

RS2

2D finite element program for stress analysis and support design
around excavations in soil and rock

Groundwater Verification Manual

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Table of Contents

1	Shallow Unconfined Flow With Rainfall	6
1.1	Problem Description.....	6
1.2	Analytical Solution.....	7
1.3	Results	7
1.4	References	8
1.5	Data Files.....	8
2	Flow Around a Cylinder	9
2.1	Problem Description.....	9
2.2	Analytical Solution.....	10
2.3	Results	10
2.4	References	11
2.5	Data Files.....	11
3	Confined Flow Under Dam Foundation	12
3.1	Problem Description.....	12
3.2	Analytical Solution.....	13
3.3	Results	13
3.4	References	15
3.5	Data Files.....	15
4	Shallow Unconfined Flow Through Earth Dam	16
4.1	Problem Description.....	16
4.2	Analytical Solution.....	17
4.3	Results	17
4.4	References	19
4.5	Data Files.....	19
5	Unsaturated Flow Behind an Embankment	20
5.1	Problem Description.....	20
5.2	Analytical Solution.....	20
5.3	Results	21
5.4	References	21
5.5	Data Files.....	22
6	Steady-State Seepage Analysis Through Saturated-Unsaturated Soils	23
6.1	Problem Description.....	23
6.1.1	Isotropic earth dam with a horizontal drain	23
6.1.2	Anisotropic earth dam with a horizontal drain	24
6.1.3	Isotropic earth dam with a core and horizontal drain	24
6.1.4	Isotropic earth dam under steady-state infiltration	24
6.1.5	Isotropic earth dam with seepage face	24
6.2	Analytical Solution.....	24
6.3	Results	25
6.3.1	Isotropic earth dam with a horizontal drain	25
6.3.2	Anisotropic earth dam with a horizontal drain	26
6.3.3	Isotropic earth dam with a core and horizontal drain	28
6.3.4	Isotropic earth dam under steady-state infiltration	30

6.3.5	Isotropic earth dam with seepage face	32
6.4	References	35
6.5	Data Files.....	35
7	Seepage Within Layered Slope.....	36
7.1	Problem Description.....	36
7.2	Analytical Solution.....	37
7.3	Results	38
7.4	References	40
7.5	Data Files.....	41
8	Flow Through Ditch-Drained Soils	42
8.1	Problem Description.....	42
8.2	Analytical Solution.....	43
8.3	Results	44
8.4	References	45
8.5	Data Files.....	45
9	Seepage Through Dam.....	46
9.1	Problem Description.....	46
9.1.1	Homogeneous dam.....	46
9.1.2	Dam with impervious core.....	47
9.2	Analytical Solution.....	48
9.3	Results	48
9.3.1	Homogeneous dam.....	48
9.3.2	Dam with impervious core.....	49
9.4	References	50
9.5	Data Files.....	51
10	Steady-State Unconfined Flow Using Van Genuchten Permeability Function	52
10.1	Problem Description.....	52
10.2	Analytical Solution.....	52
10.3	Results	53
10.4	References	54
10.5	Data Files.....	54
11	Earth and Rock-Fill Dam Using Gardner Permeability Function.....	55
11.1	Problem Description.....	55
11.1.1	Uniform earth and rock-fill dam.....	55
11.1.2	Heterogeneous earth and rock-fill dam.....	55
11.2	Analytical Solution.....	56
11.3	Results	56
11.3.1	Uniform earth and rock-fill dam.....	56
11.3.2	Heterogeneous earth and rock-fill dam.....	57
11.4	References	58
11.5	Data Files.....	58
12	Seepage from Trapezoidal Ditch into Deep Horizontal Drainage Layer.....	59
12.1	Problem Description.....	59
12.2	Analytical Solution.....	60
12.3	Results	62
12.4	References	64

12.5	Data Files.....	64
13	Seepage from a Triangular Ditch into a Deep Horizontal Drainage Layer	65
13.1	Problem Description.....	65
13.2	Analytical Solution.....	66
13.3	Results	69
13.4	References	70
13.5	Data Files.....	70
14	Unsaturated Soil Column.....	71
14.1	Problem Description.....	71
14.2	Analytical Solution.....	72
14.3	Results	73
14.4	References	74
14.5	Data Files.....	74
15	Radial Flow to a Well in a Confined Aquifer.....	75
15.1	Problem Description.....	75
15.2	Analytical Solution.....	76
15.3	Results	77
15.4	References	77
15.5	Data Files.....	78
16	Radial Flow to a Well in an Unconfined Aquifer.....	79
16.1	Problem Description.....	79
16.2	Analytical Solution.....	80
16.3	Results	81
16.4	References	82
16.5	Data Files.....	82
17	1-D Consolidation with Uniform Initial Excess Pore Pressure	83
17.1	Problem Description.....	83
17.2	RS2 Model and Results	84
17.3	References	87
18	Pore Pressure Dissipation of Stratified Soil.....	88
18.1	Problem Description.....	88
18.2	RS2 Model Results.....	88
18.3	References	90
19	Transient Seepage Through an Earth Fill Dam with Toe Drain	91
19.1	Problem Description.....	91
19.2	RS2 Model.....	91
19.3	Results	92
19.4	References	93
20	Transient Seepage Through an Earth Fill Dam.....	94
20.1	Problem Description.....	94
20.2	RS2 Model.....	94
20.3	Results	95
20.4	References	97
21	Seepage Below a Lagoon.....	98
21.1	Problem Description.....	98
21.2	RS2 Model.....	99

21.3	Results	100
21.4	References	103
22	Seepage in a Layered Slope	104
22.1	Problem Description	104
22.2	RS2 Model	104
22.3	Results	105
22.4	References	107
23	Transient Seepage Through a Fully Confined Aquifer	108
23.1	Problem Description	108
23.2	RS2 Model	108
23.3	Results	109
23.4	References	111

1 Shallow Unconfined Flow With Rainfall

1.1 Problem Description

The problem considered in this section involves the infiltration of water downward through soil. It is characterized by a boundary of flow domain also known as a free surface. Such a problem domain is said to be unconfined.

Water may infiltrate downward through the soil due to rainfall or artificial infiltration. Rainfall can be presented as a uniform discharge P (m/s), defined as the amount of water per unit area that enters the aquifer per unit time. Figure 1-1 shows the problem of flow between two long and straight parallel rivers, separated by a section of land. The free surface of the land is subjected to rainfall. Figure 1-2 shows the problem as modeled in RS2.

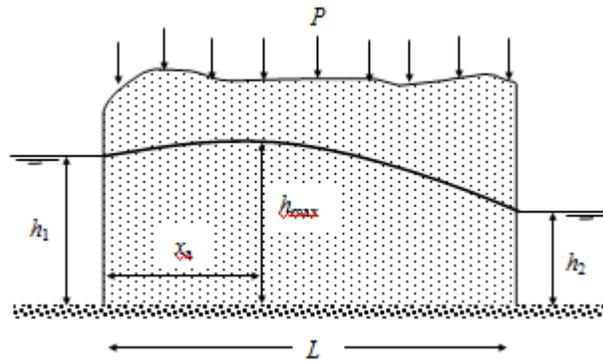


Figure 1-1: Unconfined flow under rainfall

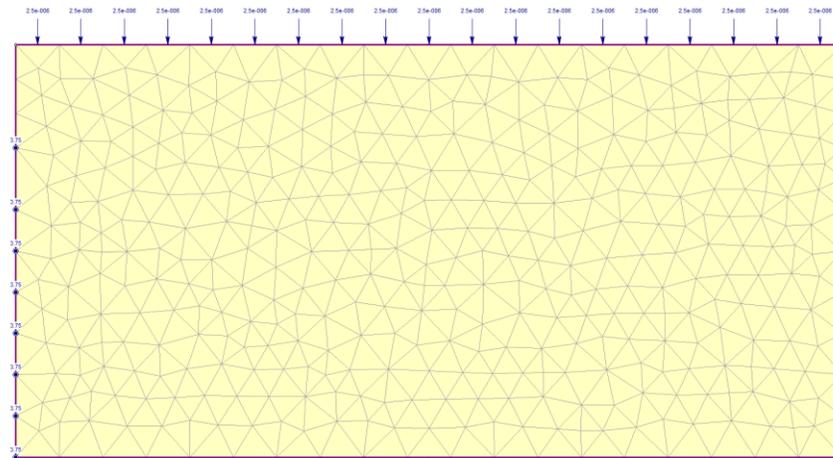


Figure 1-2: Shallow unconfined flow under rainfall as constructed in RS2

The model shown in Figure 1-2 uses 225 three-noded triangular finite elements. Table 1.1 summarizes other relevant model parameters.

Table 1.1: Model parameters

<i>Parameter</i>	<i>Value</i>
Total head at left boundary (h_1)	3.75 m
Total head at right boundary (h_2)	3.0 m
Width (L)	10.0 m
Infiltration rate (P)	2.5e-6 m/s
Hydraulic conductivity (k)	1.0e-5 m/s

1.2 Analytical Solution

The equation for flow can be expressed as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \nabla^2 \phi = -P$$

For one-dimensional flow, such as that encountered in the present example, solution of equation (1.1) after application of the appropriate boundary conditions yields the horizontal distance, x_a , at which the maximum elevation of the free surface in Figure 1-1 is located, as [1]

$$x_a = \frac{L}{2} \left(1 - \frac{k}{P} \frac{h_1^2 - h_2^2}{L^2} \right)$$

The corresponding maximum height for the free surface, h_{\max} , can be calculated as

$$h_{\max} = \sqrt{h_1^2 - \frac{x_a}{L} (h_1^2 - h_2^2) + \frac{P}{k} (L - x_a)x}$$

1.3 Results

Figure 1-3 shows contours of pressure head and the phreatic line.

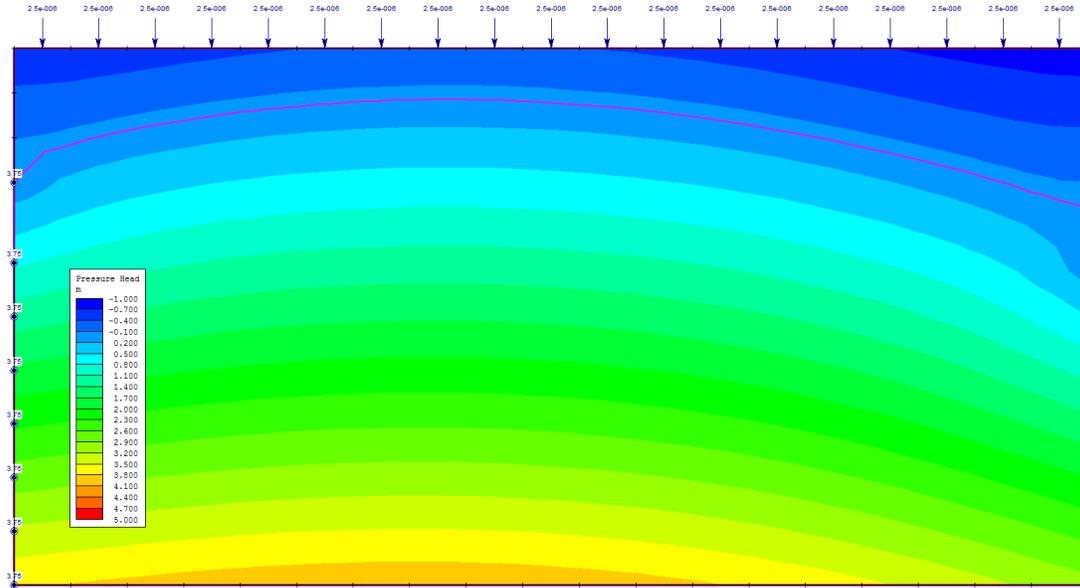


Figure 1-3: Pressure head contour plot as produced by RS2

Table 1.2 compares the co-ordinates of the maximum height of the free surface predicted by RS2 with those calculated analytically.

Table 1.2: Maximum height of free surface co-ordinates

	<i>RS2</i>	<i>Analytical</i>
x_a	4.22	3.99
h_{max}	4.52	4.25

The RS2 results are in close agreement with the analytical solution. If necessary, a finer mesh discretization could be used to improve the results.

1.4 References

1. Haar, M. E. (1990) *Groundwater and Seepage*, 2nd Edition, Dover

1.5 Data Files

The input data files **groundwater #001_01.fez** (regular mesh), and **groundwater #001_02.fez** (uniform mesh) can be downloaded from the RS2 Online Help page for Verification Manuals.

2 Flow Around a Cylinder

2.1 Problem Description

This example examines the problem of uniform fluid flow around a cylinder of unit radius as depicted in Figure 2-1. Figure 2-2 shows the problem as implemented in RS2.

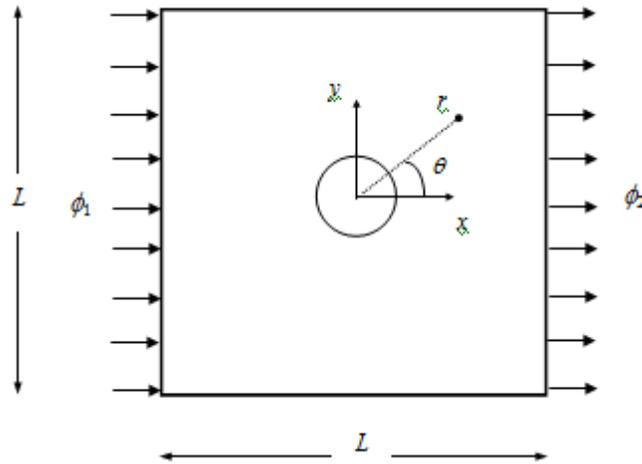


Figure 2-1: Fluid flow surrounding impermeable cylinder

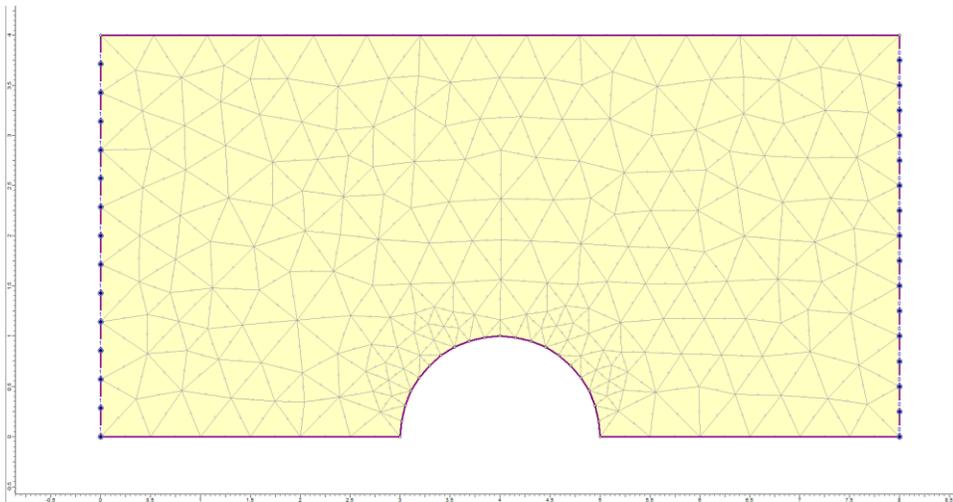


Figure 2-2: RS2 model geometry

Owing to the symmetry of the problem around the x -axis, only one half of the domain is discretized in the RS2 model. The half domain is represented with 442 six-noded triangular elements.

Table 2.1 summarizes the material and model properties used.

Table 2.1: Model parameters

<i>Parameter</i>	<i>Value</i>
Head at left boundary (ϕ_1)	1.0 m
Head at right boundary (ϕ_2)	0 m
Domain length (L)	8.0 m
Hydraulic conductivity (k)	10^{-5} m/s
Cylinder radius (a)	1 m

2.2 Analytical Solution

The closed form solution for this problem is given in [1]. This analytical solution gives the total head values at any point (r, θ) in the problem domain as

$$\phi = U \left(r + \frac{a^2}{r} \right) \cos \theta + 0.5$$

where U is the uniform undisturbed velocity = $\frac{\phi_1 - \phi_2}{L}$

2.3 Results

Figure 2-3 shows contours of total head with the values at a number of specified locations in the domain. These results from RS2 are compared with those provided in [2]. The RS2 results were within 4% of those provided in [2], as well as analytical values.

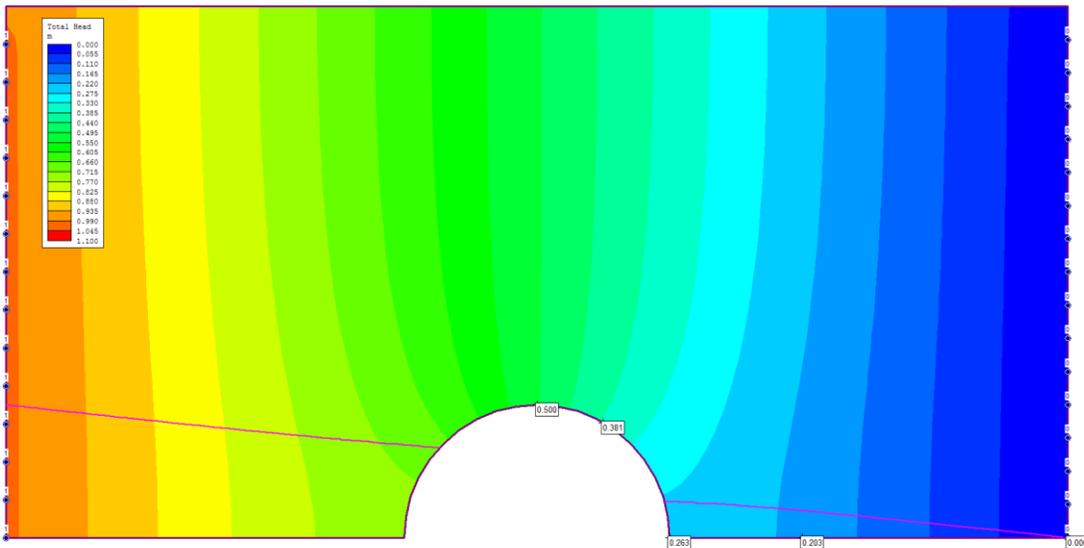


Figure 2-3: Total head contour plot produced by RS2

Table 2.2 compares the results from RS2 with those calculated using the equations in 2.2 and those presented in [2].

Table 2.2: Total head at selected points in problem domain

Coordinate		Total Head		
x	y	RS2	Analytical	Ref. [2]
4	1	0.5	0.5	0.5
4.5	0.866	0.381	0.374997	0.378
5	0	0.263	0.25	0.2765
6	0	0.203	0.1875	0.2132
8	0	0	-0.03125	0

2.4 References

1. Streeter, V.L. (1948), *Fluid Dynamics*, McGraw Hill
2. Desai, C. S., Kundu, T. (2001), *Introductory Finite Element Method*, Boca Raton, Fla. CRC Press

2.5 Data Files

The input data file **groundwater #002.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

3 Confined Flow Under Dam Foundation

3.1 Problem Description

The problem considered is a simple example of confined flow. It was selected to help assess the performance of RS2 on confined flow problems.

Figure 3-1 shows a dam that rests upon a homogeneous isotropic soil [1]. In the example, the walls (entity 1) and base (entity 2) of the dam are assumed to be impervious. The water level is 5 m upstream of the dam and 0 m downstream. The coordinates for point A are (0,0).

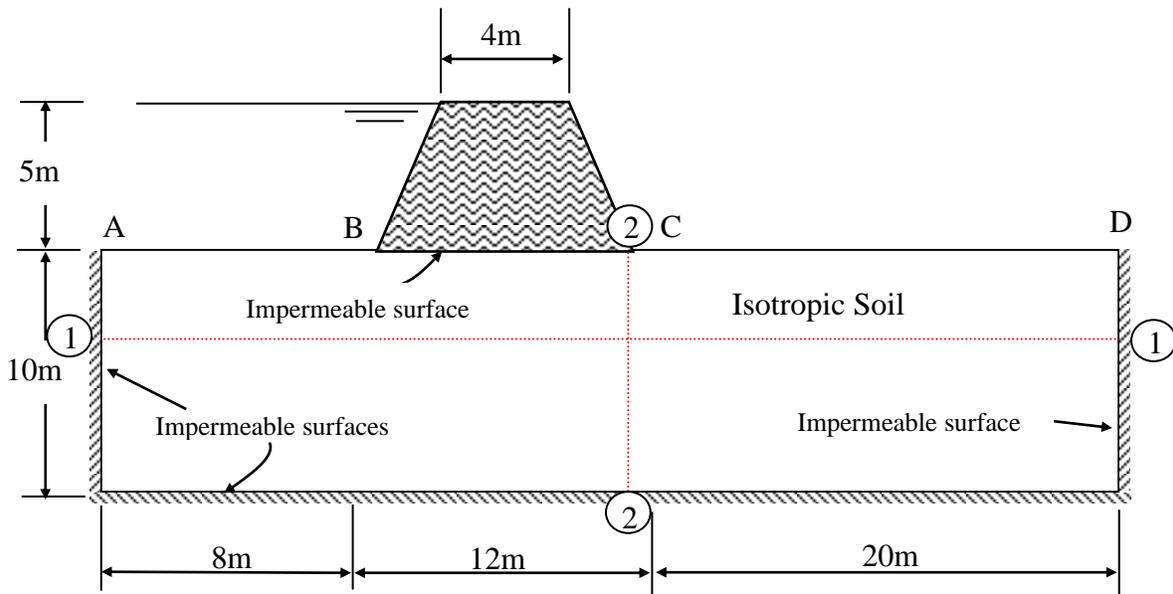


Figure 3-1: Model geometry

The model created in RS2 for this problem, with the mesh used, is shown in Figure 3-2.

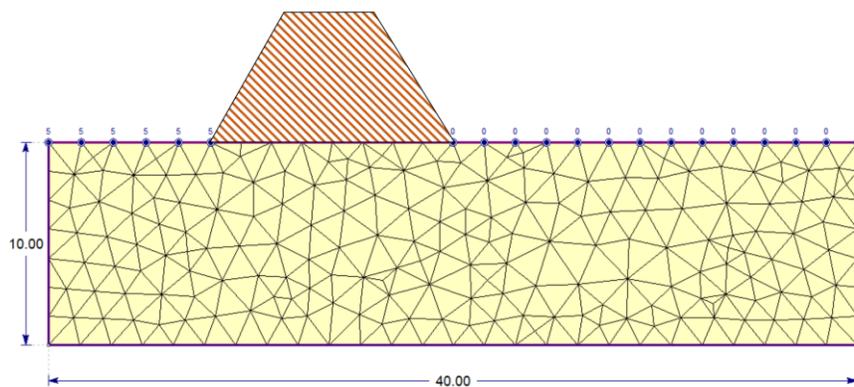


Figure 3-2: Confined flow under dam foundation as modeled in RS2

The following boundary conditions were used for the model:

- The total head along the line segment, upstream of the dam, that lies between points A and B (Figure 3-1), is equal to 5 m
- The total head along the line segment, downstream of the dam, that lies between points C and D, is equal to 0 m

The RS2 model was discretized using 398 three-noded triangular finite elements.

3.2 Analytical Solution

The flow is considered to be two-dimensional with negligible flow in the lateral direction. The flow equation for isotropic soil can be expressed as

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

This equation can be solved either using a numerical procedure or a flow net. Flow net techniques are well documented in groundwater references.

The accuracy of numerical solutions for the problem is dependent on how the boundary conditions are applied. For the particular example in this document, two boundary conditions are applied:

- No flow occurs across the impermeable base, and
- The pressure heads at the ground surface upstream and downstream of the dam are solely due to water pressure

3.3 Results

Figure 3-3 and Figure 3-4 show contours of pressure head and total pressure head, respectively.

