frequencies and the damping ratio they are to have. The program will then calculate the alpha and beta values. Alternatively the user may specify the alpha and beta values explicitly. Setting alpha and beta to zero will produce a system that is undamped resulting in the transient response of the system to never dissipate.

Time History

This section allows the user to control the time step used in the dynamic simulation.

Auto

Under the auto option, *RS3* determines the most appropriate time step given the duration of the dynamic stage and the dynamic load present in the model. The engine will typically try to provide at least 200 time steps in the simulation in order to generate response curves with an appropriate amount of definition. If any of the dynamic loads are more defined than this default than the program will utilize the time step used in the dynamic loads.

Time Step

With this option the user may specify exactly the default length of each time step.

Num Time Steps

Alternatively, the number of time steps in each dynamic stage can be specified. The time step for each dynamic stage would be calculated then from taking the duration and dividing it by the specified number of time steps.

Integration Methods

The dynamic integration scheme utilized in *RS3* is based on the N.M. Newmark family of time-stepping methods which are characterized by the following equations, where *u*, *v* and *a* represent displacement, velocity and acceleration respectively.

$$v_{i+1} = v_i + [(1 - \beta)\Delta t]a_i + (\beta \Delta t)a_{i+1}$$

$$u_{i+1} = u_i + (\Delta t)v_i + \left[\left(\frac{1}{2} - \gamma\right)(\Delta t)^2\right]a_i + \left[\gamma(\Delta t)^2\right]a_{i+1}$$

The beta and gamma parameters define how the acceleration over any given time step varies and influence greatly the stability and accuracy of the algorithm. The table below are standard values that are typically used for this integration scheme. It is recommended to use the average *acceleration configuration* since it results in an implicit scheme that is unconditionally stable. The other schemes may result in an unstable simulation.

Scheme	γ	β
Average Acceleration	½	¼
Linear Acceleration	½	⅙
Numerically Damped	0.6	0.3025

Dynamic Boundary Conditions

If <u>Dynamic Analysis</u> activated in the <u>Project Settings</u>, then the dynamic boundary conditions are specified with the **Define Dynamic Boundary Conditions** option in the **Dynamic** menu.

The following Dynamic Boundary Condition Types can be chosen:

- None (i.e. remove boundary condition)
- Absorb
- Transmit
- Damper (enter damper values in the X and Y directions)
- Nodal Mass (enter a mass value)
- Hydro Mass

Absorb and Transmit Boundary Conditions

The absorb and transmit boundary conditions provide the line segment with what is sometimes called a Lysmer-Kuhlemeyer dashpot boundary. It is an artificial boundary condition that attempts to reproduce the infinite boundary behavior of the soil medium. That is to say that the absorb and transmit boundary absorb incoming shear and pressure waves as if the model was not actually bounded.

The assumption of these boundaries is that the waves present in the system will propagate according to the soil material's shear and pressure wave velocities. The boundary therefore is constructed from three dampers at the external boundary, one perpendicular and the others tangential to the boundary orientation, whose damping coefficient is proportional to the wave velocities. The wave velocities are calculated using the following equations.

$$V_S = \sqrt{\frac{E}{2(1+v)\rho}}$$

$$V_P = \sqrt{\frac{E(1+v)}{2(1-v)(1-2v)\rho}}$$

Vs and Vp are the shear wave and pressure wave velocities respectively. Obtaining these velocities the damping coefficients can be determined.

$$C_P = \rho \cdot l_0 \cdot V_P$$

$$C_T = \rho \cdot l_0 \cdot V_S$$

The subscripts P and T signify perpendicular and tangential directions. ρ is the soil mass density and l_0 is the length of external boundary that is attributed to that absorbing boundary element.
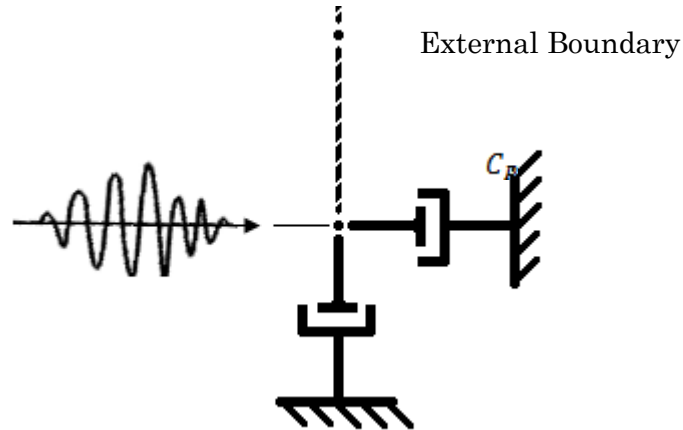
Since the dashpot damper's coefficient is solely dependent on the material properties of the solid element the boundary is attached too, these boundaries require no input values. If an absorb or transmit boundary is applied on a plane segment that borders elements with different material properties, an average value for modulus and density will be used in the damping coefficient calculation.