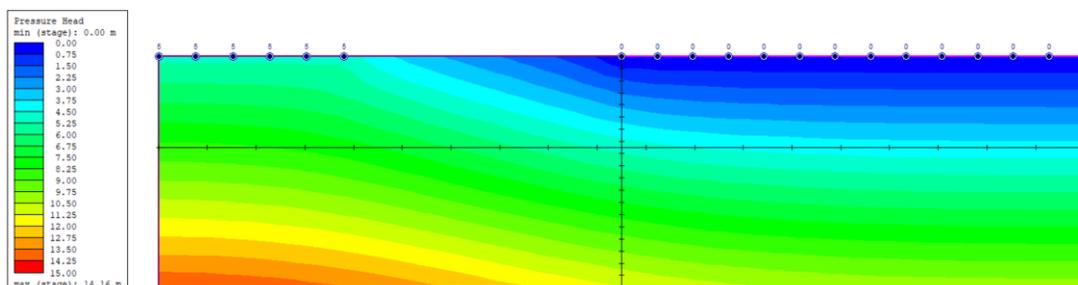


Figure 3-3: Pressure head contour plot produced using RS2

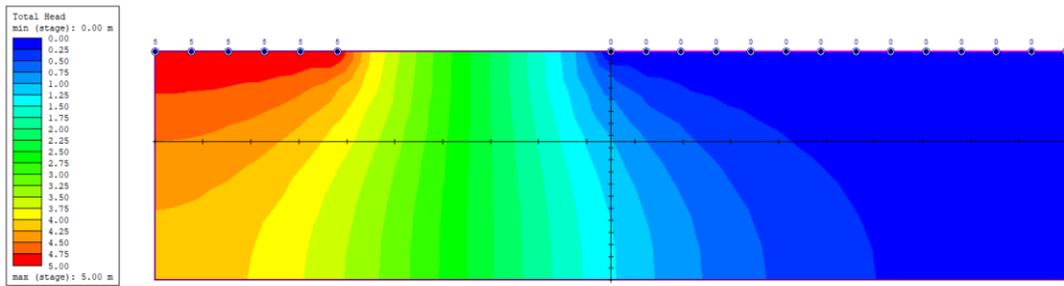


Figure 3-4: Total head contour plot produced using RS2

Figure 3-5 and Figure 3-6 compare total head values from RS2 with those obtained from [1]. These head pressures are calculated at points along line 1-1, which is located 4 m below the dam base (see Figure 3-1), and along segment 2-2, a vertical cross section passing through the rightmost base of the dam.

The results from RS2 agree closely with those provided in [1].

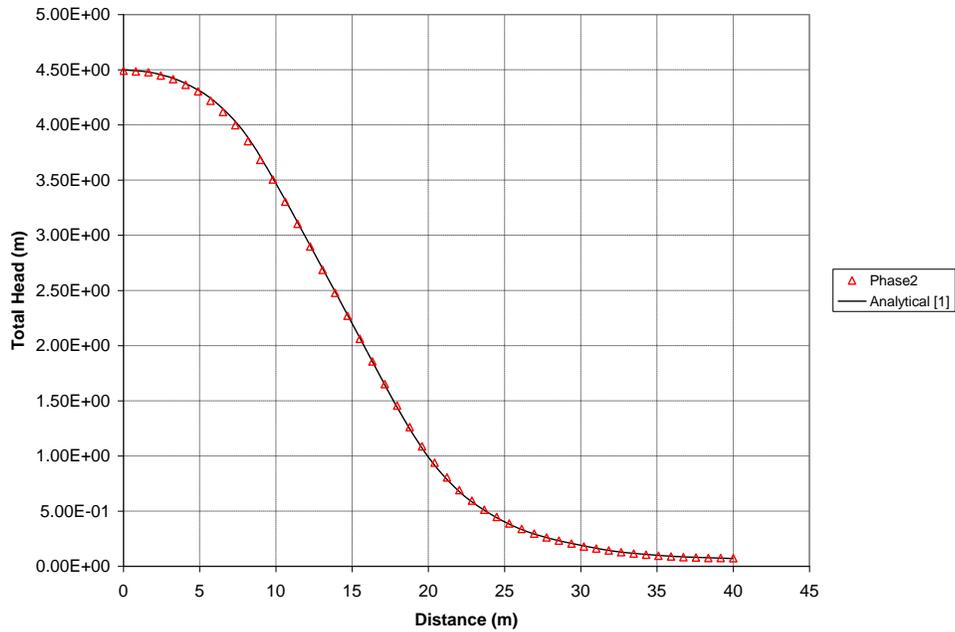


Figure 3-5: Total head variation along line 1-1

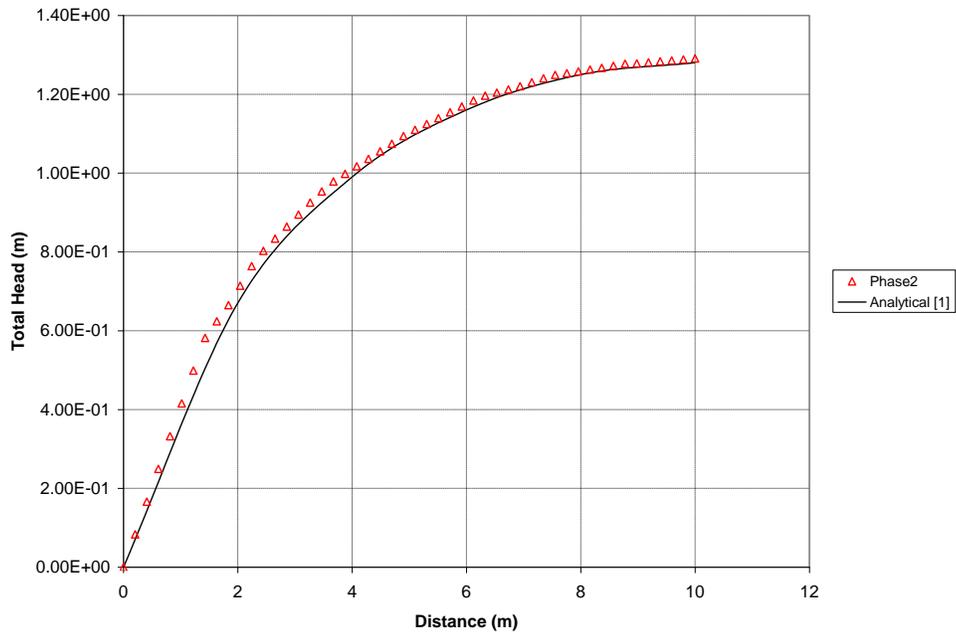


Figure 3-6: Total head variation along line 2-2

3.4 References

1. Rushton, K. R., Redshaw, S.C. (1979), *Seepage and Groundwater Flow*, John Wiley & Sons, U.K.

3.5 Data Files

The input data file **groundwater #003.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

4 Shallow Unconfined Flow Through Earth Dam

4.1 Problem Description

This example considers the problem of seepage through an earth dam. The task of calculating the shape and length of the free surface (line of seepage) is quite complicated. Some analytical solutions based on presenting flow nets as confocal parabolas are available in [1] and [2].

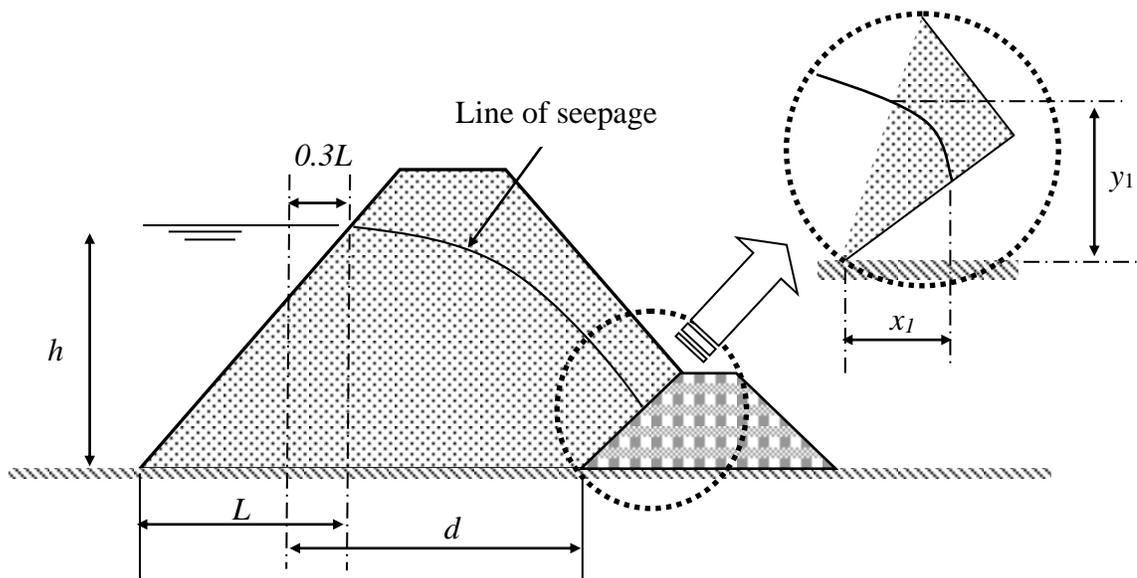


Figure 4-1: Earth dam with trapezoidal toe drain

Figure 4-1 shows a dam that has a trapezoidal toe drain.

The RS2 model geometry and boundary conditions used in this example are shown in Figure 4-2.

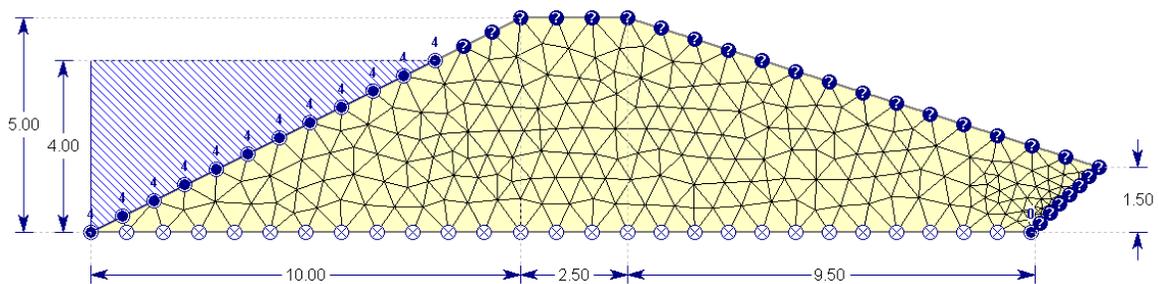


Figure 4-2: Earth dam as modeled in RS2

The total head on the upstream face of the dam was taken to be 4 m, and the toe drain was located at the downstream toe of the dam, i.e. total head at location (22,0) was taken to be 0. The boundary condition at the toe was assumed to be undefined, meaning that it initially either had flow, Q , or pressure head, P , equal to 0. A total number of three-noded triangular finite elements were used to model the problem.

4.2 Analytical Solution

By defining the free surface as Kozney's basic parabola [1], we can evaluate y_1 , the vertical height of the underdrain, as

$$y_1 = \sqrt{d^2 + L^2} - d$$

Then the minimum horizontal length of the underdrain, x_1 , equals

$$x_1 = \frac{y_1}{2}$$

4.3 Results

Figure 4-3 and Figure 4-4 show contours of pressure head and total head, respectively.

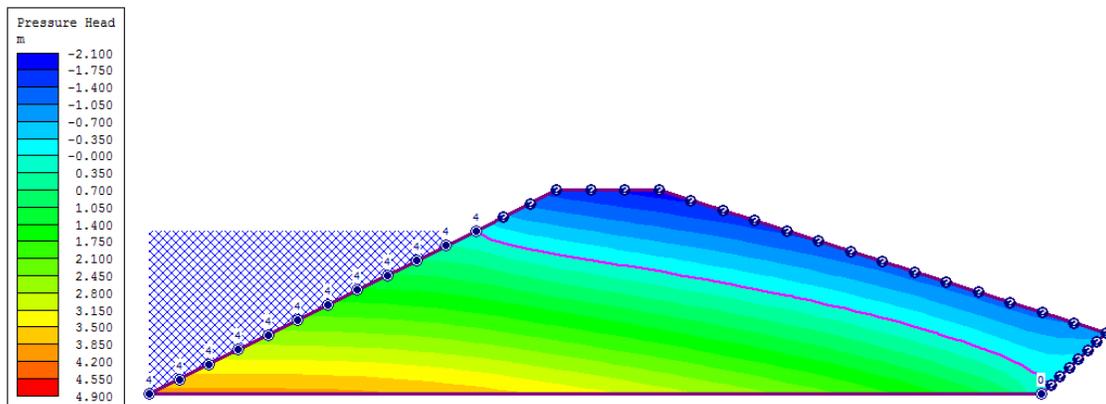


Figure 4-3: Pressure head contour plot produced by RS2

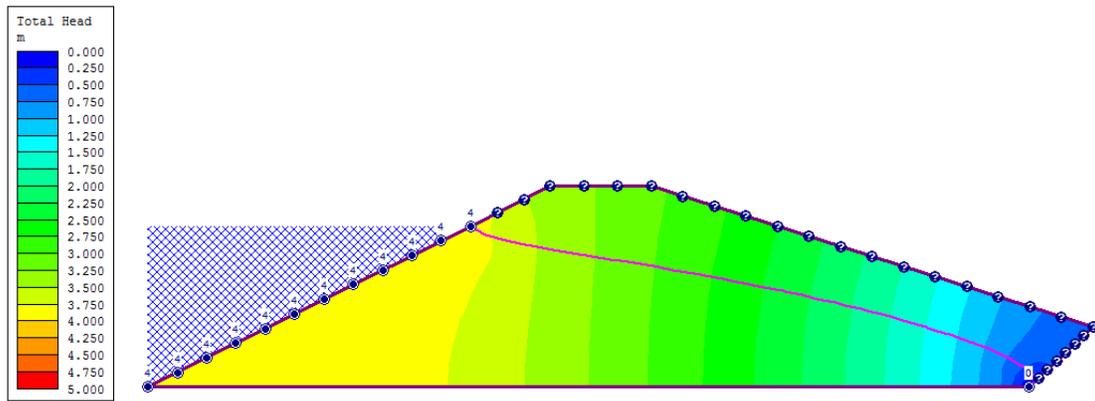


Figure 4-4: Total head contour plot produced by RS2

The minimum length and height of the underdrain were measured in RS2 and the results are shown in Figure 4-5.

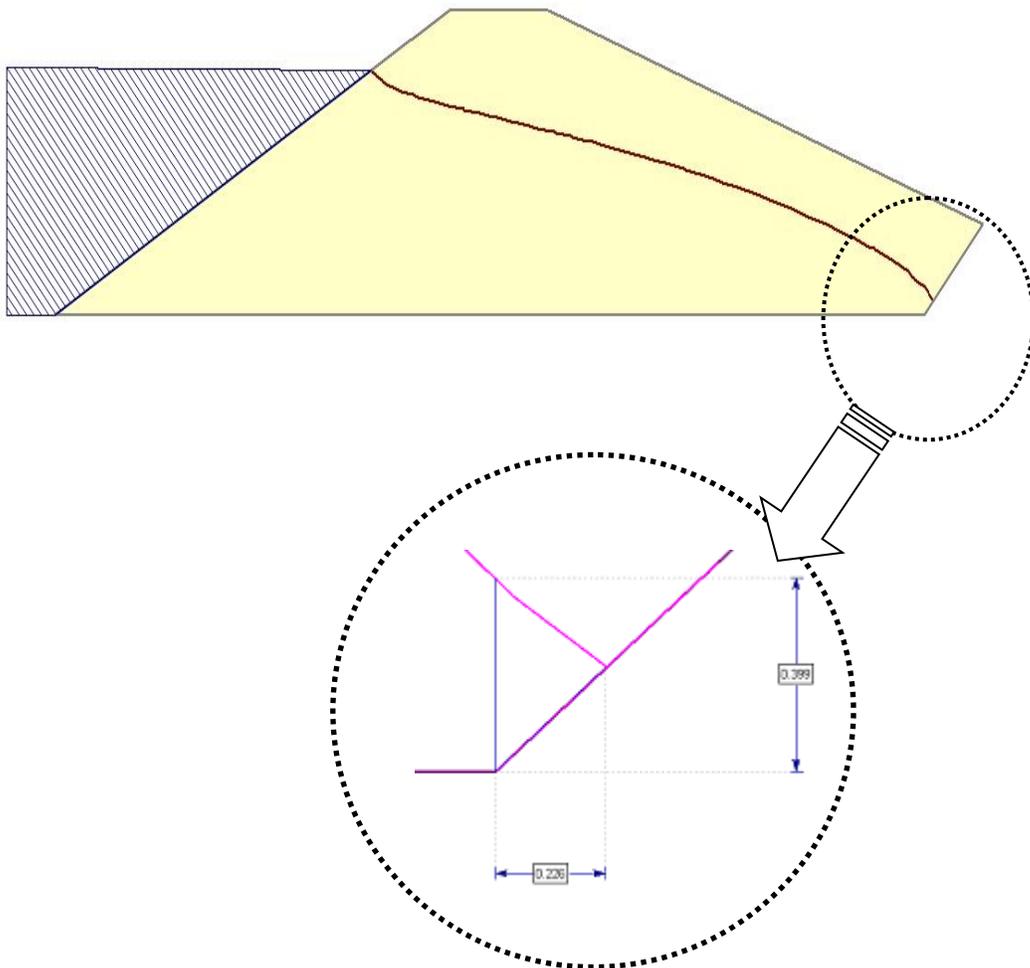


Figure 4-5: Length and height of minimum underdrain

The following table compares the minimum drain dimensions from RS2 with theoretical results.

Table 4.1: Minimum drain dimensions (m)

	<i>RS2</i>	<i>Analytical</i>
x_1	0.226	0.242
y_1	0.395	0.484

As can be seen, the RS2 results are in good agreement with theory.

4.4 References

1. Haar, M. E. (1990), *Groundwater and Seepage*, 2nd edition, Dover.
2. Raukivi, A.J., Callander, R.A. (1976), *Analysis of Groundwater Flow*, Edward Arnold.

4.5 Data Files

The input data file **groundwater #004.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

5 Unsaturated Flow Behind an Embankment

5.1 Problem Description

The geometry of the problem considered in this section is taken from the FLAC manual [1]. The example is modified slightly to handle two different materials. Two materials with different coefficients of permeability are considered. Figure 5-1 shows the geometry of the proposed model.

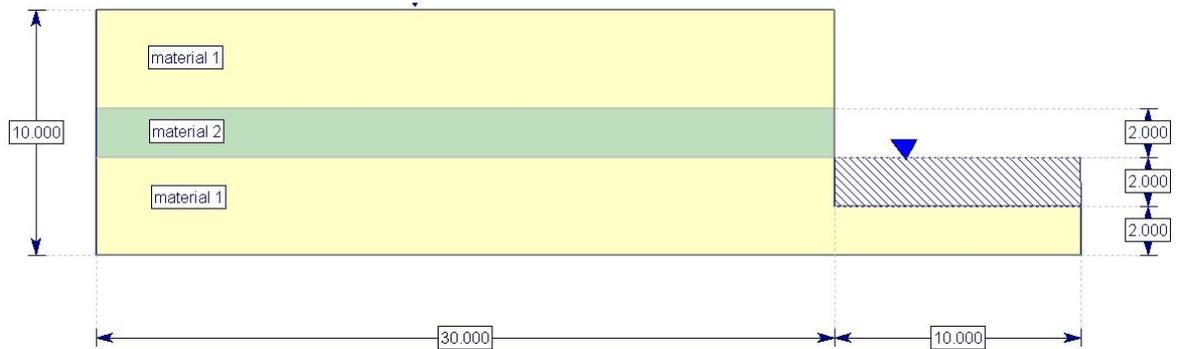


Figure 5-1: Embankment model in RS2

The saturated hydraulic conductivities of material 1 and material 2 are 1×10^{-10} m/s and 1×10^{-13} m/s respectively. RS2 model geometry is presented in Figure 5-1. The problem is discretized into 746 6-noded triangular finite elements. Total head boundary conditions of 10 m and 4 m are applied to the left and right boundaries of the model, respectively. Zero flow (impermeability) is assumed at the top and at the bottom of the embankment.

5.2 Analytical Solution

For this problem, RS2 results are compared with those from FLAC presented in [1]. Figure 5-2 and Figure 5-3 show the contours of pressure head and flow lines produced by FLAC.

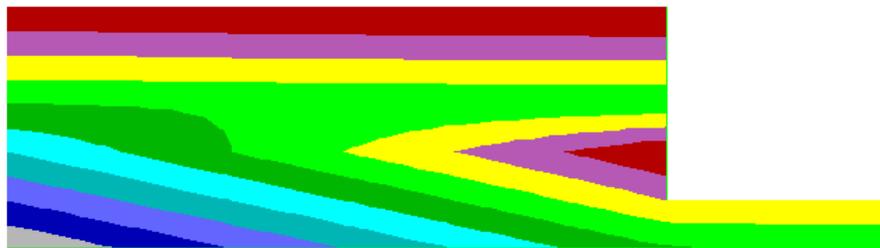


Figure 5-2: Pressure head contour plot produced by FLAC

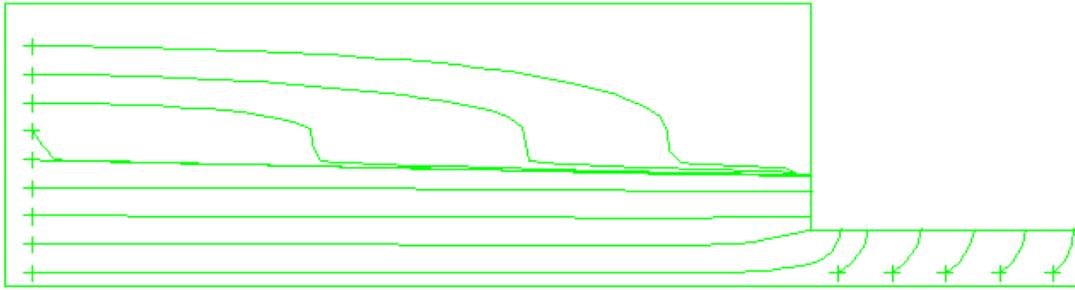


Figure 5-3: Flow lines produced by FLAC

5.3 Results

Figure 5-4 and Figure 5-5 show the pressure head contours and flow lines produced by RS2. Both are in close agreement with FLAC.

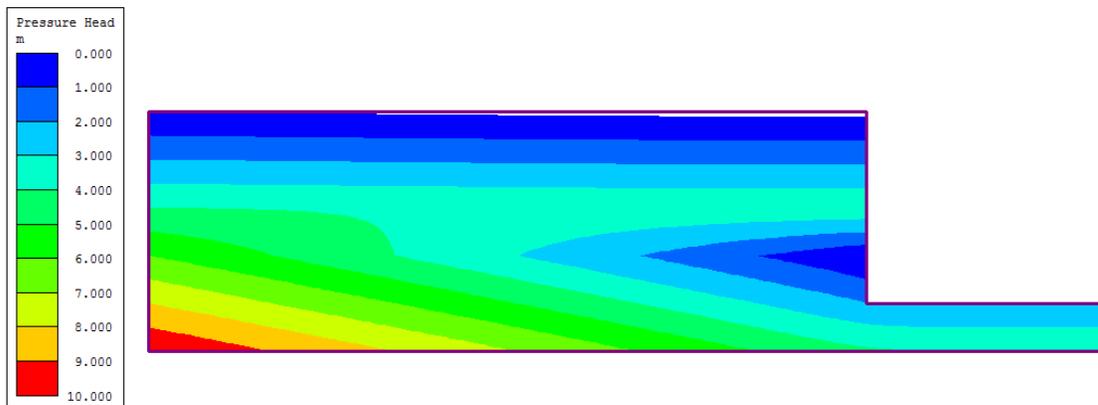


Figure 5-4: Pressure head contour plot produced by RS2

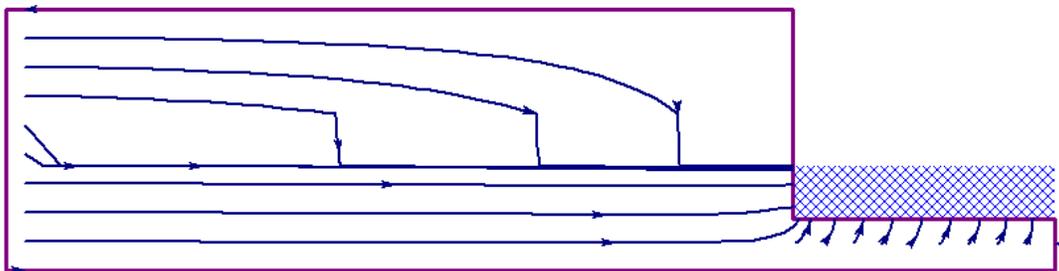


Figure 5-5: Flow line plot produced by RS2

5.4 References

1. Coetzee, Hart, et al. (1995), *Flac Basics: An introduction to FLAC and a guide to its practical application in geotechnical engineering*. Minneapolis, MA.: Itasca Consulting Group, Inc.

5.5 Data Files

The input data file **groundwater#005.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

6 Steady-State Seepage Analysis Through Saturated-Unsaturated Soils

In this verification example, five earth dams with various properties are modeled using RS2. Pressure head contours for each example are compared to the flownets presented in Fredland & Rahardjo [1].

6.1 Problem Description

This problem concerns seepage through an unsaturated earth dam. The geometry of the problem considered in this section, which is shown in Figure 6-1, is taken from [1].

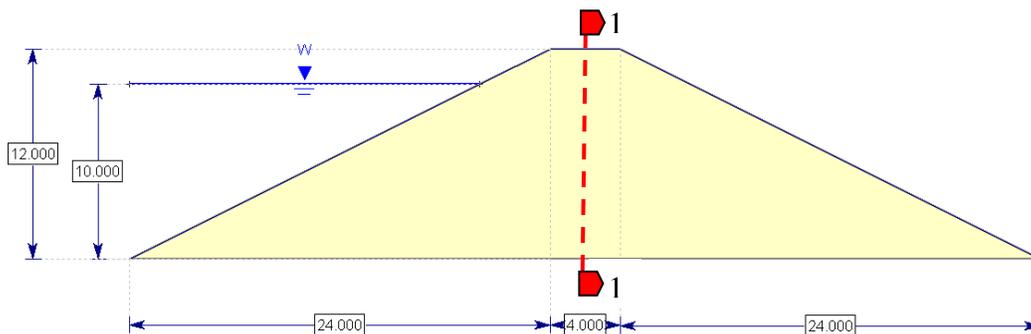


Figure 6-1: Isotropic earth dam with horizontal drain

In RS2, the problem is discretized into 336 3-noded triangular finite elements. The mesh used for this example was created using mapped mesh option to nearly replicate that used in [1]. The five different cases studied are as follows:

6.1.1 Isotropic earth dam with a horizontal drain

The first case considers an isotropic earth dam with 12 m horizontal drain. The permeability function used in the analysis is shown in Figure 1-2.

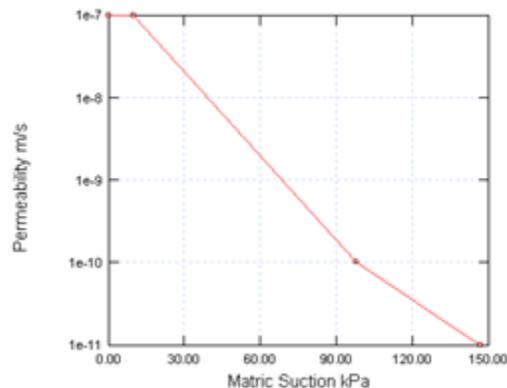


Figure 6-2: Permeability function for the isotropic earth dam

6.1.2 Anisotropic earth dam with a horizontal drain

For the second case, the dam is modeled with anisotropic soil. The water coefficient permeability in the horizontal direction is assumed to be nine times larger than in the vertical direction.

6.1.3 Isotropic earth dam with a core and horizontal drain

The third case considers an isotropic dam having a core with a lower coefficient of permeability. Figure 6-3 shows the permeability function used for the core material.

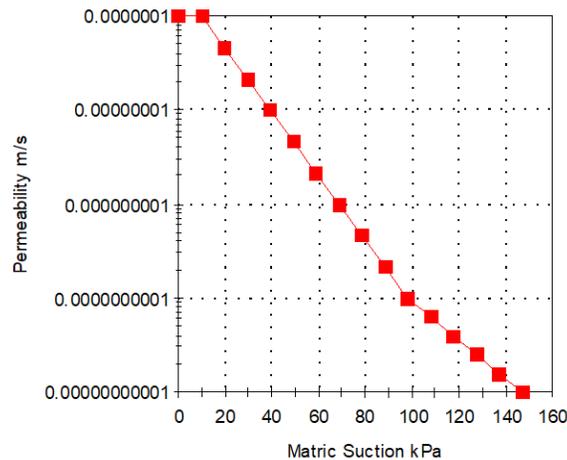


Figure 6-3: Permeability function for the core material

6.1.4 Isotropic earth dam under steady-state infiltration

The fourth case considers the effect of infiltration on the dam shown in Figure 6-17. Infiltration is simulated by applying a flux boundary of 1×10^{-8} m/s along the boundary of the dam.

6.1.5 Isotropic earth dam with seepage face

The fifth case demonstrates the use of an unknown boundary condition, which is usually used for the case of developing seepage faces.

6.2 Analytical Solution

The results of finite-element analysis by Lam (1984) [2] are presented in [1] in the form of two-dimensional contour charts. Figure 6-4 shows the results in [1] for the first case.

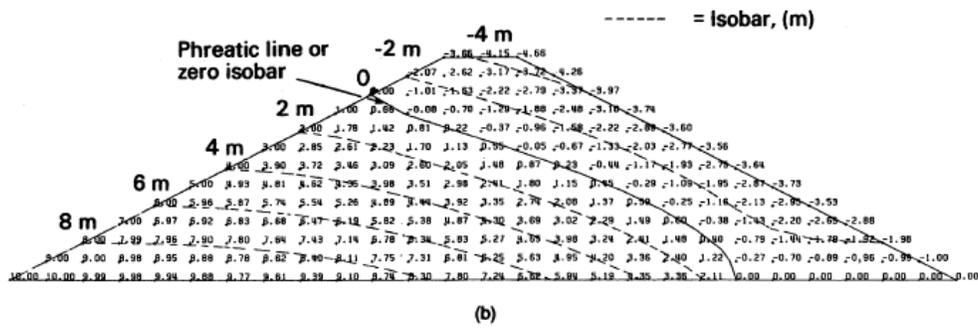


Figure 7.19 Seepage through an isotropic earth dam with a horizontal drain. (a) Equipotential lines and nodal flow rate vectors through the dam; (b) contours of pore-water pressure head (isobars) through the dam.

Figure 6-4: Pressure head contour data for isotropic earth dam [1]

6.3 Results

6.3.1 Isotropic earth dam with a horizontal drain

Figure 6-5, Figure 6-6, and Figure 6-7 show the flow vectors, pressure head and total head fields calculated by RS2 for the first case.

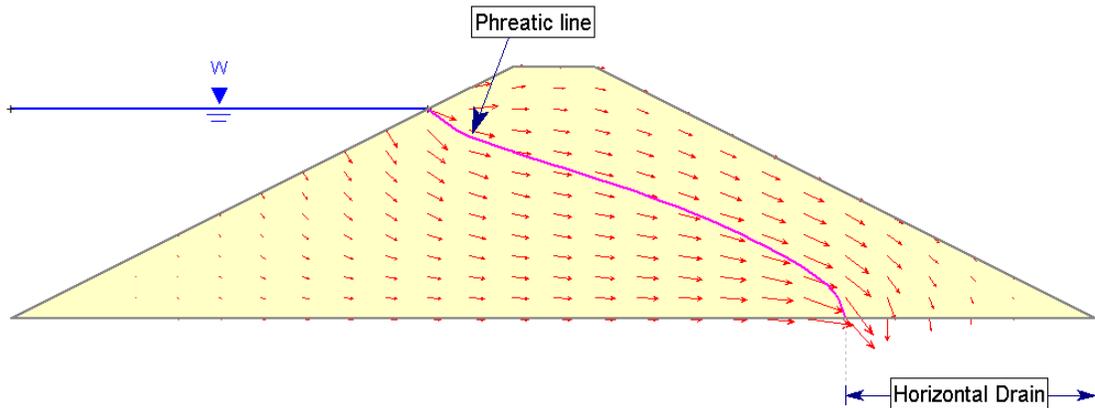


Figure 6-5: Flow vector plot produced by RS2 for first case

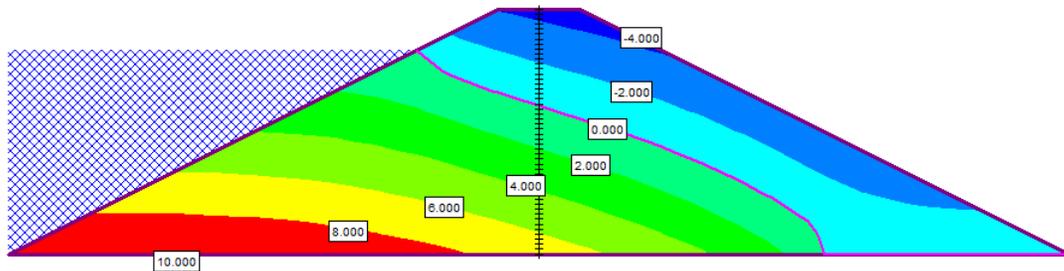


Figure 6-6: Pressure head contour plot produced by RS2 for first case

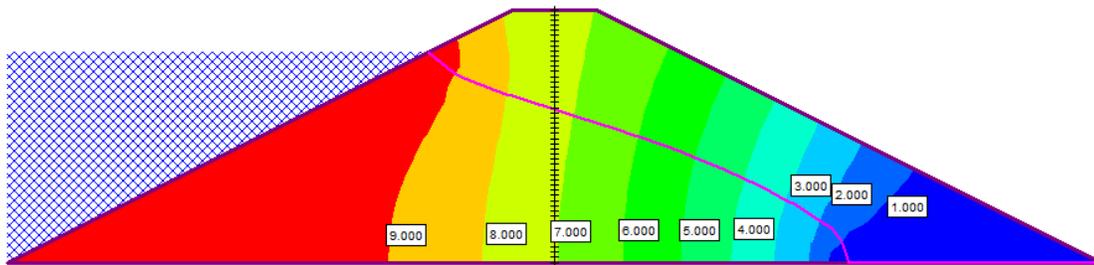


Figure 6-7: Total head contour plot produced by RS2 for first case

Figure 6-8 shows a comparison between RS2 results and results from [1] for the pressure head distribution along line 1-1 (see Figure 6-1).

Figure 6-8: Pressure head distributions along line 1-1 for isotropic earth dam

6.3.2 Anisotropic earth dam with a horizontal drain

Figure 6-9 presents the flow vectors and the location of the phreatic line from the RS2 groundwater model.

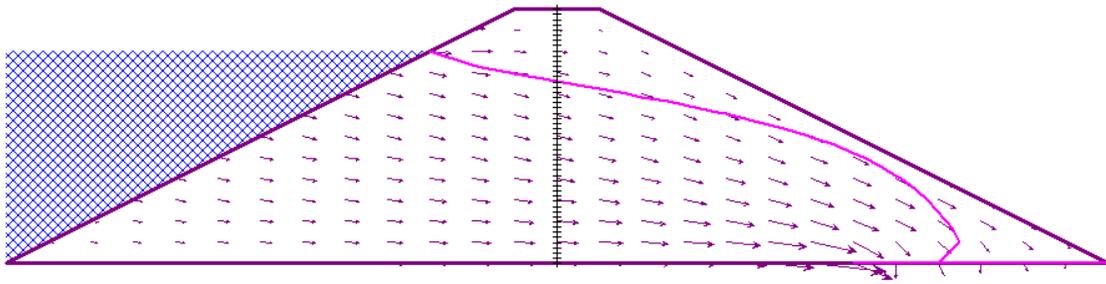


Figure 6-9: Flow vector plot and phreatic surface for earth dam with anisotropic permeability

Figure 6-10 and Figure 6-11 show the contours for pressure head and total head throughout the dam.

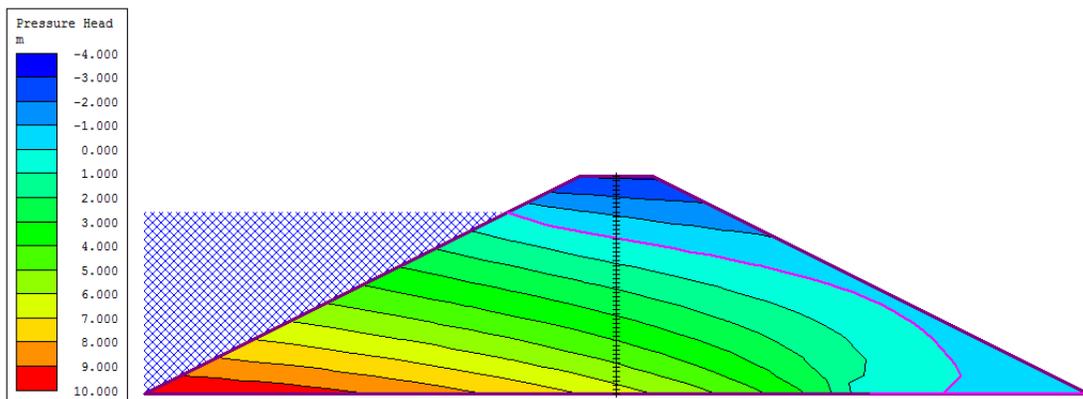


Figure 6-10: Pressure head contour plot for earth dam with anisotropic permeability

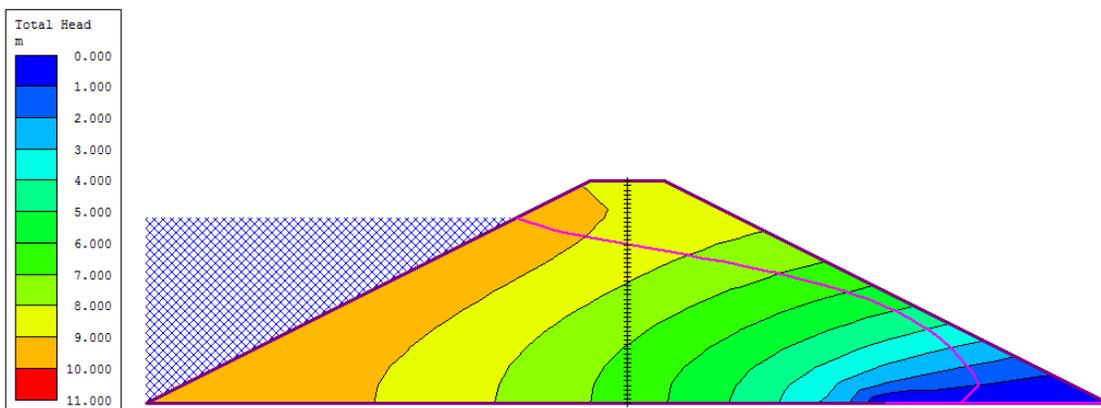


Figure 6-11: Total head contour plot for earth dam with anisotropic permeability

Figure 6-12 compares RS2 results to those from [1] for the pressure head distribution along line 1-1.

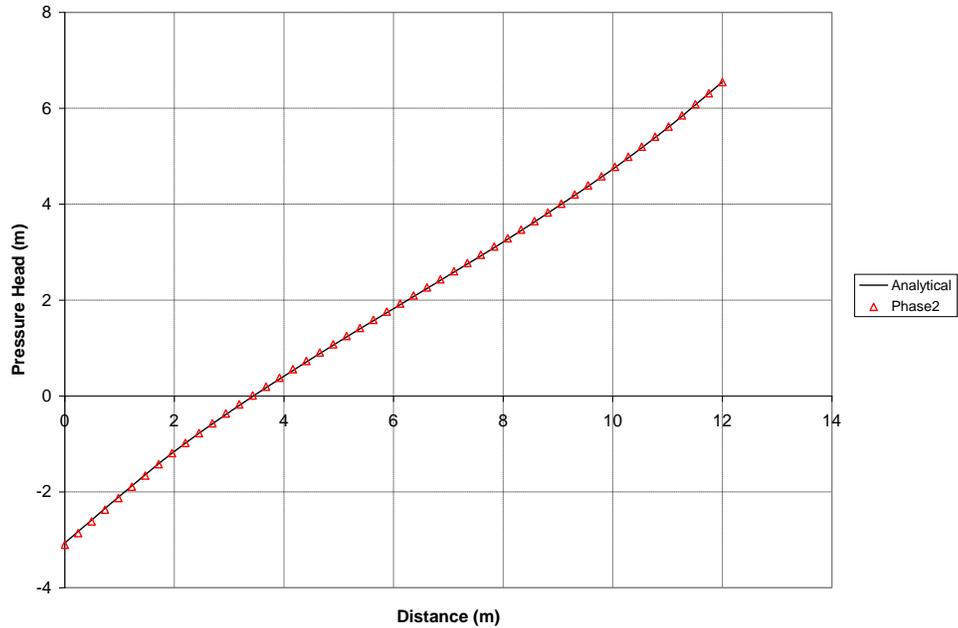


Figure 6-12: Pressure head distributions from RS2 and [1] along line 1-1

6.3.3 Isotropic earth dam with a core and horizontal drain

The results from the third case show that the hydraulic head change takes place largely in the zone around the core. The flow vectors show that the water flows upward into the unsaturated zone and around the core zone as shown in Figure 6-13. Pressure head and total head contours are presented in Figure 6-14 and Figure 6-15.

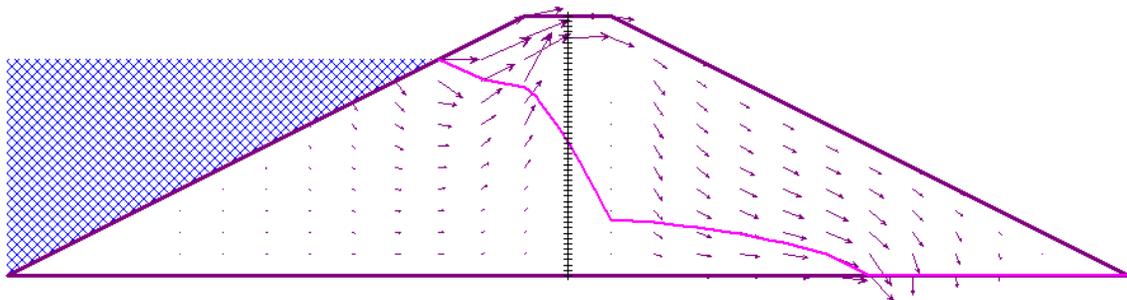


Figure 6-13: Flow vectors and phreatic surface for isotropic earth dam with core and horizontal drain

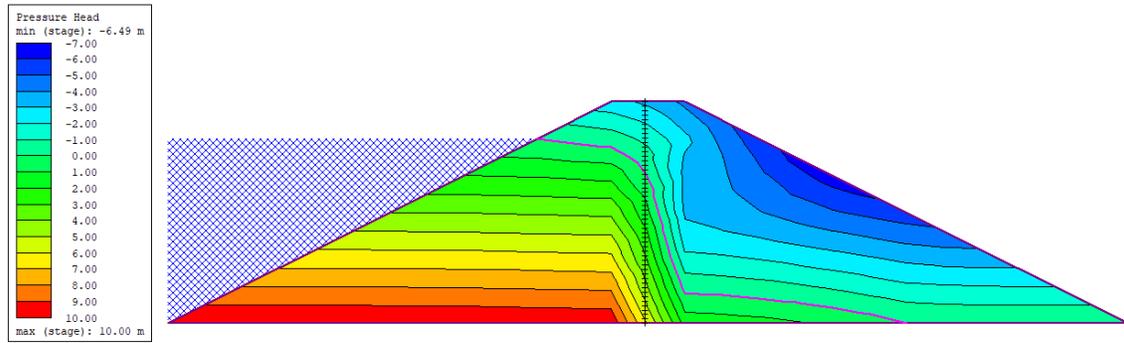


Figure 6-14: Pressure head contour plot for isotropic earth dam with core and horizontal drain

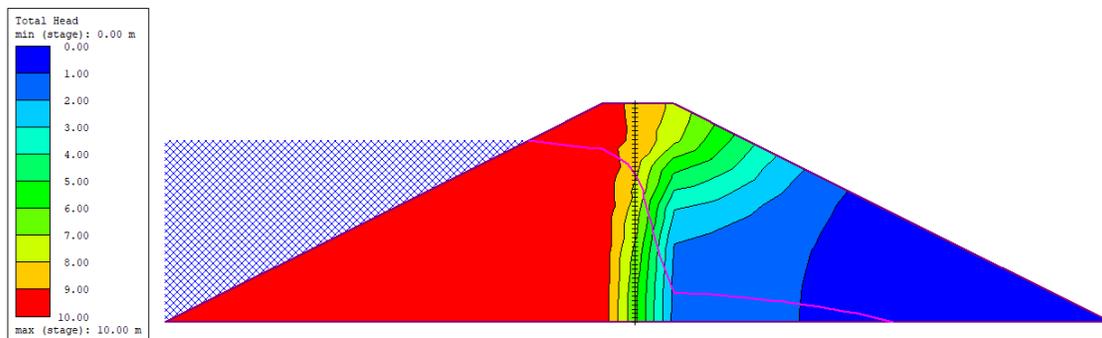


Figure 6-15: Total head contour plot for isotropic earth dam with core and horizontal drain

Figure 6-16 compares RS2 results and those from [1] for the pressure head distribution along line 1-1.

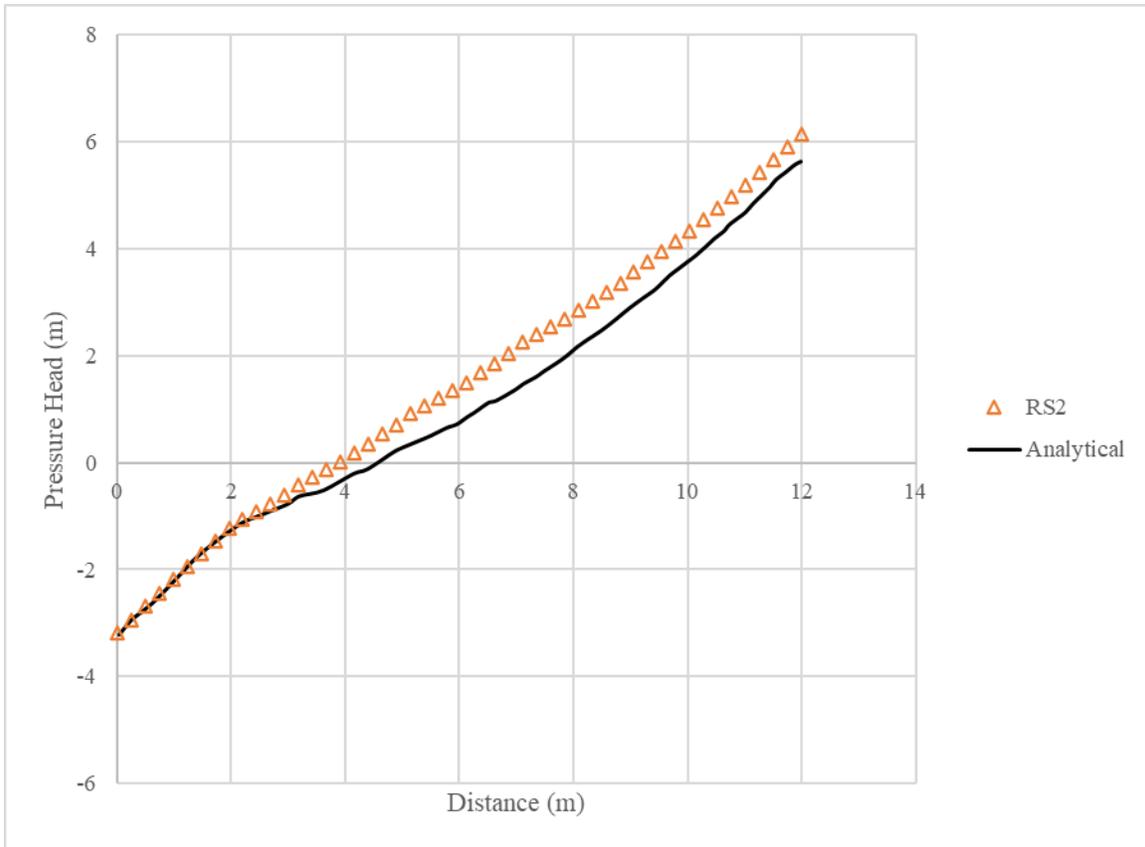


Figure 6-16: Pressure head distributions along line 1-1 from RS2 and [1]

6.3.4 Isotropic earth dam under steady-state infiltration

Figure 6-17 plots the flow vectors and phreatic line calculated by RS2 for the fourth case. Pressure head and total head contours are presented in Figure 6-18 and Figure 6-19 respectively.