The dampers that are created in the absorb boundary are attached to the node on the external boundary and to a rigid base. Transmit boundaries on the other hand are attached to a restrained external virtual node rather than being rigid. This is done so that if applied motion is prescribed to an external boundary with a transmit boundary, that applied motion is provided to the external virtual node. In this way the transmit boundary allows the input wave motion to enter the soil system while absorbing shear and pressure waves that would be leaving the soil domain.
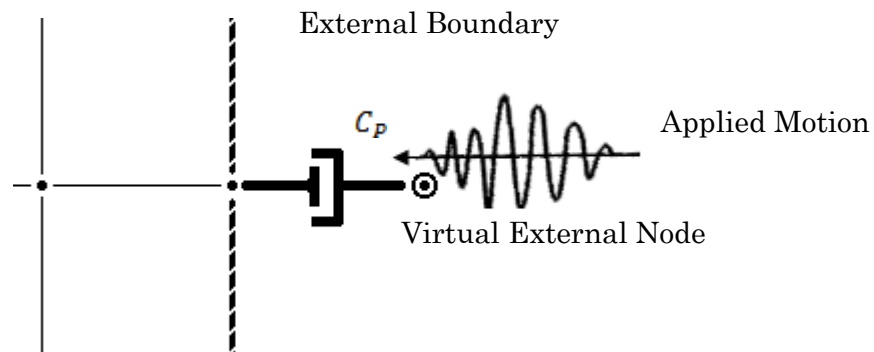
Damper Boundary Condition

Should the user wish to provide dashpot dampers in the model with values that differ from the absorb boundaries they may do so using the damper boundary. The x, y and z direction damping coefficients may be inputted individually and the boundary need not be applied on the entire face of a geometry.

Nodal Mass

Typically the mass of the system is being contributed solely from the solid elements in the model. The mass at the nodes is derived from the density of the material attributed to an element and from the element's area. Nodal mass elements allow extra mass to be introduced into the model. The user specifies the mass and then selects which nodes will receive the additional mass.

NOTE: dynamic boundary conditions are actually always applied to nodes of the mesh, regardless of the type of selected boundary (vertex, edge or face).

Dynamic Load Types

The load types available in RS3 can be divided into external force loads and prescribed motion loads. The force type is applied to the model similar to static line loads and are essentially external forces that can vary over time and are applied at nodes.

Displacement, velocity and acceleration loads are prescribed motion loads because they define the motion nodes are to have during the dynamic simulation. In *RS3*, nodes with a dynamic load that prescribes motions are restrained in the applicable direction and they are translated the necessary displacement amount as dictated by the loading function. It should be noted that applying these dynamic loads therefore restrain portions of the model and thus increase the overall stiffness of the system.

If the dynamic load type is velocity or acceleration, the inputted load histories need to be integrated in order to obtain the displacement history that will be applied to the restrained nodes. Since the loading function is always discrete, numerical integration is performed using the trapezoid rule.

Prescribed motion loads that are defined in only one direction, either X, Y or Z, is restrained in the direction the load is defined and free to move in the other direction.

Drift Correction

If an acceleration is provided as prescribed motion, the time history will be integrated twice in order to obtain the displacement that will be applied to the restrained node. The double integration however is sometimes problematic because typically the acceleration history does not contain equal area below and above the horizontal axis. This inequality will result in a velocity history that has nonzero residual constant velocity. This residual velocity will in turn result in a steadily increasing displacement history. This phenomenon is displayed visually in the graph below that overlays normalized acceleration, velocity and displacement histories.
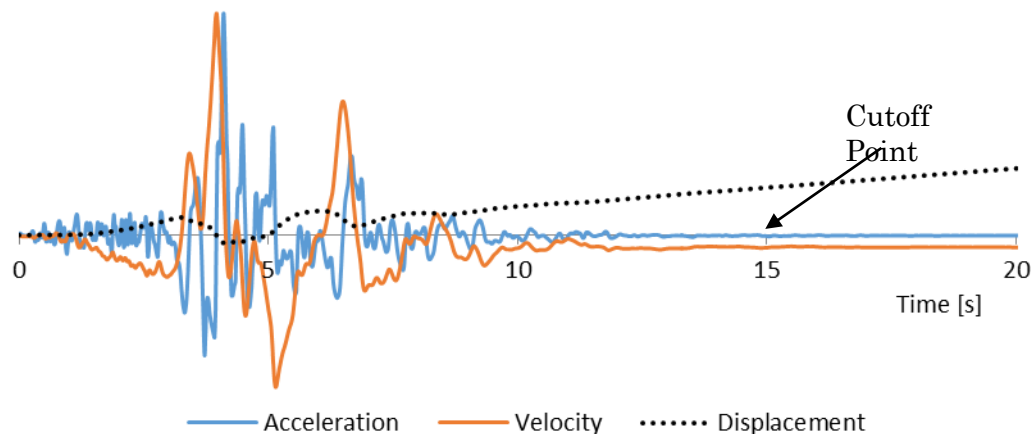
A number of procedures exist to correct this problem. One approach is based on offsetting the velocity by a constant value so that it may terminate on a value of zero. This offset is essentially the constant of integration that is ignored when calculating the integral of the acceleration numerically.