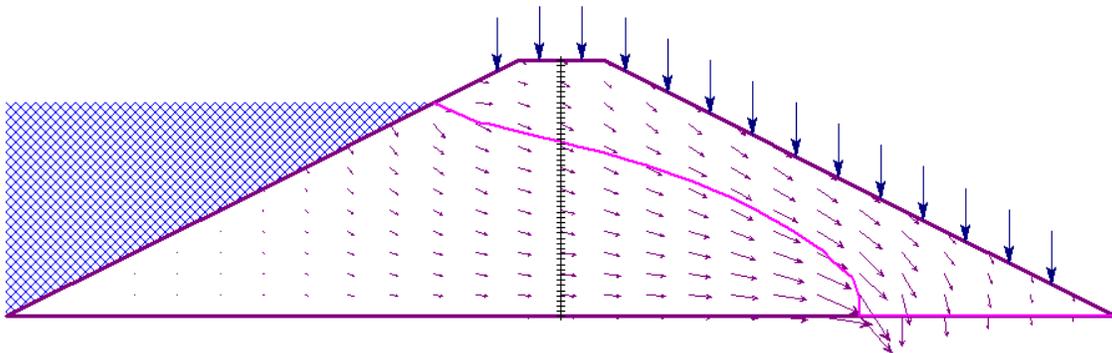


Figure 6-17: Flow vectors and phreatic surface for isotropic earth dam under steady-state infiltration

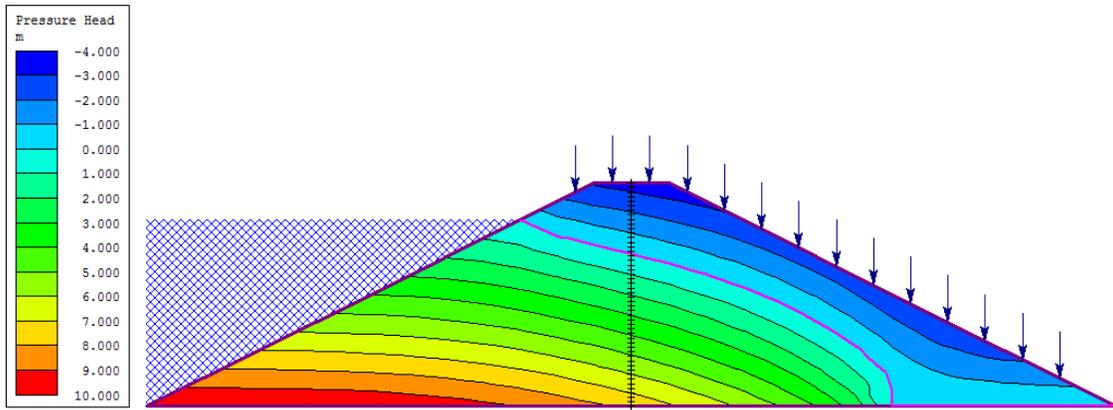


Figure 6-18: Pressure head contour plot for isotropic earth dam under steady-state infiltration

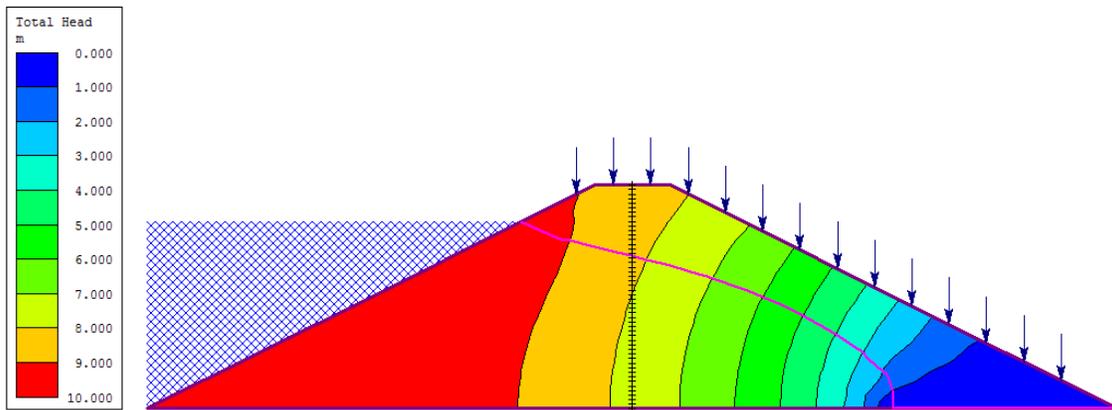


Figure 6-19: Total head contour plot for isotropic earth dam under steady-state infiltration

Figure 6-20 compares RS2 results to those from [1] for pressure head distribution along line 1-1.

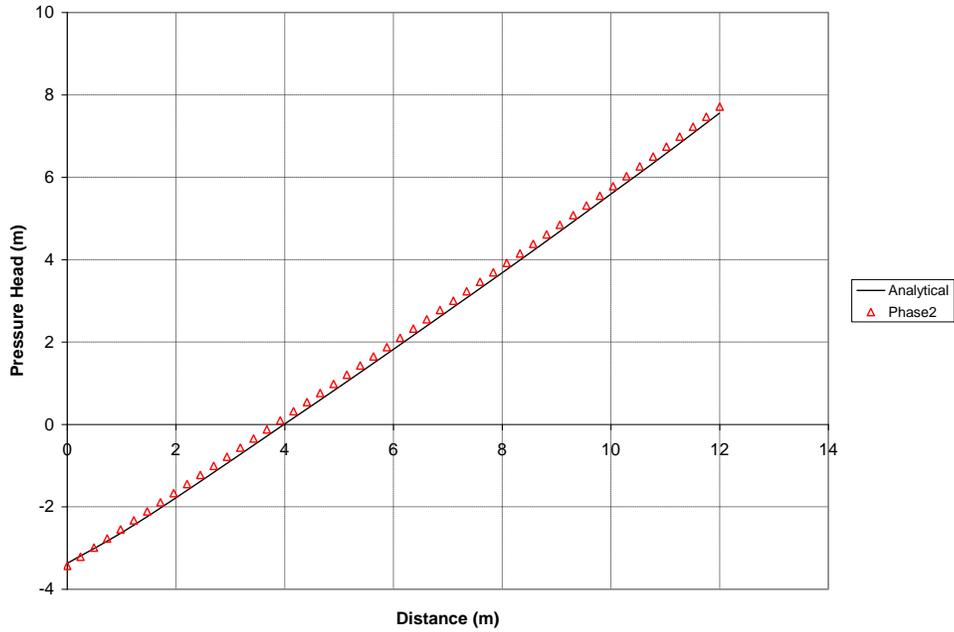


Figure 6-20: Comparison of pressure head distributions along line 1-1 from RS2 and [1]

6.3.5 Isotropic earth dam with seepage face

The boundary conditions and the phreatic surface for the fifth case are presented in Figure 6-21. Pressure head and total head contours are presented in Figure 6-22 and Figure 6-23 respectively.

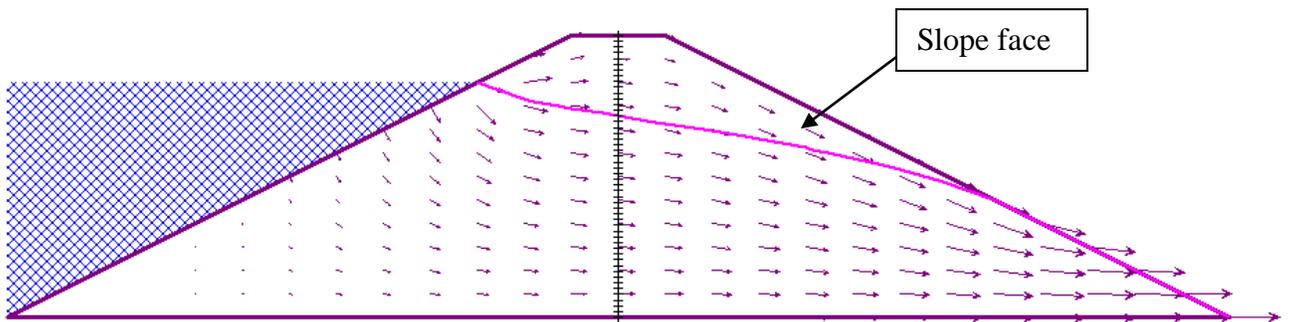


Figure 6-21: Flow vectors and phreatic surface for isotropic earth dam with seepage face

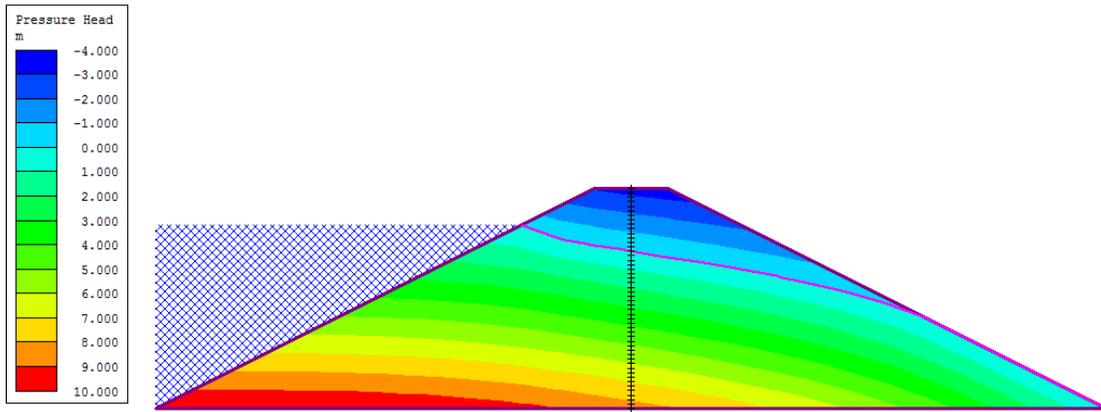


Figure 6-22: Pressure head contour plot for isotropic earth dam with seepage face

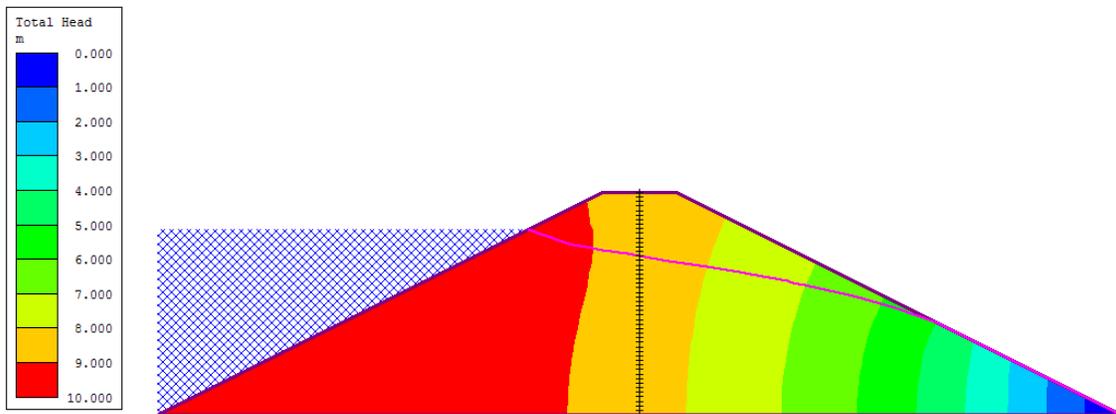


Figure 6-23: Total head contour plot for isotropic earth dam with seepage face

Figure 6-24 compares RS2 results to those from [1] for the pressure head distribution along the slope face.

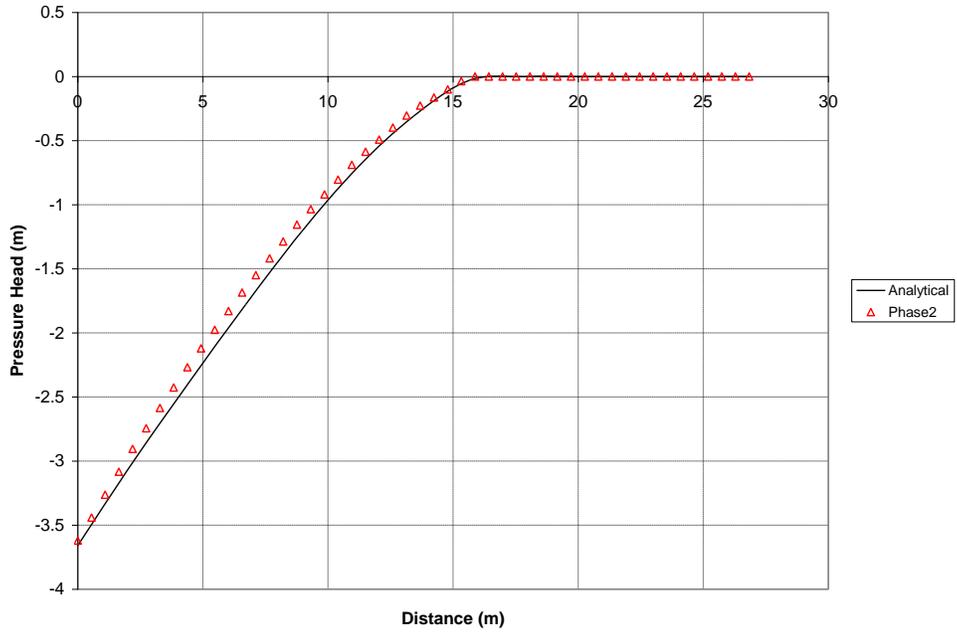


Figure 6-24: Pressure head distributions along seepage face from RS2 and [1]

Figure 6-24 compares RS2 results to those from [1] for the pressure head distribution along line 1-1.

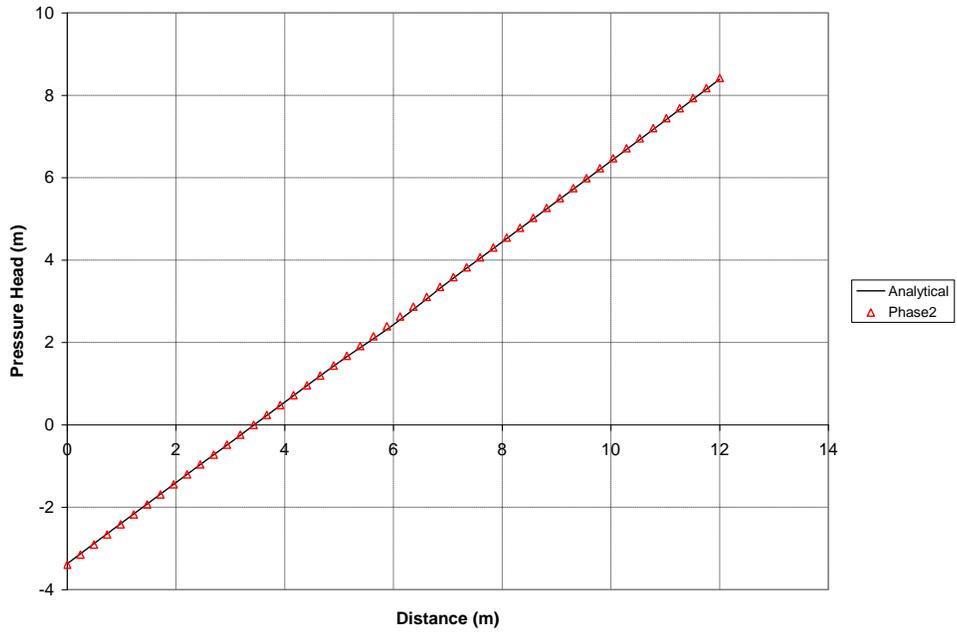


Figure 6-25: Pressure head distributions along line 1-1 from RS2 and [1]

6.4 References

1. Fredlund, D.G. and H. Rahardjo (1993), *Soil Mechanics for Unsaturated Soils*, John Wiley.
2. L. Lam and D. G. Fredlund (1984), "Saturated-Unsaturated Transient Finite Element Seepage Model for Geotechnical Engineering," *Adv. Water Resources*, vol. 7, pp. 132-136.

6.5 Data Files

The input data files **groundwater#006_01.fez** to **groundwater#006_05.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

7 Seepage Within Layered Slope

7.1 Problem Description

This example considers the problem of seepage through a layered slope. Rulan and Freeze [1] studied this problem using a sandbox model. The material of the slope consisted of medium sand and a fine sand with relatively lower permeability. The geometry of the problem is shown in Figure 7-1 and the two permeability functions used to model the soil are shown in Figure 7-2. These permeability functions are similar to those presented by Fredlund and Rahardjo [2].

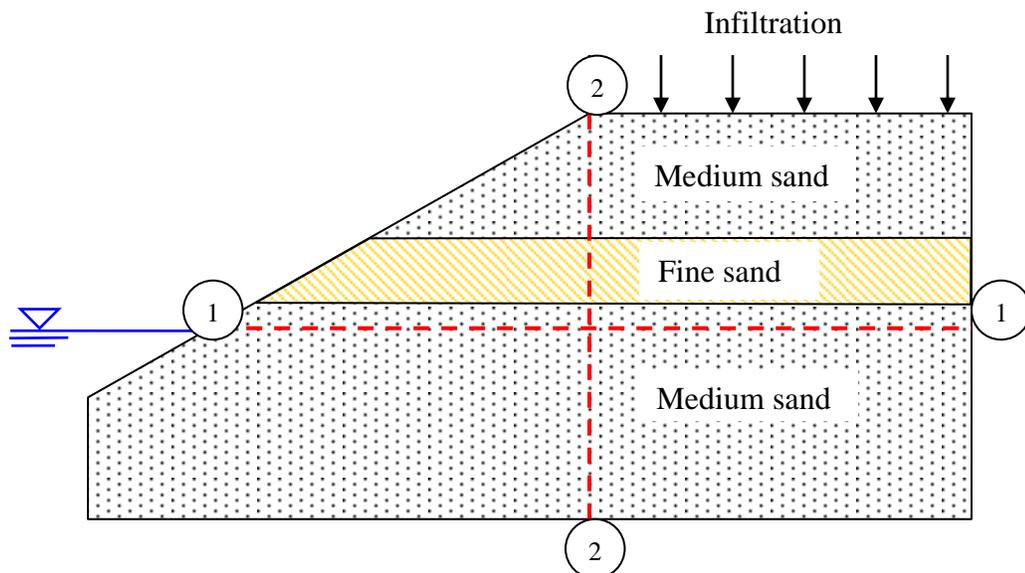


Figure 7-1: Layered slope problem geometry

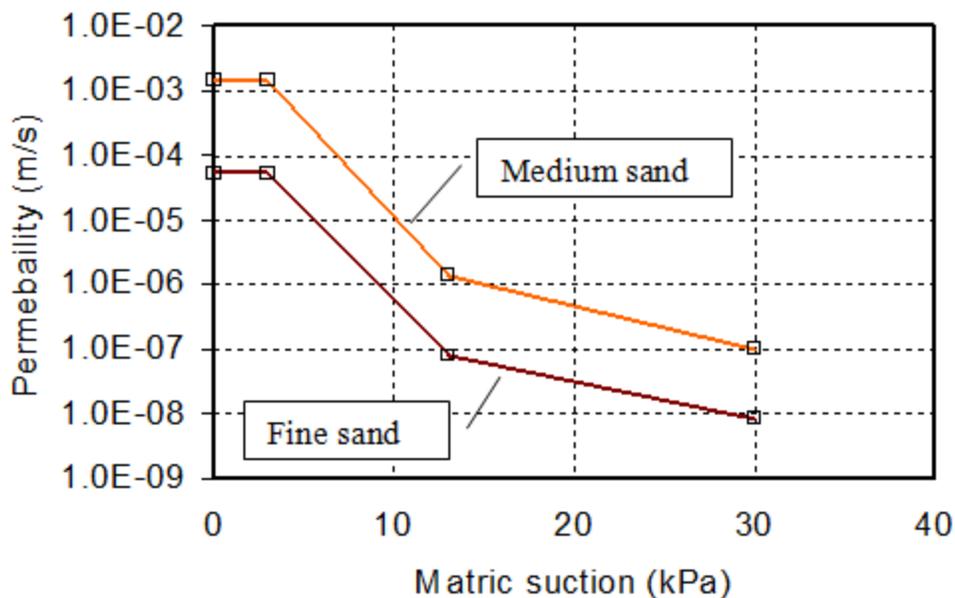


Figure 7-2: Permeability functions for materials used in model

The RS2 model geometry used in this example is shown in Figure 7-3.

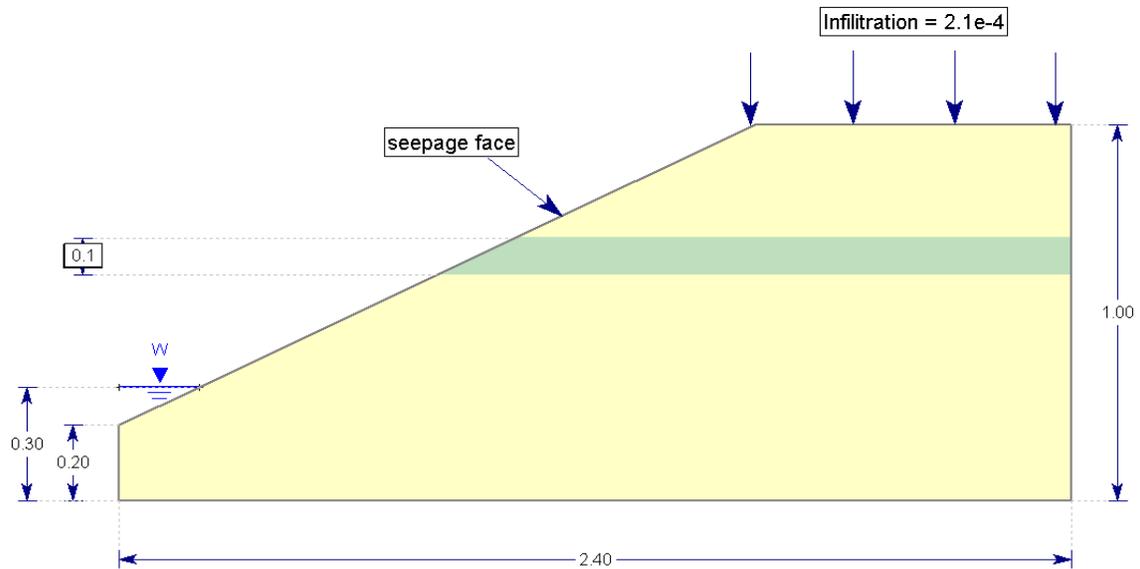


Figure 7-3: Layered slope model in RS2

A constant infiltration rate of $2.1 \cdot 10^{-4}$ m/s is applied to the top of the side of the slope. The water table is located at 0.3 m from the toe of the slope. The boundary condition at the slope face was assumed to be undefined, meaning that it initially either had flow, Q , or pressure head, P , equal to 0.

7.2 Analytical Solution

Fredlund and Rahardjo present their own finite element analysis for this problem in [2]. The resultant pressure head data are shown in the figure below.

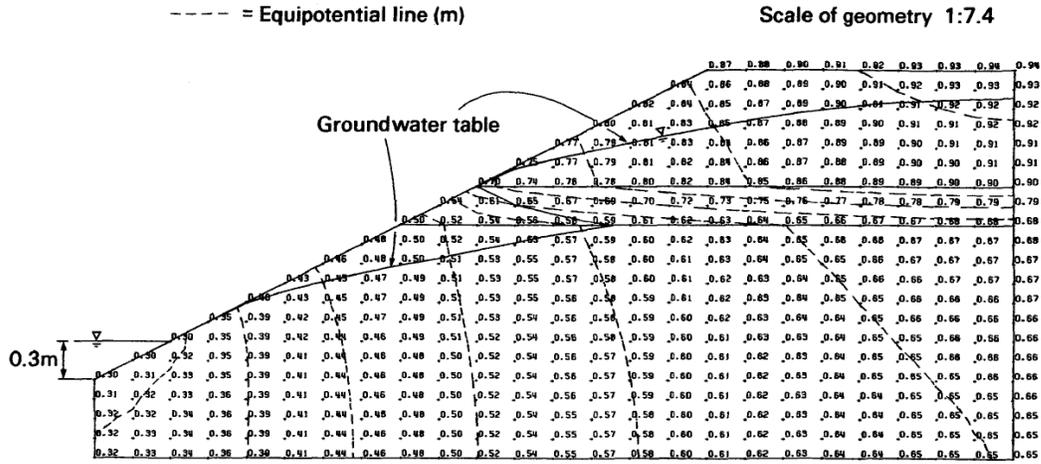


Figure 7-4: Hydraulic head data at $t = 208$ s for unsteady-state flow analysis in [1]

7.3 Results

Figure 7-5 shows the location of the calculated water table location and the direction of the flow vectors.

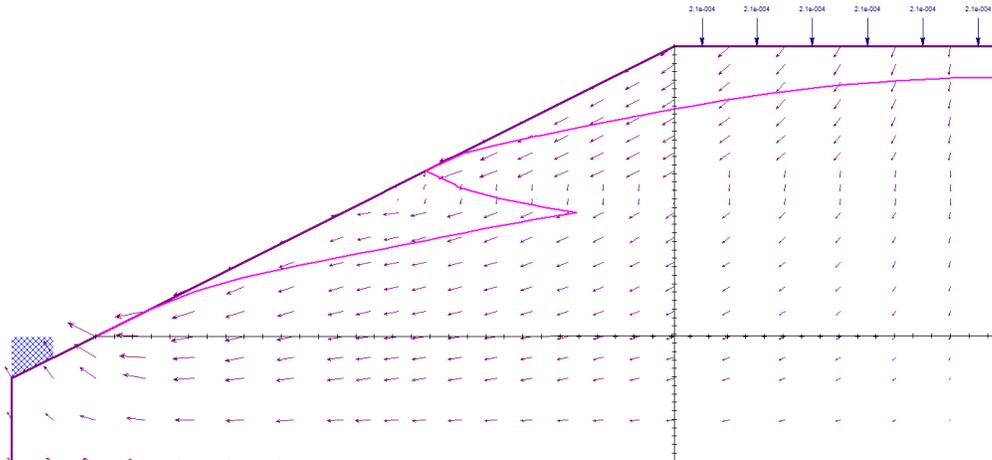


Figure 7-5: Flow vectors and phreatic surface as calculated by RS2 for isotropic earth dam with constant infiltration

Figure 7-6 and Figure 7-7 show contours of pressure head and total head from RS2, respectively.

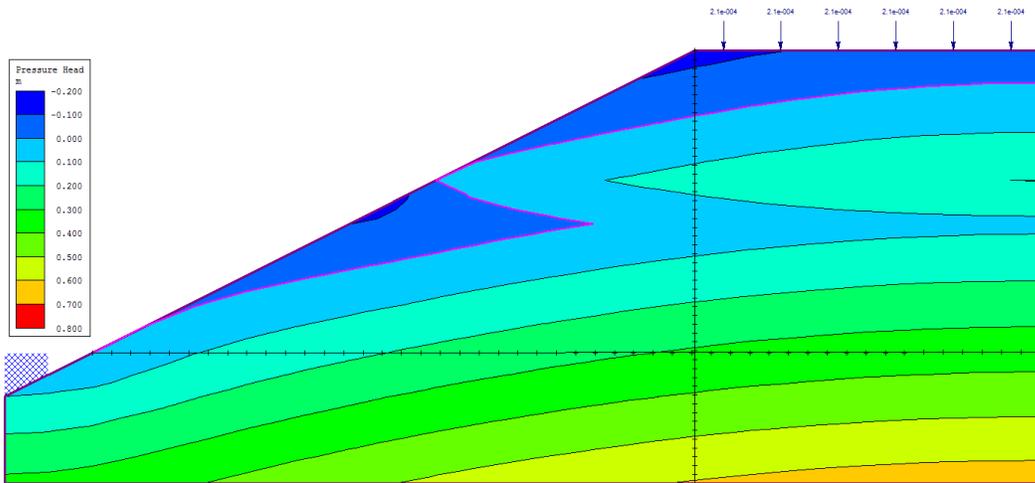


Figure 7-6: Pressure head contour plot for isotropic earth dam with constant surface infiltration

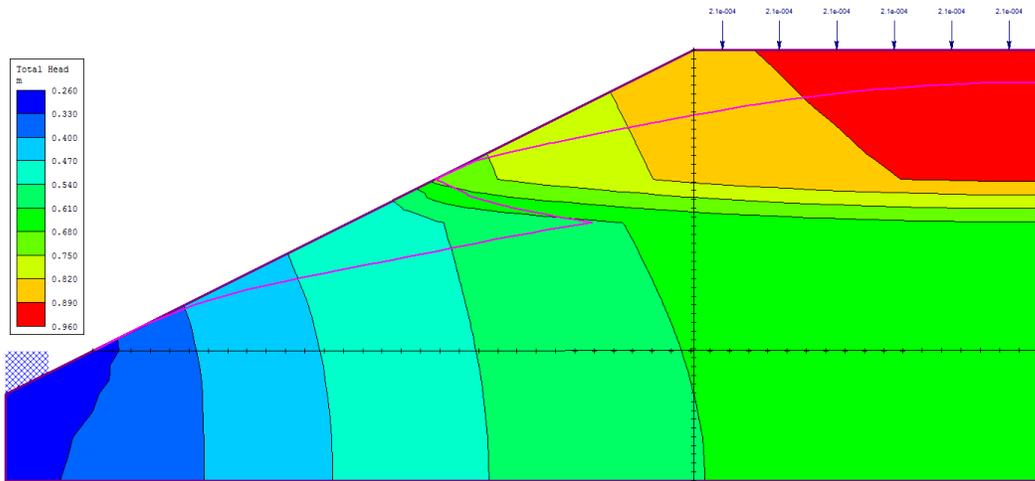


Figure 7-7: Total head contour plot for isotropic earth dam with constant surface infiltration

Figure 7-8 and Figure 7-9 compare the total head distributions along sections 1-1 and 2-2 (see Figure 7-1). RS2 results are in good agreement with [2].

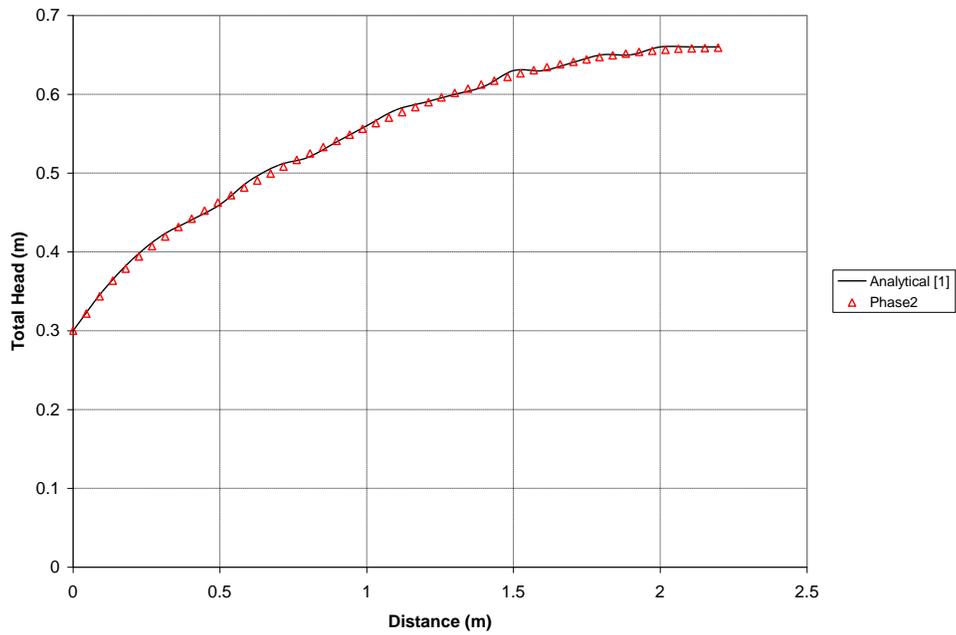


Figure 7-8: Total head variation along line 1-1

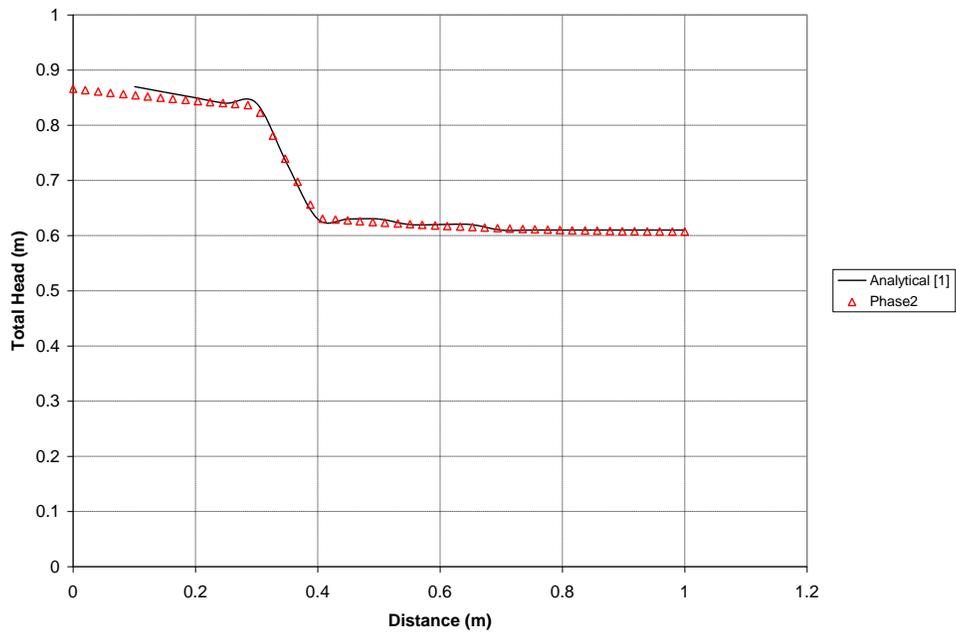


Figure 7-9: Total head variation along line 2-2

7.4 References

1. Fredlund, D.G. and H. Rahardjo (1993) *Soil Mechanics for Unsaturated Soils*, John Wiley

7.5 Data Files

The input data file **groundwater#007.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

8 Flow Through Ditch-Drained Soils

8.1 Problem Description

In problems related to ditch-drained aquifers, numerical solutions are often used to predict the level of the water table and the distribution of soil-water pressure. The problem considered in this section involves the infiltration of water downward through two soil layers.

The depth of the soil to the impermeable level is 0.5 m. The ditch is assumed to be water free. Figure 8-1 illustrates the problem.

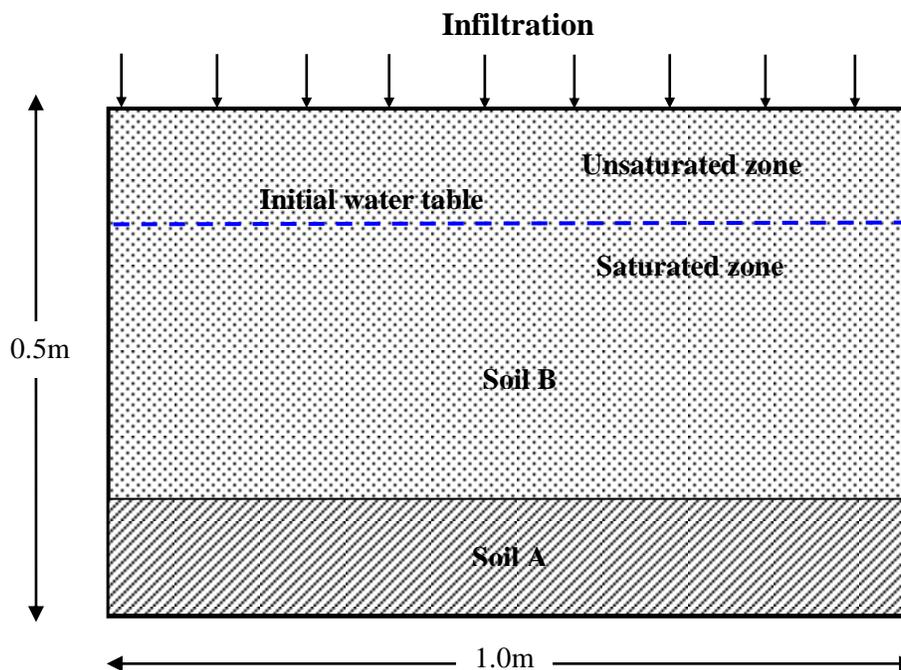


Figure 8-1: Drainage through multi-layered soil

The soil properties of the layered system are given in the following table, simulating a coarse and a fine soil. The lower layer has a thickness of 0.1 m. The rate of incident rainfall (infiltration) is taken to be equal to $4.4e-6$ m/s.

Table 8.1 summarizes the soil parameters used.

Table 8.1: Soil conductivity and Gardner's parameters

Soil A	<i>Relative Conductivity</i>	1.11e-3 (m/s)
	<i>Gardner's parameters</i>	$a = 1000, n = 4.5$
Soil B	<i>Relative Conductivity</i>	1.11e-4 (m/s)
	<i>Gardner's parameters</i>	$a = 2777.7, n = 4.2$

The RS2 model for the problem is shown in Figure 8-2.

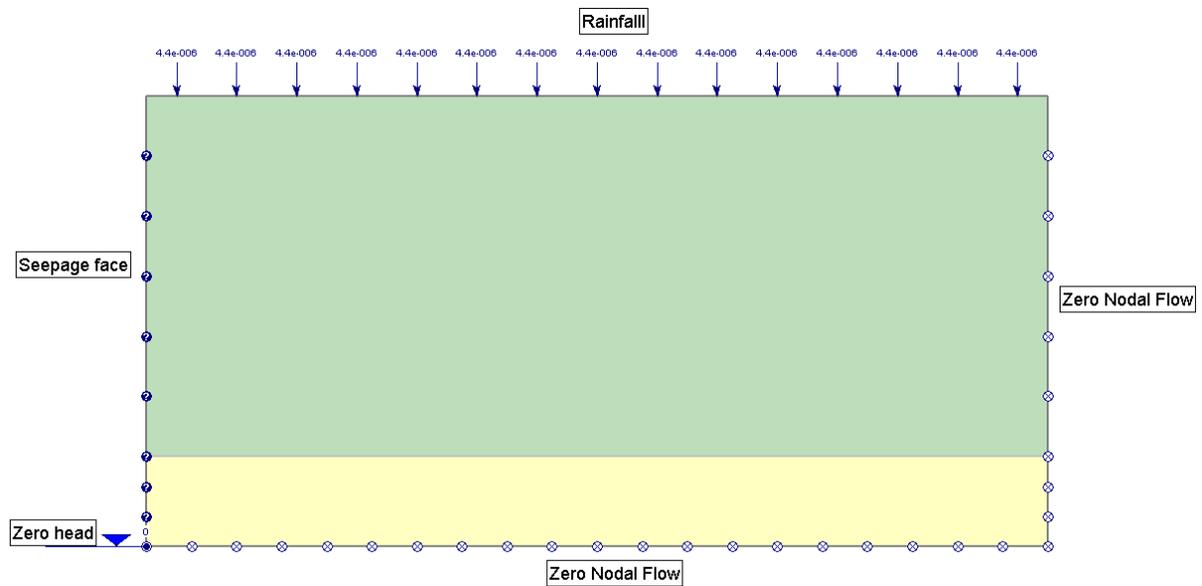


Figure 8-2: Multi-layered soil model in RS2

The problem is modeled using 459 three-noded triangular finite elements.

8.2 Analytical Solution

An alternative finite element solution for this problem can be found in Gureghian (1981) [1]. A sketch of the problem with pressure head contours is shown in Figure 8-3.

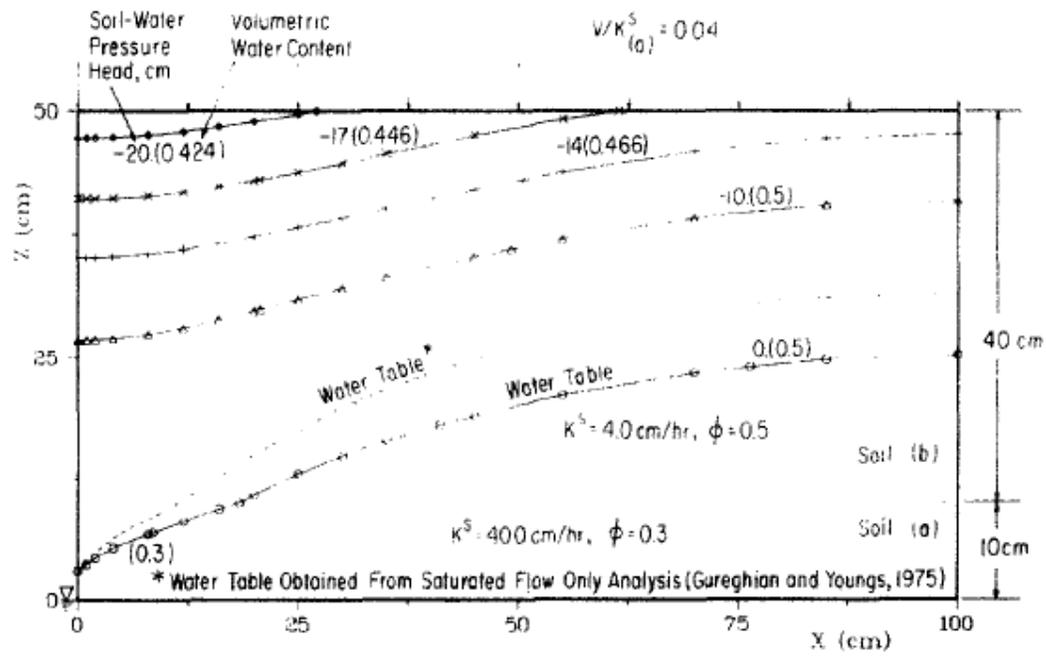


Figure 8-3: Pressure head contours for layered soil problem, as developed in [1]

8.3 Results

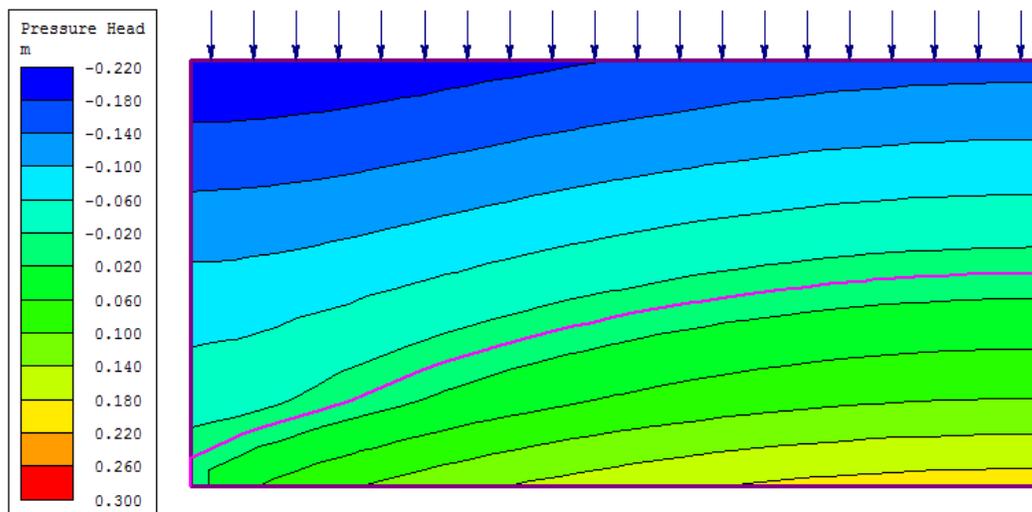


Figure 8-4: Pressure head contour plot for multi-layered soil in RS2

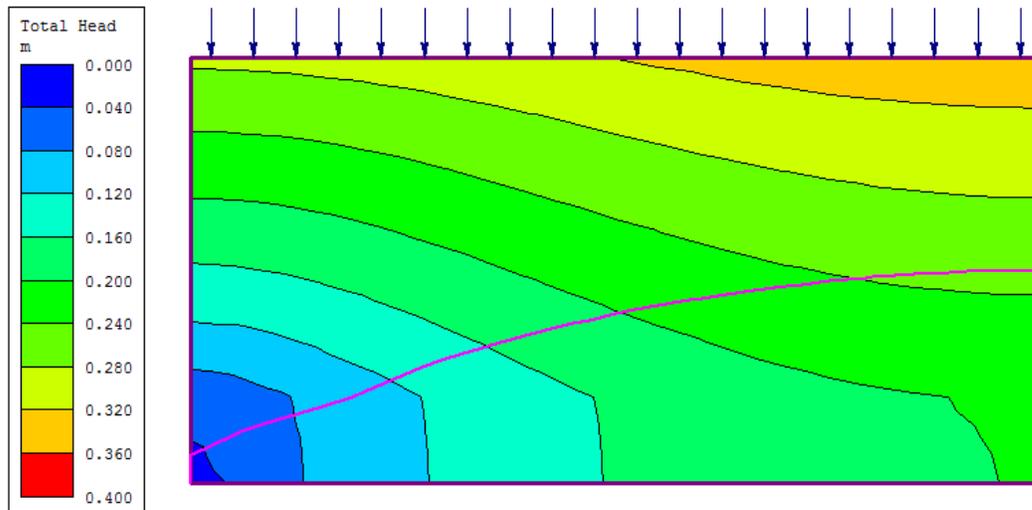


Figure 8-5: Total head contour plot for multi-layered soil in RS2

Figure 8-4 gives the distribution of the soil-water pressure head for the unsaturated regime above the water table. The computed total head contours are presented Figure 8-5. The RS2 results are in close agreement with the solution provided in [1].

8.4 References

1. Gureghian A. (1981), "A two dimensional finite element solution scheme for the saturated-unsaturated flow with application to flow through ditch drained soils:" J. Hydrology. (50), 333-353.

8.5 Data Files

The input data file **groundwater#008.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

9 Seepage Through Dam

9.1 Problem Description

Seepage flow rate through earth dams is examined in this section. The geometry and material properties for two earth dams are taken from Bowels' *Physical and geotechnical properties of soils* [1].

9.1.1 Homogeneous dam

The seepage rate through a homogeneous dam is verified in this section. This problem is presented on p. 295 of [1]. Figure 9-1 shows detailed geometry of the dam. A total head of 18.5 m is applied on the left side of the dam and the seepage flow rate is calculated on the right side of the dam. A customized permeability function is used to model the material conductivity for the saturated-unsaturated zone (Figure 9-2). This hydraulic conductivity function is similar to the one presented in Chapuis et al. [2]. The dam is discretized using 4-noded quadrilateral finite elements. A total of 391 finite elements are used for the mesh.