$$\int a(t)dt = v(t) + C$$

The method implemented in RS3 begins by determining the cutoff time where acceleration history stops undulating and remains close to zero. The acceleration is integrated twice to determine the displacement history. The velocity offset is taken to be the negative value of the displacement at the cutoff time divided by the cutoff time.

$$C = -\frac{d(t_{cutoff})}{t_{cutoff}}$$

The velocity history then is modified by subtracting this offset to all data entries and velocity beyond the cutoff point is taken to be zero. Integrating the modified velocity produces a displacement history that starts and ends at zero and retrains the general displacement shape.

.... Displacement   ——— Corrected Displacement
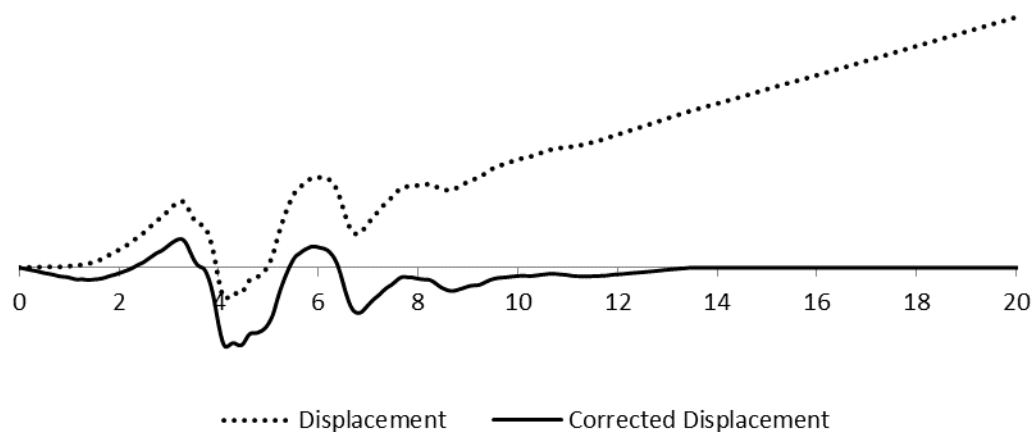
Using the corrected displacement the effective acceleration time history can be generated by applying numerical differentiation twice. The comparison is displayed below and it apparent that there is some amount amplitude loss but it is not significant.

—— Acceleration   —— Acceleration Derived From Corrected Data
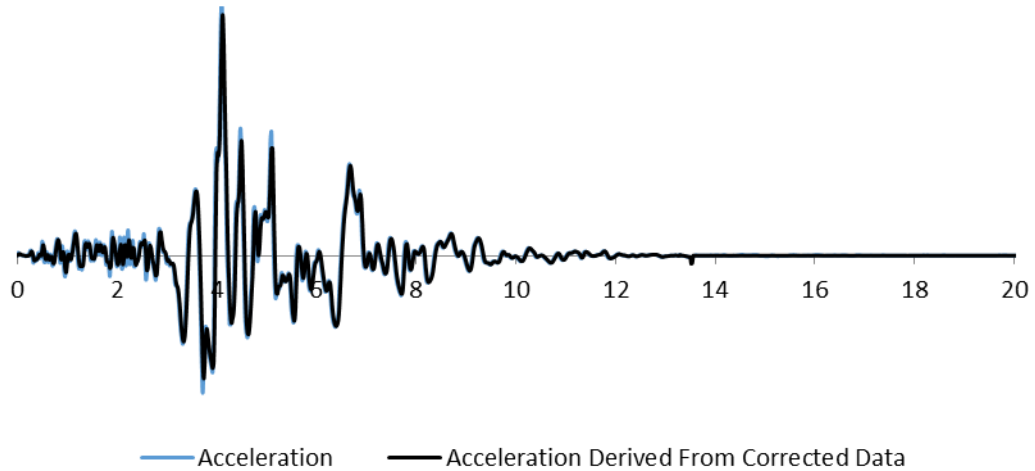
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