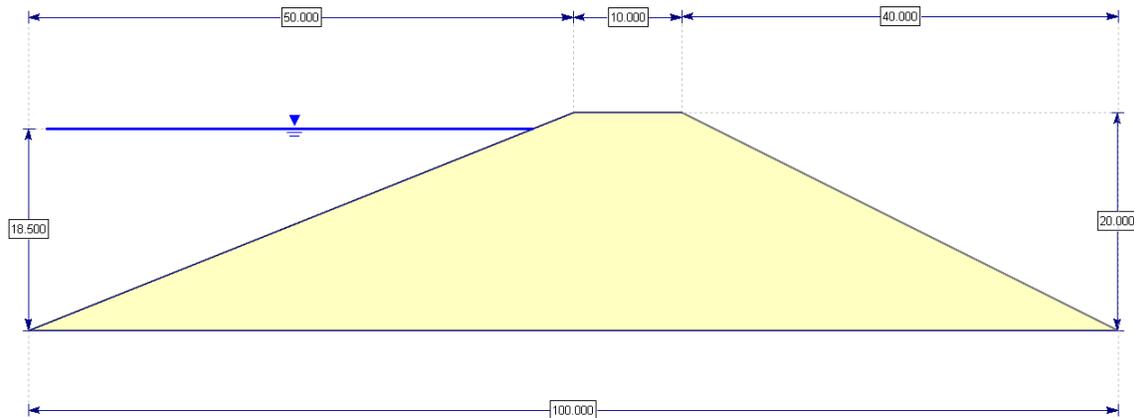


Figure 9-1: Homogenous earth dam as modeled in RS2

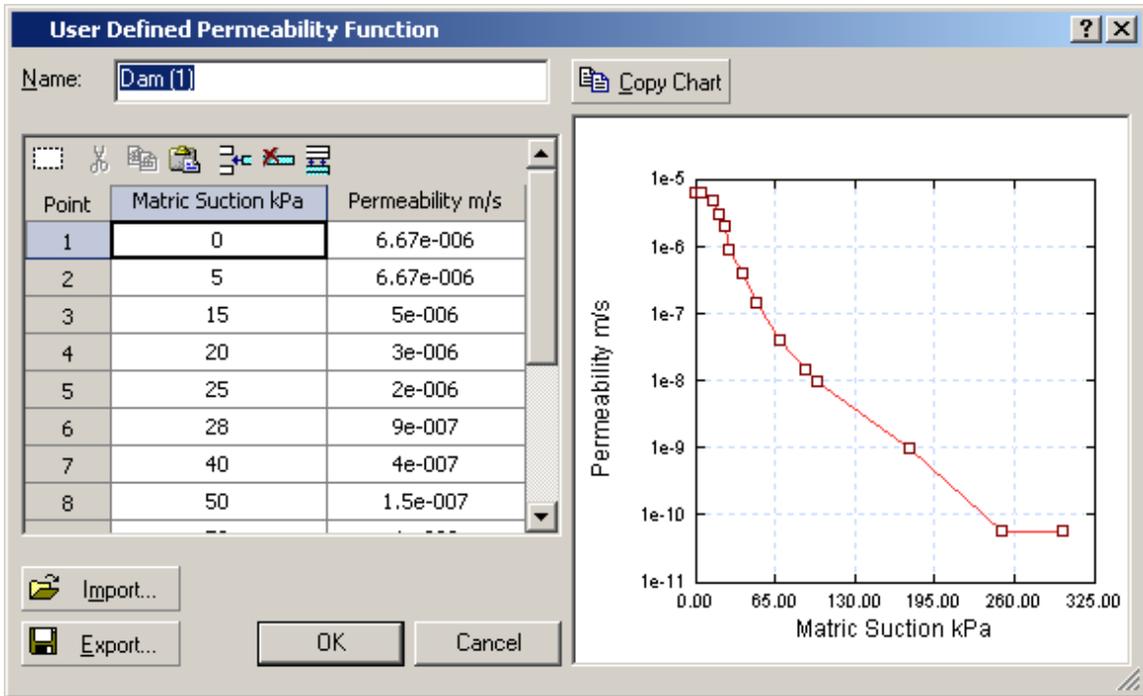


Figure 9-2: Permeability function for the isotropic earth dam

9.1.2 Dam with impervious core

The second problem in this section considers a dam with an impervious core (Figure 9-3). The hydraulic permeability for the dam and the drain material are assumed to follow the functions shown in Figure 9-4.

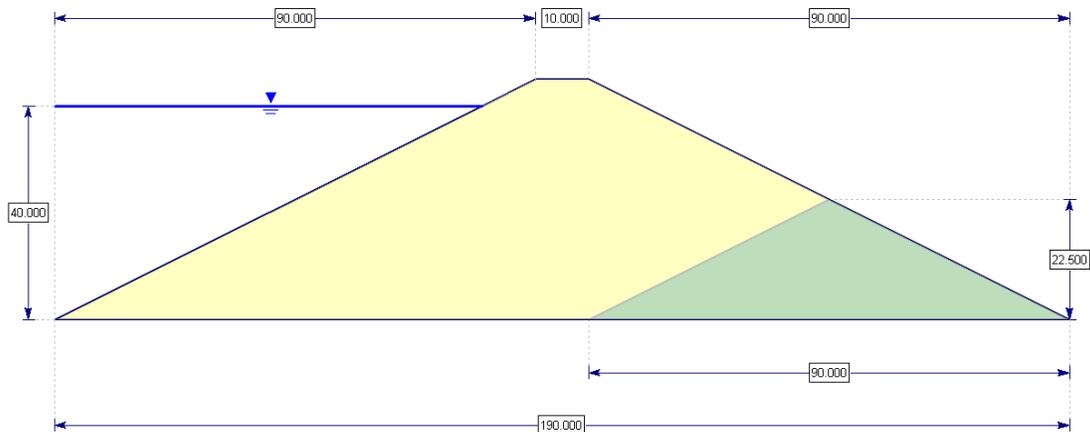


Figure 9-3: Dam with impervious core geometry detail

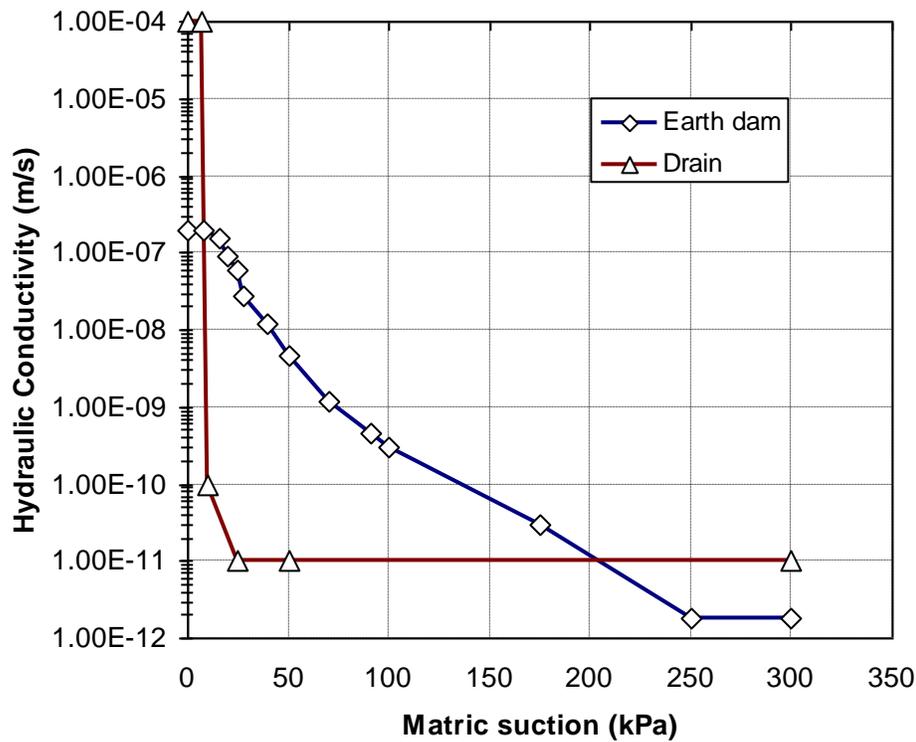


Figure 9-4: Permeability function for isotropic earth dam and drain

9.2 Analytical Solution

Bowles calculated the leakage flow rate through these dams using flow net techniques, which neglect the unsaturated flow. Chapuis et. al. [2] solved the same examples using SEEP/W, a finite element software package. RS2 results are compared with both Bowles [1] and SEEP/W [2] results.

9.3 Results

9.3.1 Homogeneous dam

RS2 predicts a flow rate of $Q = 1.378 \cdot 10^{-3} \text{ m}^3/(\text{min} \cdot \text{m})$ which compares well with the flow rate estimated by Bowles [1], which used two approximate methods that neglect the unsaturated flow. Bowles' two methods gave $Q = 1.10 \cdot 10^{-3}$ and $1.28 \cdot 10^{-3} \text{ m}^3/(\text{min} \cdot \text{m})$. Chapuis et al. [2] solved the same example using finite element software SEEP/W. The flow rate calculated using SEEP/W was $1.41 \cdot 10^{-3} \text{ m}^3/(\text{min} \cdot \text{m})$ for a mesh of 295 elements and a flow rate of $1.37 \cdot 10^{-3} \text{ m}^3/(\text{min} \cdot \text{m})$ for a mesh of 1145 elements.

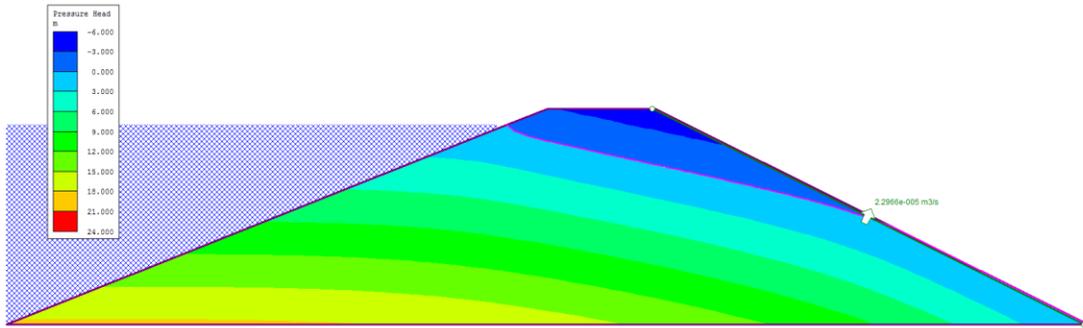


Figure 9-5: Pressure head contours for homogenous dam in RS2

Figure 9-5 presents the flow vectors and the location of the phreatic line from RS2 ground water model. Figure 9-6 shows the contours of total head with flow lines in the homogenous dam.

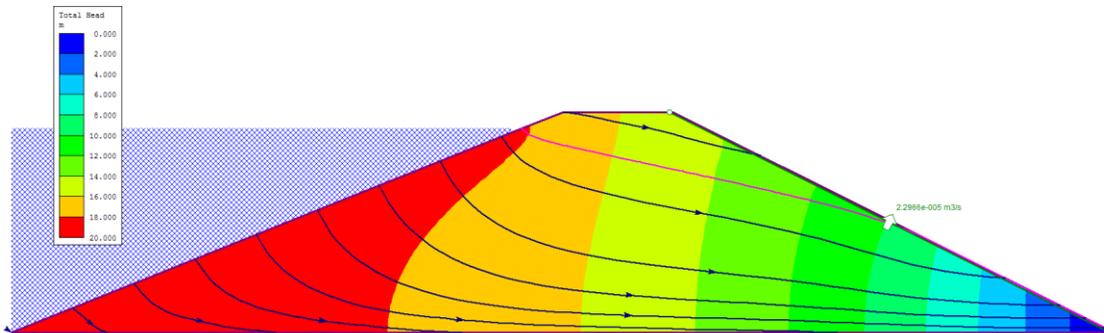


Figure 9-6: Total head contours and flow lines for homogenous dam in RS2

9.3.2 Dam with impervious core

RS2 predicts a flow rate of $Q = 4.23 \cdot 10^{-6} \text{ m}^3/(\text{min} \cdot \text{m})$ which compares well with the flow rate estimated by Bowels [1], $Q = 3.8 \cdot 10^{-6} \text{ m}^3/(\text{min} \cdot \text{m})$. Chapuis et al. [2] solved the same example using finite element software SEEP/W. The flow rate calculated using SEEP/W was $5.1 \cdot 10^{-6} \text{ m}^3/(\text{min} \cdot \text{m})$ for a coarse mesh and $4.23 \cdot 10^{-6} \text{ m}^3/(\text{min} \cdot \text{m})$ for a finer mesh of 2328 elements.

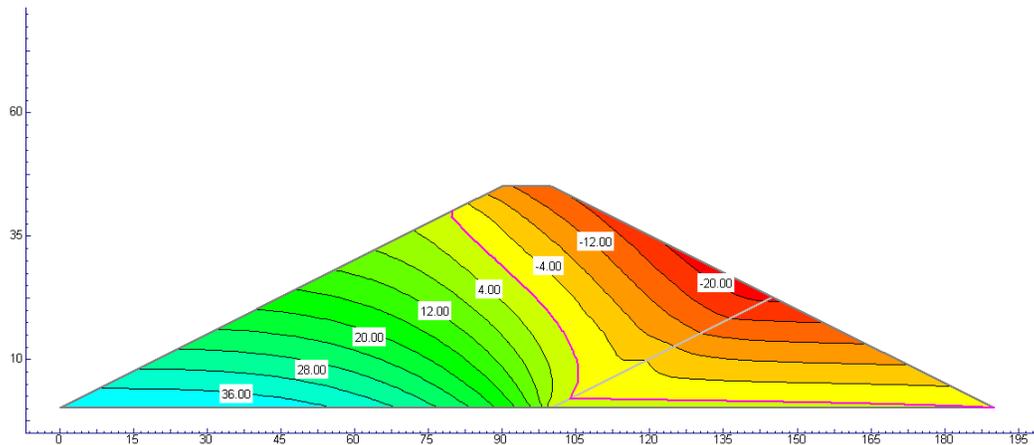


Figure 9-7: Pressure head contours for isotropic dam with impermeable core

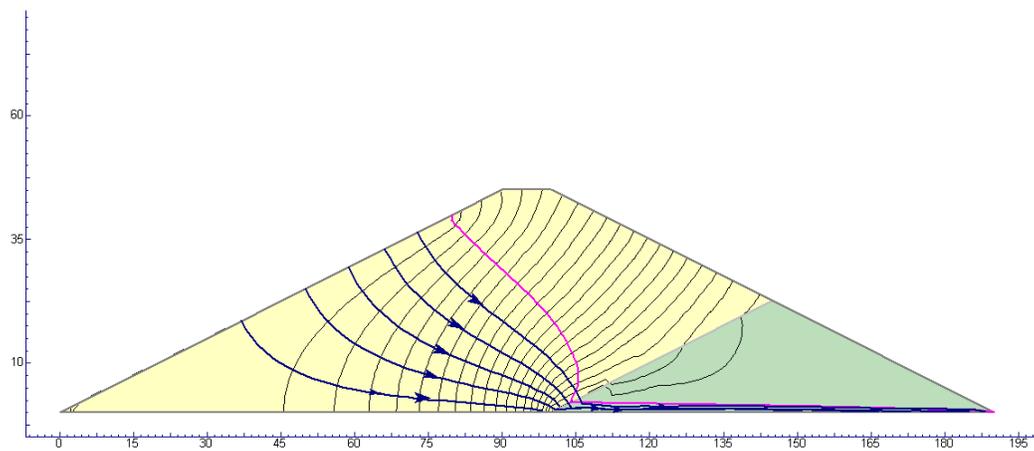


Figure 9-8: Total head contours and flow lines for isotropic dam with impermeable core

9.4 References

1. Bowles J.E. (1984), *Physical and geotechnical properties of soils*. 2nd Ed. McGraw Hill, New York.
2. Chapuis, R., Chenaf D, Bussiere, B. Aubertin M. and Crespo R. (2001), "A user's approach to assess numerical codes for saturated and unsaturated seepage conditions", *Can Geotech J.* **38**: 1113-1126.

9.5 Data Files

The input data files **groundwater#009_01.fez** and **groundwater#009_02.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

10 Steady-State Unconfined Flow Using Van Genuchten Permeability Function

10.1 Problem Description

Unconfined flow in a rectangular domain was analyzed in this section. The sensitivity of seepage face height to the downstream head is examined. The Van Genuchten [1] closed form equation for the unsaturated hydraulic conductivity function is used to describe the soil properties for the soil model. A Dupuit-Forcheimer model [2], which assumes equipotential surfaces are vertical and flow is essentially horizontal, is also used for comparison.

A 10 m x 10 m square embankment has no-flow boundary conditions on the base and at the top. The water level at the left boundary is 10 m. Four different water levels (2, 4, 6 and 8 m) at the downstream boundary are considered. The soil has a saturated conductivity of $K_s = 1.1574 \cdot 10^{-5}$ m/s. The values of the Van Genuchten soil parameters are $\alpha = 0.64\text{m}^{-1}$, $n = 4.65$. The geometry and the mesh discretization are shown in Figure 10-1.

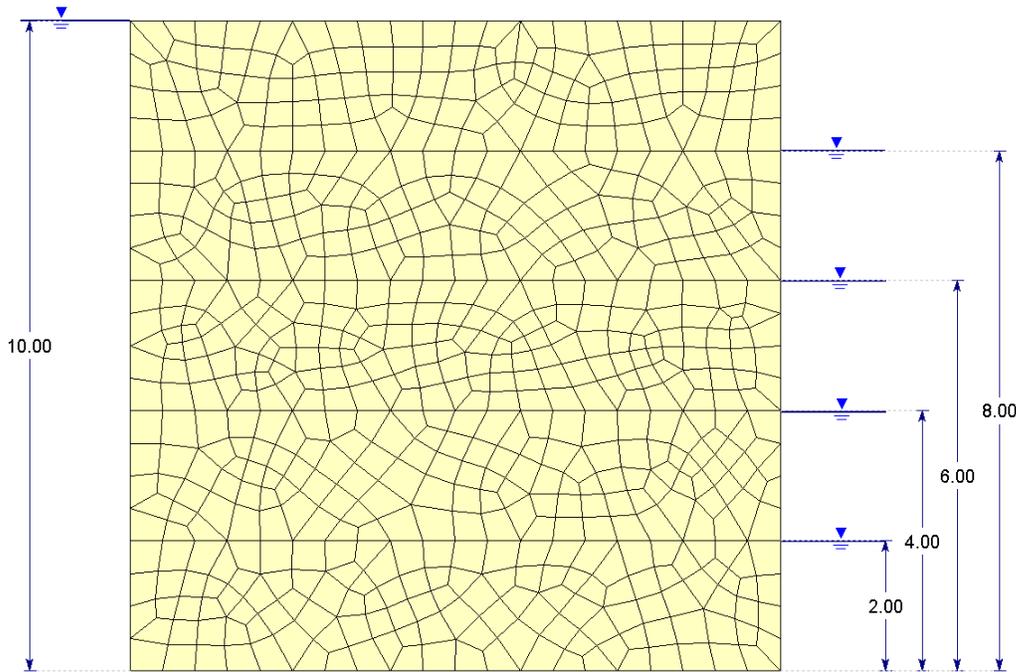


Figure 10-1: RS2 model and meshing for rectangular embankment

10.2 Analytical Solution

Figure 10-2 shows the theoretical phreatic lines for each downstream head developed in [2] using the Dupuit-Forcheimer model.

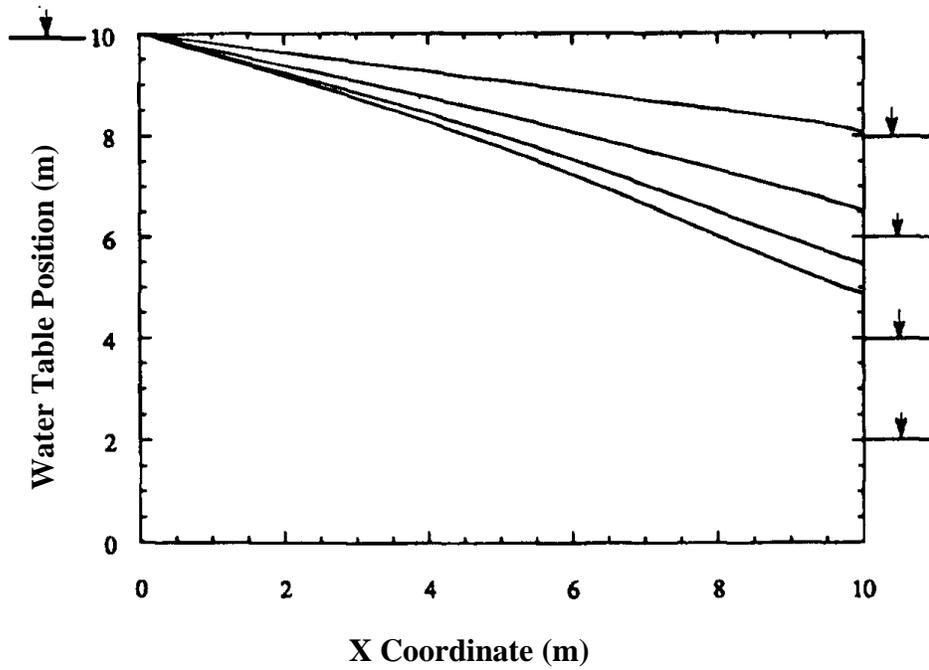


Figure 10-2: Phreatic surfaces with variable downstream head [2]

10.3 Results

Figure 10-3 shows the variation of the phreatic surface with changing downstream water level predicted by RS2. It can be seen that the absolute length of the seepage face decreases significantly with an increase in the water level at the downstream boundary.

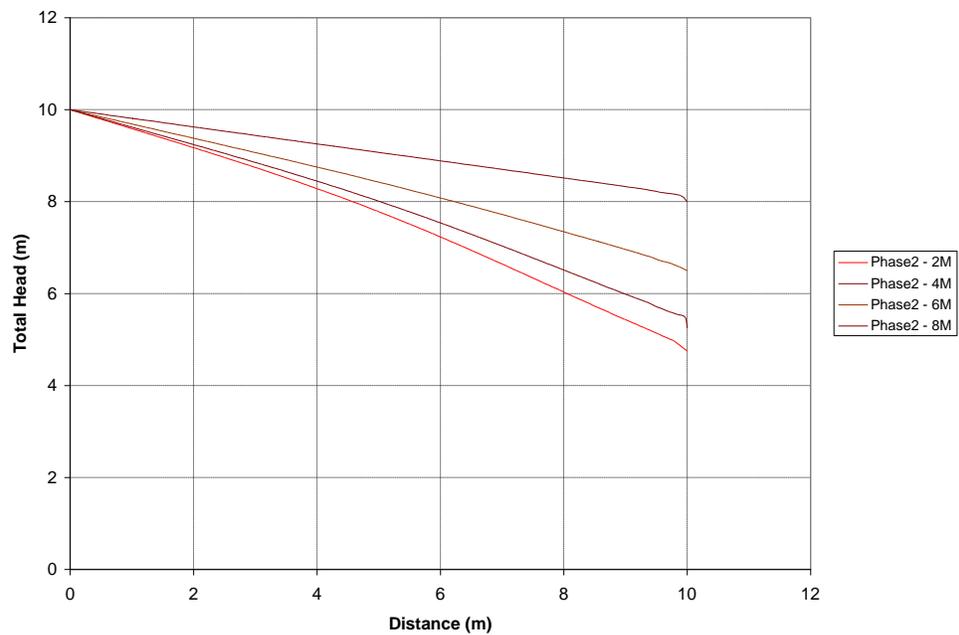


Figure 10-3: Phreatic surfaces for various downstream water levels in RS2

Table 10.1 presents comparison of discharge values and seepage face from [2] and RS2.

Table 10.1: Discharge velocities and seepage face dimensions

	<i>Model Dimensions</i>	<i>Downstream Water Level (m)</i>	<i>Discharge (m/sec)</i>	<i>Seepage face (m)</i>
Clement et. al. [2]	10m x 10m	2	6.0764×10^{-5}	4.8
RS2	10m x 10m	2	6.0659×10^{-5}	5.0

10.4 References

1. Genuchten, V. M (1980), "A closed equation for predicting the hydraulic conductivity of unsaturated soils" , Soils Sci Soc Am J. **44**: 892-898
2. Clement, T.P, Wise R., Molz, F. and Wen M. (1996), "A comparison of modeling approaches for steady-state unconfined flow", J. of Hydrology **181**: 189-209

10.5 Data Files

The data input file **groundwater#010.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

11 Earth and Rock-Fill Dam Using Gardner Permeability Function

11.1 Problem Description

Seepage in a uniform earth and rock-fill dam is examined in this section. Nonlinear modeling is used to represent the seepage flow above and below the free surface. Gardner's nonlinear equation [1] between permeability function k_w and pressure head is used in this section and it can be presented as

$$k_w = \frac{k_s}{1 + ah^n}$$

where a and n are the Gardner parameters

h = pressure head (suction)

k_w = permeability

k_s = saturated permeability

11.1.1 Uniform earth and rock-fill dam

Figure 11-1 shows detailed geometry of the first dam studied. The upstream elevation head is 40 m and the downstream elevation head is 0 m. The geometry of the dam is taken from [2]; the slope of the upstream face is 1:1.98 and the slope of the downstream face is 1:1.171 (Figure 11-1). Gardner's parameters are assigned values of $a = 0.15$ and $n = 6$.

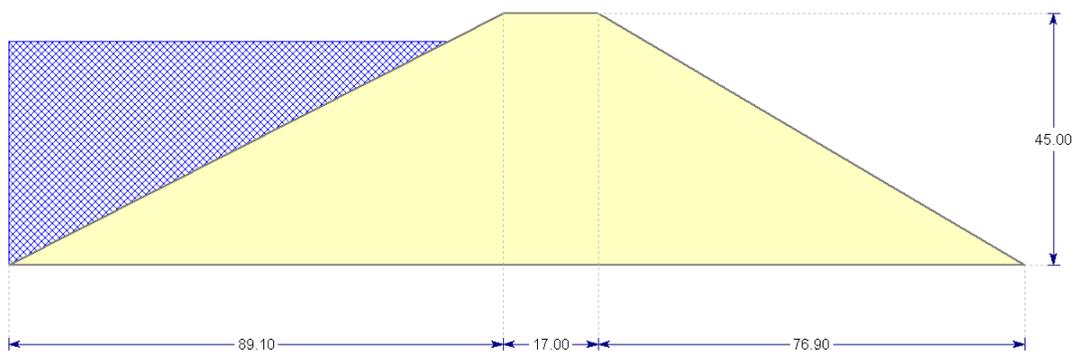


Figure 11-1: Uniform earth and rock-fill dam model geometry

11.1.2 Heterogeneous earth and rock-fill dam

Figure 11-2 shows a dam with a permeable foundation and toe drain [2]. The permeability coefficient of the foundation of sand layer is 125 times that of the earth dam

and blanket. The toe drain has a permeability coefficient 10000 times larger than that of the dam. Table 11.1 shows the Gardner’s parameters for the different model layers.

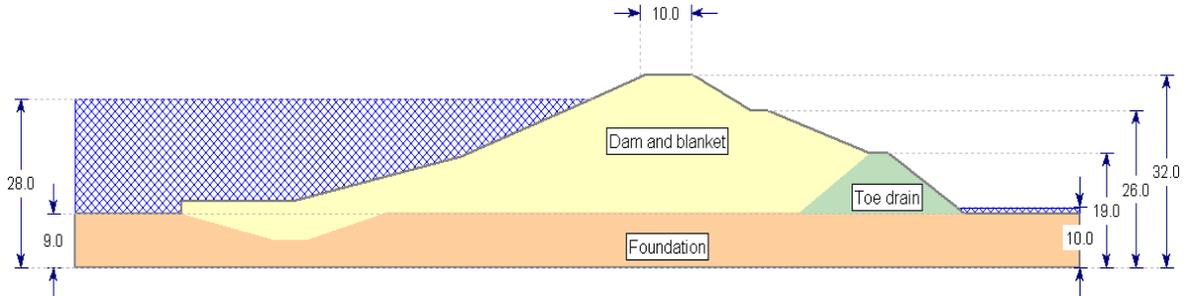


Figure 11-2: Heterogeneous dam with permeable foundation and toe drain

Table 11.1: Material parameters for heterogeneous dam

Layer	K_s (m/s)	a	n
Dam	1×10^{-7}	0.15	2
Foundation	1.25×10^{-5}	0.15	6
Toe drain	1×10^{-3}	0.15	6

11.2 Analytical Solution

For this problem, RS2 results are compared to those obtained using ABAQUS commercial software, which are presented by Zhang et al. in [2].

11.3 Results

11.3.1 Uniform earth and rock-fill dam

Figure 11-3 shows the pressure head contour plot produced by RS2, which indicates that the elevation of the release point on the downstream face is 19.404 m. This compares well to ABAQUS results from [2], which predict an elevation of 19.64 m for identical dam geometry.

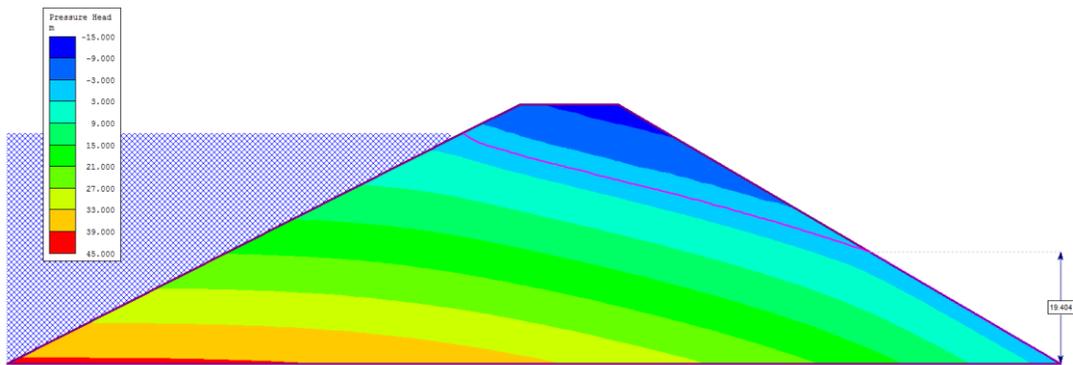


Figure 11-3: Pressure head contour plot in RS2, indicating elevation of release point

11.3.2 Heterogeneous earth and rock-fill dam

Figure 11-4 and Figure 11-5 show the distribution of the total head contours from [2] and RS2 respectively. RS2 results were in a good agreement with those obtained from ABAQUS.

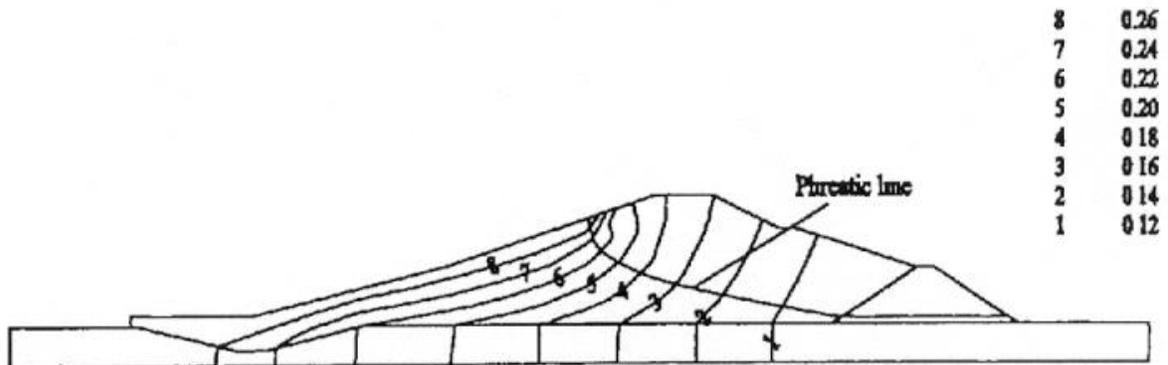


Figure 11-4: Total head contours for heterogeneous dam [2]. Units in $m \cdot 10^2$.

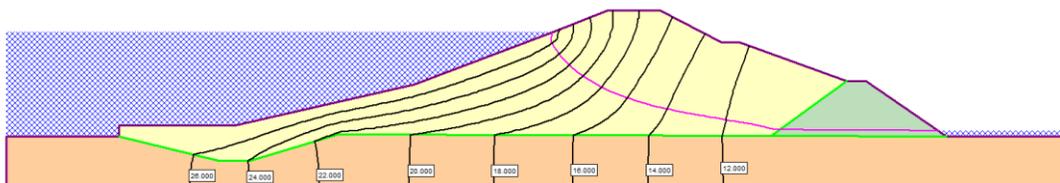


Figure 11-5: Total head contour plot in RS2

11.4 References

1. Gardner, W. (1956), "Mathematics of isothermal water conduction in unsaturated soils." Highway Research Board Special Report 40 International Symposium on Physico-Chemical Phenomenon in Soils, Washington D.C. pp. 78-87.
2. Zhang, J, Xu Q. and Chen Z. (2001), "Seepage analysis based on the unified unsaturated soil theory", Mechanics Research Communications, **28** (1) 107-112.

11.5 Data Files

The input data files **groundwater#011_01.fez** and **groundwater#011_02.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

12 Seepage from Trapezoidal Ditch into Deep Horizontal Drainage Layer

12.1 Problem Description

Seepage from a trapezoidal ditch into a deep horizontal drainage layer is analyzed in this section. The geometry of the problem is depicted in Figure 12-1.

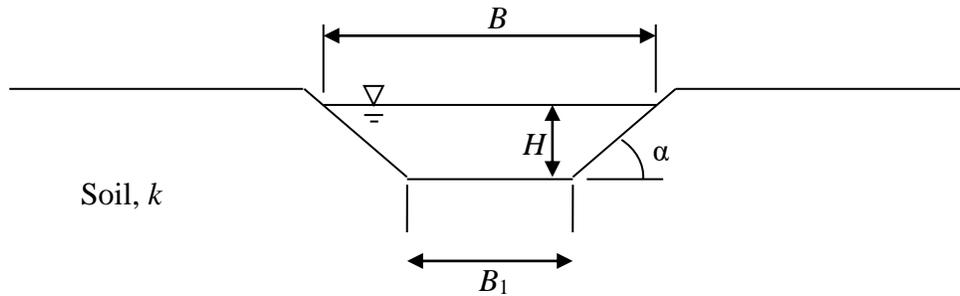


Figure 12-1: Seepage from a trapezoidal ditch

The RS2 model for the problem described in the previous section is shown in Figure 12-2. Owing to symmetry, only half of the problem was modeled.

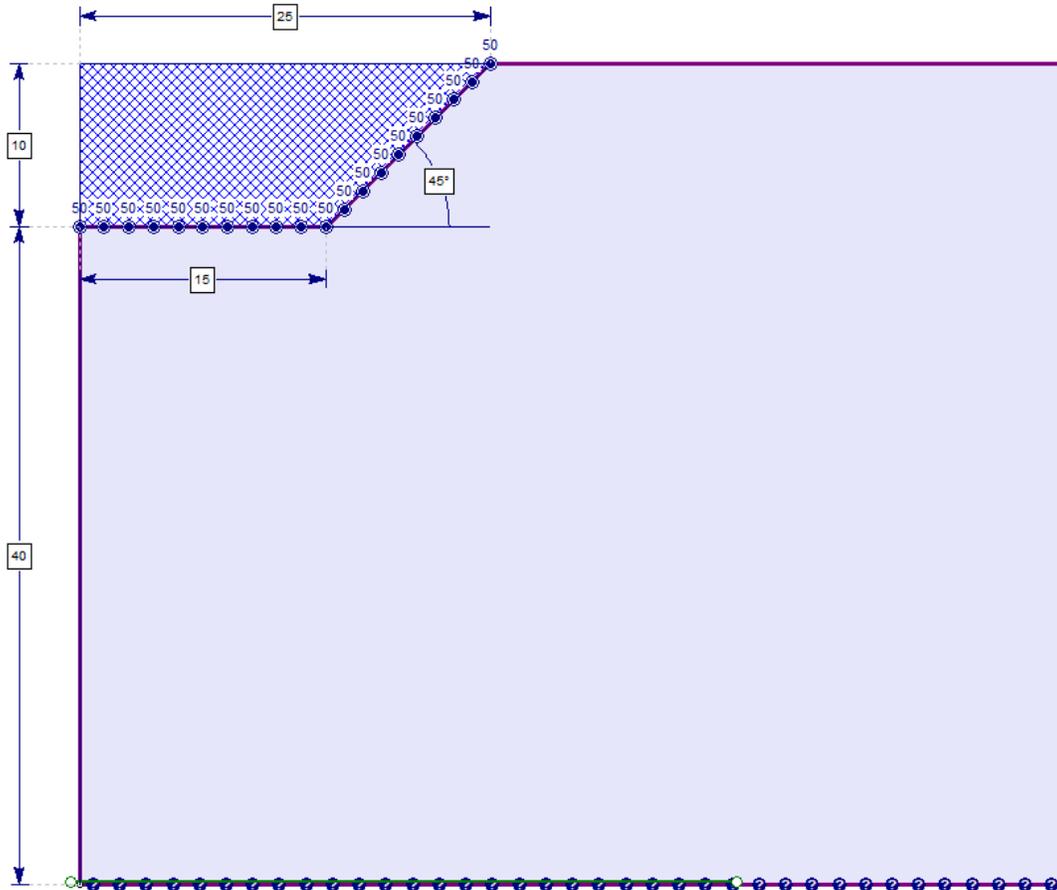


Figure 12-2: RS2 model of trapezoidal ditch and deep drainage layer

The RS2 model uses the following input parameters:

- Ditch half-width $B/2 = 25$ m
- Ditch depth $H = 10$ m
- Bank angle $\alpha = 45^\circ$
- Soil hydraulic conductivity $k = 10^{-5}$ m/s

12.2 Analytical Solution

Vedernikov (1934) proposed a direct method to solve for the seepage from such a ditch. He proposed the following equation for calculating the flow:

$$q = k(B + AH)$$

where A is a function of B/H and $\cot \alpha$. In this example, we will use $B = 50$ m, $H = 10$ m and $\alpha = 45^\circ$ which will yield a value of $A = 3$ [1].

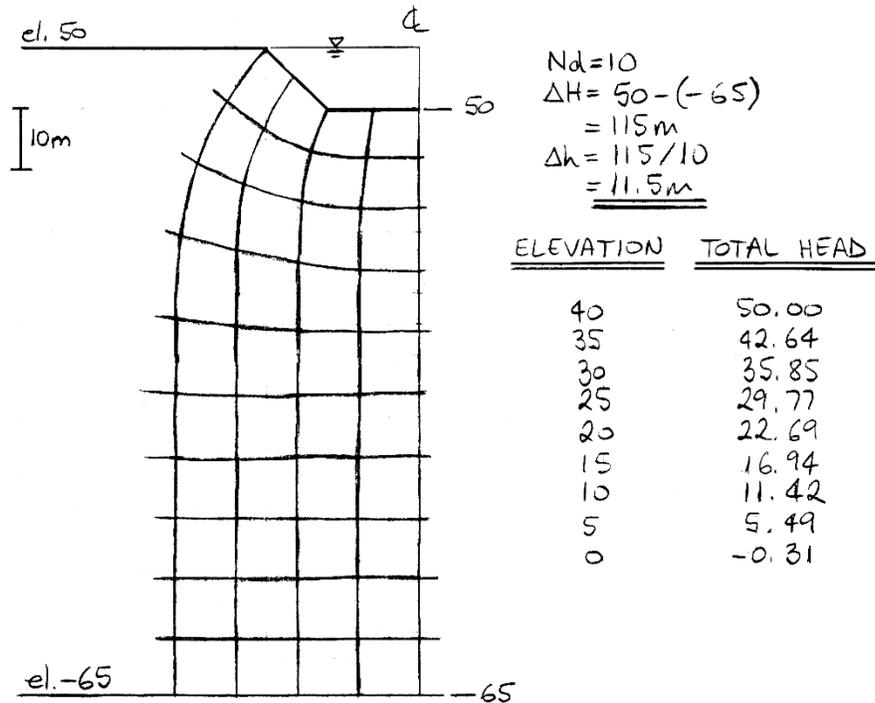


Figure 12-4: Hand-drawn flow net according to Vedernikov's boundary conditions

12.3 Results

A discharge section was added to the RS2 model to compute the flow at the lower boundary and compare it to the Vedernikov solution. The results are depicted in Figure 12-5. RS2 is in good agreement with the flow net shown in Figure 12-3.

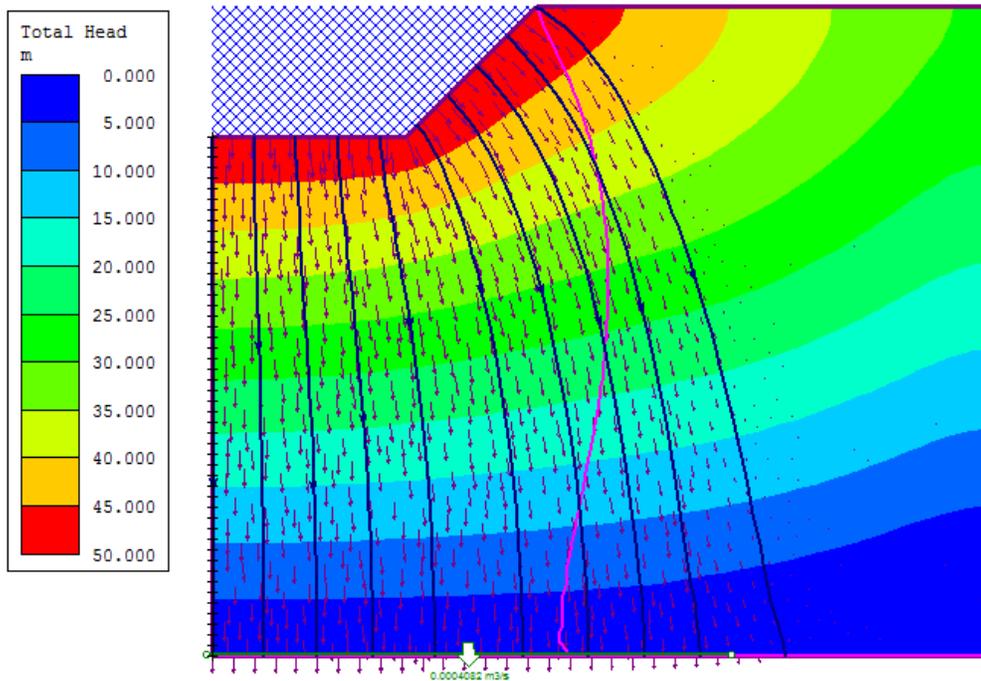


Figure 12-5: Flow net and flow vector plot generated by RS2

The discharge section shows a flow of 0.0004082 m/s through the model. The total flow from the trapezoidal ditch is thus 0.000816 m³/s. Upon analysis of the flow vectors, the seepage zone appears to be approximately 42 m wide, equivalent to an 84 m seepage zone when symmetry is accounted for. These results are similar to those in [1], which predict a seepage zone 80 m wide and a flow of 0.008 m³/s.

A material query was added at the center of the ditch to obtain the total head distribution along the vertical cross-section immediately underlying the ditch. Figure 12-6 plots total head as a function of depth and compares RS2 results to those drawn from the flow net in Figure 12-4.

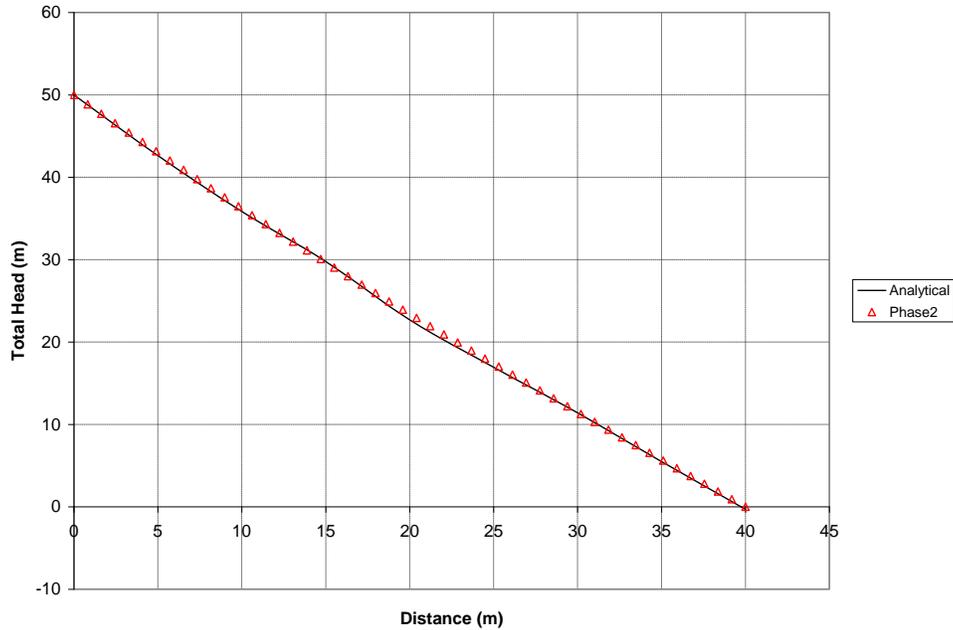


Figure 12-6: Comparison of RS2 and analytical solutions for total head distribution below centre of ditch

12.4 References

1. Haar, M. E. (1990), *Groundwater and Seepage*, 2nd Edition, Dover.

12.5 Data Files

The input data file **groundwater#012.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

13 Seepage from a Triangular Ditch into a Deep Horizontal Drainage Layer

13.1 Problem Description

Seepage from a triangular ditch into a deep horizontal drainage layer is analyzed in this section. The geometry of the problem is depicted in Figure 13-1.

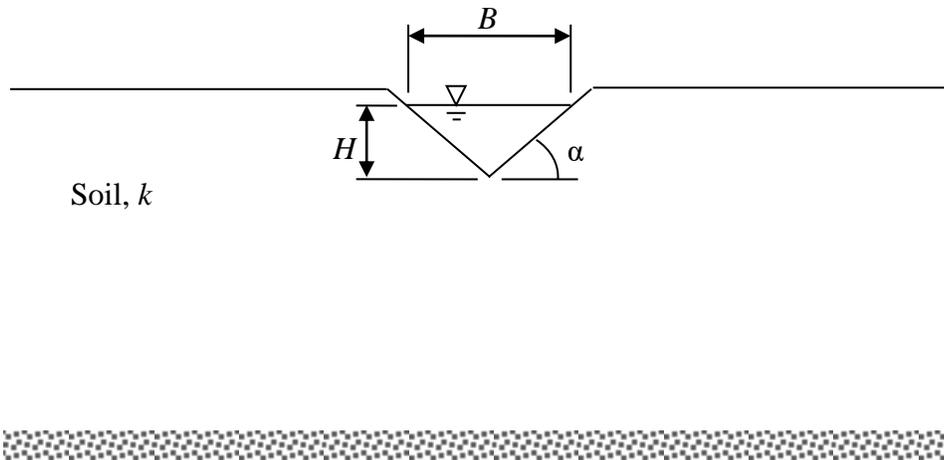


Figure 13-1: Triangular ditch with deep underlying drainage layer

The RS2 model for the problem described in the previous section is shown in Figure 13-2. Only half of the problem was modeled due to symmetry.

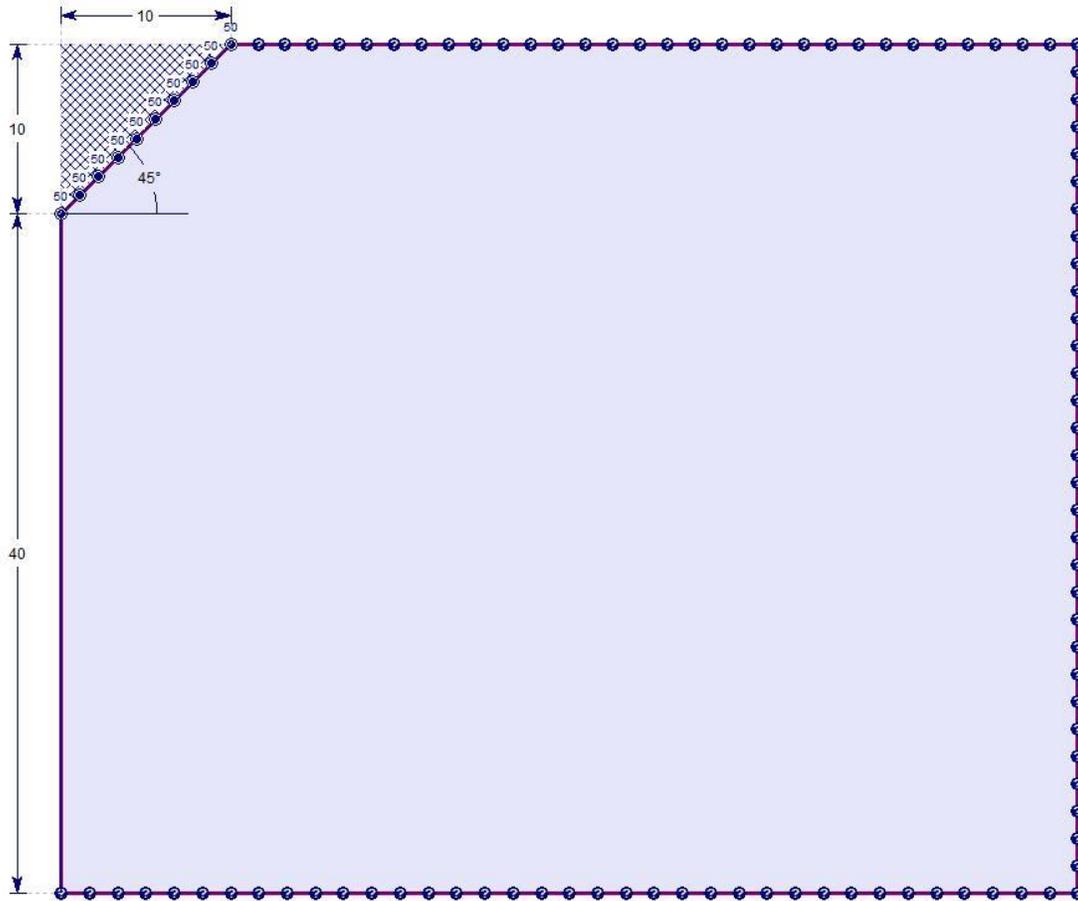


Figure 13-2: Triangular ditch as constructed in RS2

The RS2 model uses the following input parameters:

- Ditch half-width $B/2 = 10$ m
- Ditch depth $H = 10$ m
- Bank slope $\alpha = 45^\circ$
- Soil conductivity $k = 10^{-3}$ m/s

13.2 Analytical Solution

Vedernikov (1934) proposed a direct method to solve for the seepage from such a ditch. He proposed the following equation for calculating the flow:

$$q = k(B + AH)$$

where A is a function of α . In this example, we will use $B = 20$ m, $H = 10$ m and $\alpha = 45^\circ$, which will yield a value of $A = 2$ [1].

He also proposed the following equation for calculating the width of the flow at an infinite distance under the bottom of the ditch:

$$L = B + AH$$

Using these equations, the flow through the system was calculated to be $0.04 \text{ m}^3/\text{s}$. The width of the seepage zone was calculated to be 40 m.

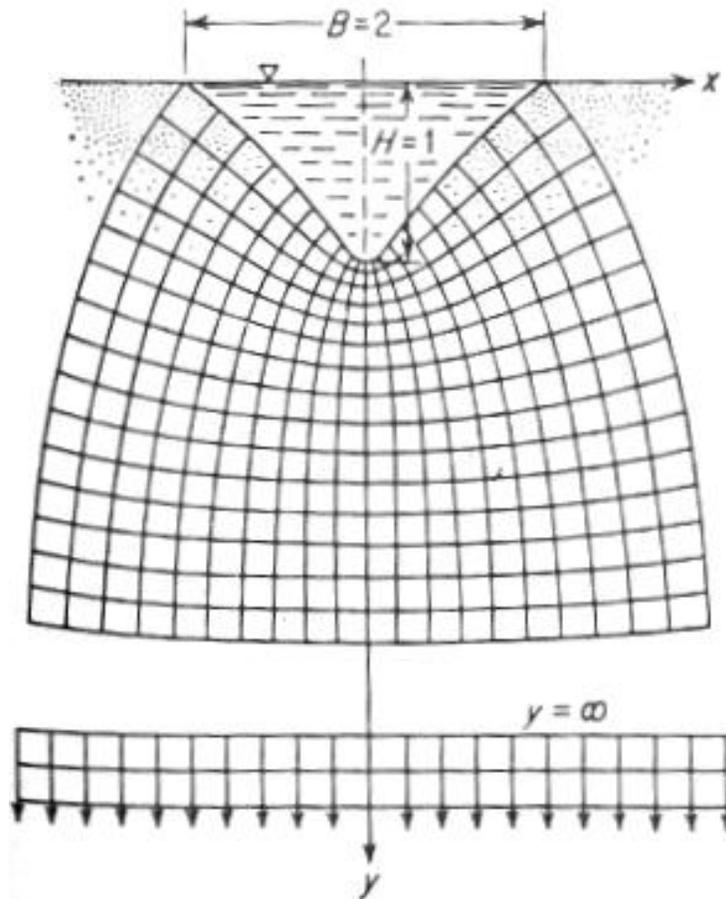


Figure 13-3: Theoretical flow net beneath triangular ditch from Harr (1990) [1]

To determine the total head variation with depth immediately beneath the ditch, a flow net was drawn by hand using Vedernikov's boundary conditions (width of seepage zone, depth to horizontal equipotential lines). Figure 13.5 shows the flow net used to obtain this analytical solution.

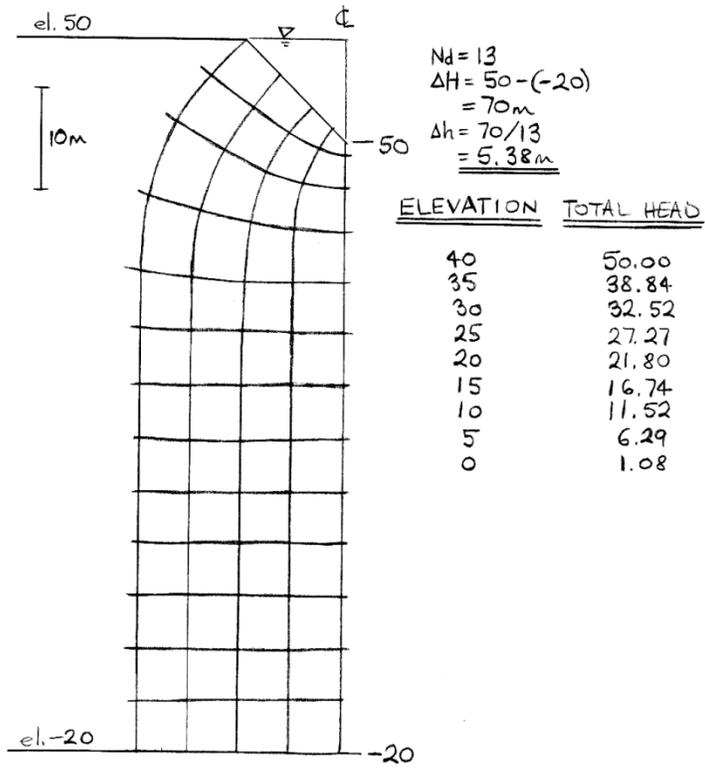


Figure 13-4: Hand-drawn flow net indicating total head along vertical axis of problem space

13.3 Results

A discharge section was added to the model to compute the total flow and compare it to the Vedernikov solution. The output is depicted in Figure 13.3.

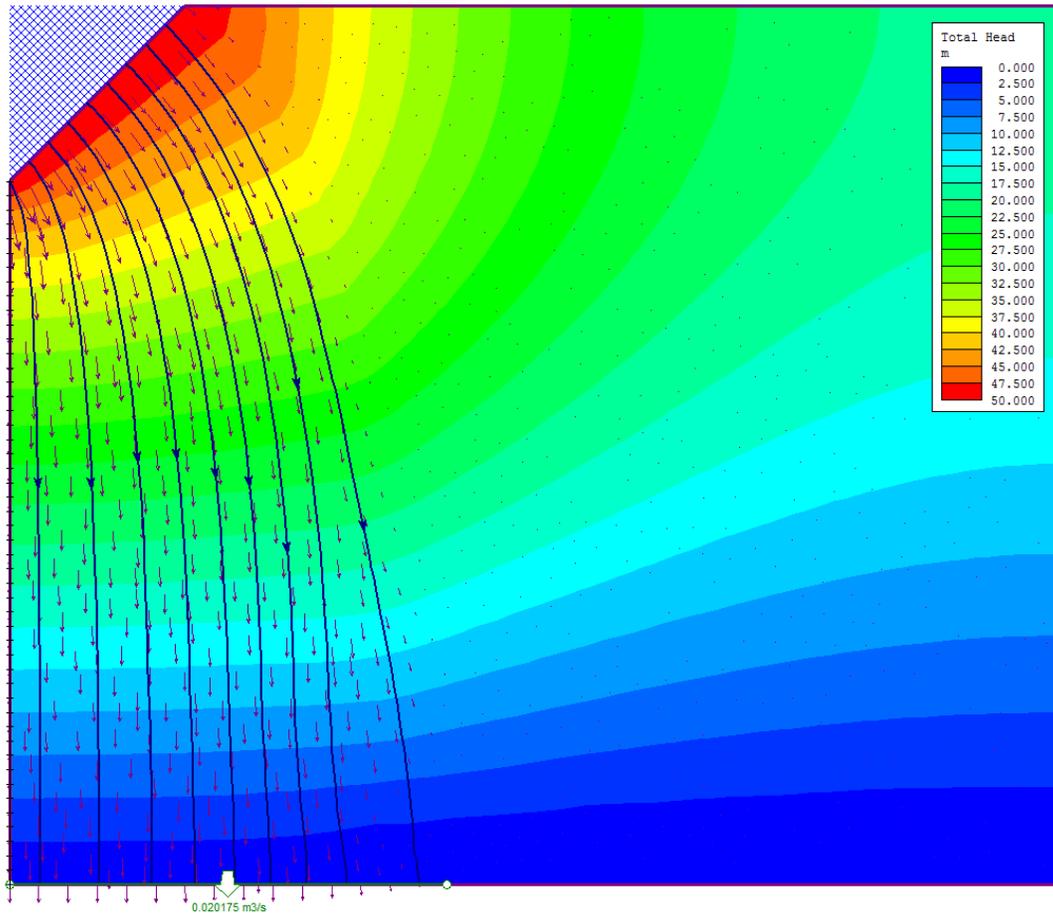


Figure 13-5: Flow net and flow vectors produced by RS2

The discharge section shows a flow of $0.020175 \text{ m}^3/\text{s}$ through the model, which equates to a total flow from the trapezoidal ditch of $0.0404 \text{ m}^3/\text{s}$. This is very similar to the analytical Vedernikov solution.

Upon inspection of the flow vectors, the seepage zone appears to be approximately 21 m wide, which equates to a total seepage zone of 42 m when symmetry is accounted for. This is in close accordance with Vedernikov's solution.

A material query was added at the center of the ditch to obtain the total head values along the vertical axis of the model. Figure 13 compares the head distribution obtained using RS2 with the analytical solution.

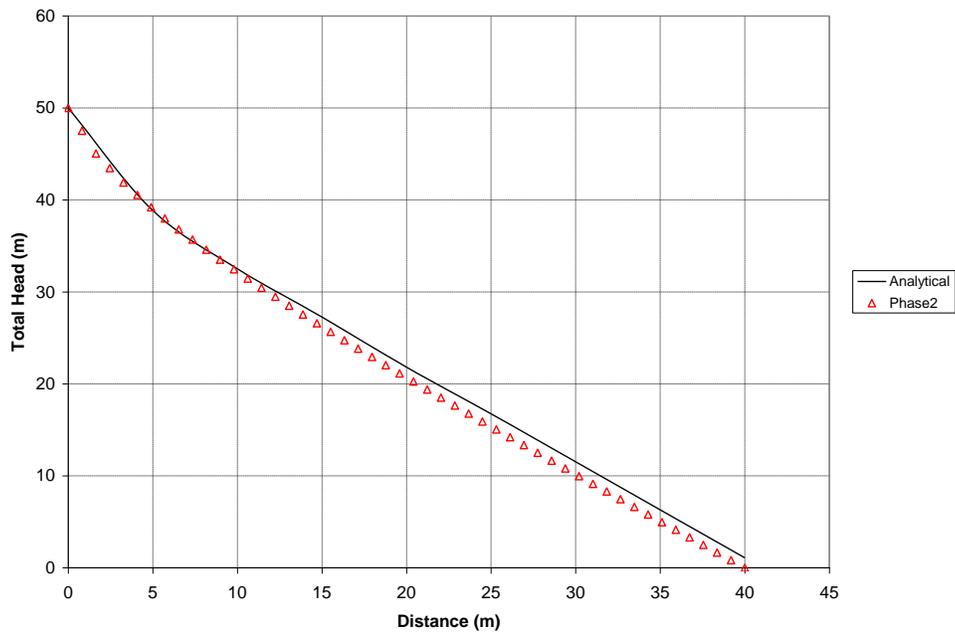


Figure 13-6: Comparison of RS2 and analytical solutions for total head beneath ditch. Note that $x = 40$ m corresponds to the bottom of the ditch.

13.4 References

1. Haar, M. E. (1990), *Groundwater and Seepage*, 2nd Edition, Dover.

13.5 Data Files

The input data file **groundwater#013.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

14 Unsaturated Soil Column

14.1 Problem Description

Steady-state capillary head distribution above the water table in a narrow soil column is analyzed in this example. The geometry of the problem is depicted in Figure 14.1.

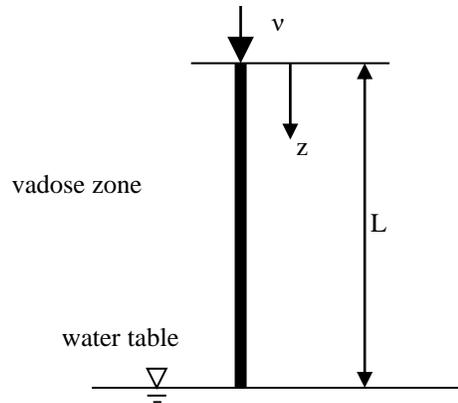


Figure 14-1: Narrow soil column above the water table

Table 1 summarizes the model and material parameters used in this instance.

Table 14.1: Model parameters

<i>Parameter</i>	<i>Value</i>
Column height (L)	1 m
Saturated soil conductivity (K_s)	10^{-7} m/s
Infiltration/Exfiltration rate (v)	$\pm 8.64 \cdot 10^{-4}$ m/day
Sorptive number (α)	1 m^{-1}

The RS2 model for the problem is shown in Figure 14-2. The model is a very thin soil column (2 mm wide), 1 meter deep to the water table.

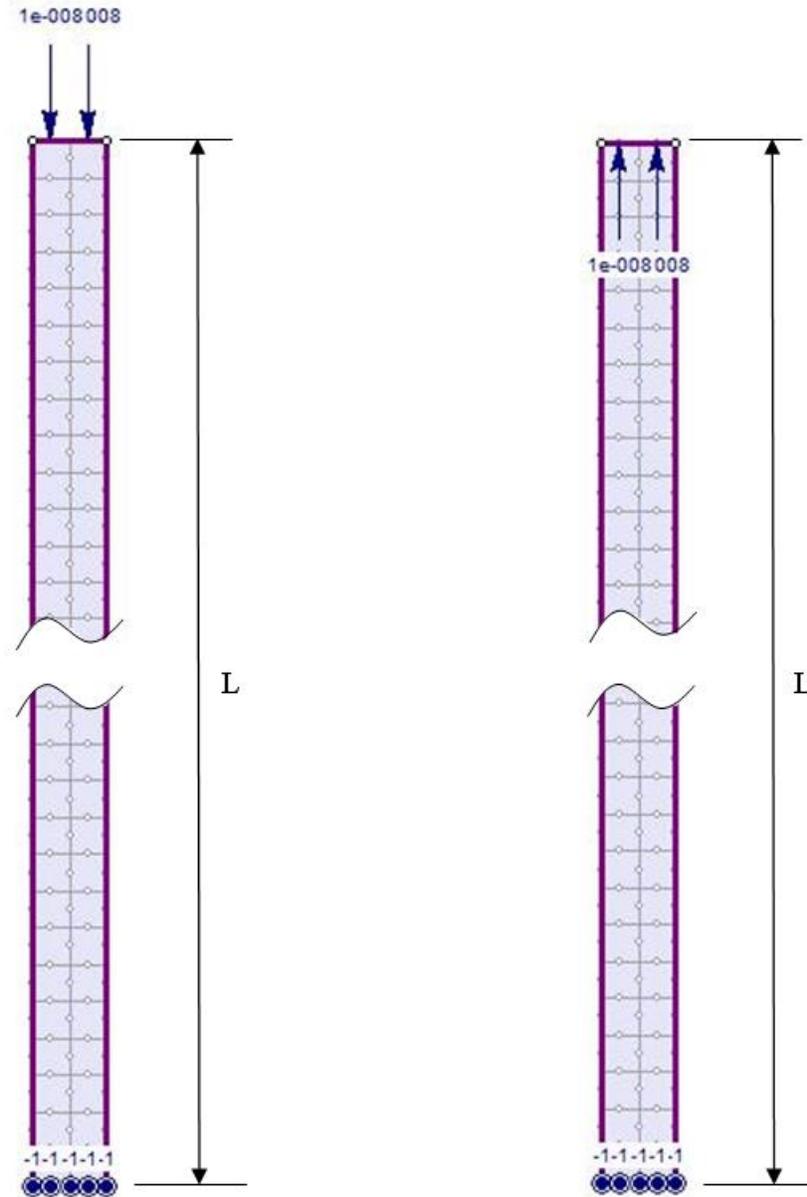


Figure 14-2: Infiltration and exfiltration in a narrow column as modeled in RS2

14.2 Analytical Solution

Gardner (1958) [1] proposed an analytical solution to this problem. He proposed the following equation for calculating capillary head:

$$\psi(z) = -\frac{1}{\alpha} \ln \left[\frac{1}{K_s} \left(\nu + (K_s - \nu) e^{-\alpha(L-z)} \right) \right]$$

where z is the vertical coordinate (m) and other parameters are as defined in Table 14.1.

14.3 Results

A material query was added throughout the depth of the column to plot the pressure head values. The output is depicted in Figure 14-3 for the constant infiltration case and Figure 14-4 for the constant exfiltration case. The RS2 results are in good agreement with the analytical solution presented by Gardner.

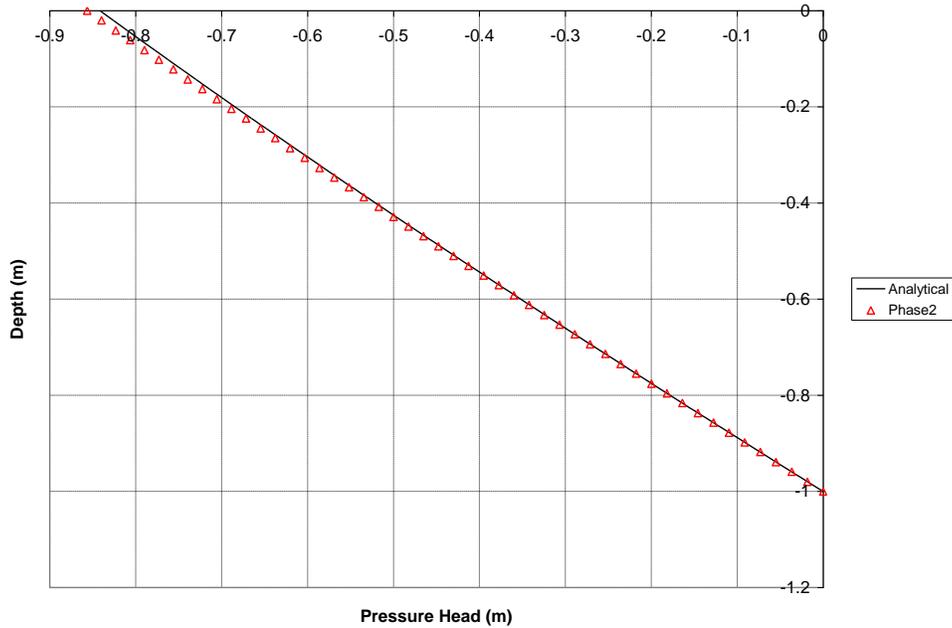


Figure 14-3: Plot of pressure head against depth comparing the Gardner analytical results to the results from RS2 for the constant infiltration case

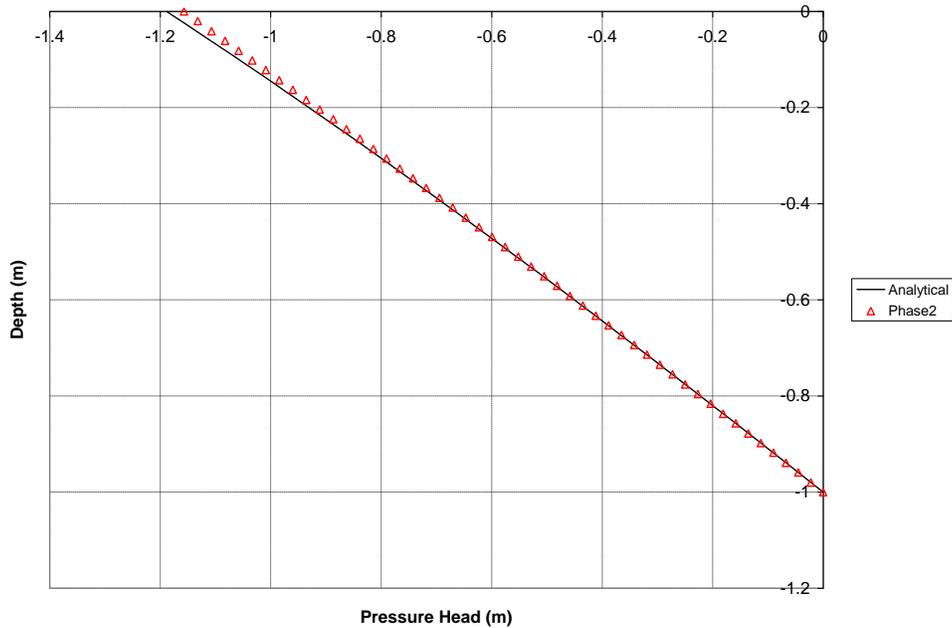


Figure 14-4: Plot of pressure head against depth comparing the Gardner analytical results to the results from RS2 for the constant exfiltration case

14.4 References

1. Gardner, W.R. (1959), *Some Steady-State Solutions of the Unsaturated Moisture Flow Equation with Application to Evaporation from a Water Table*, Soil Science 35 (1958) 4, 228-232.

14.5 Data Files

The input data file **groundwater#014.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

15 Radial Flow to a Well in a Confined Aquifer

15.1 Problem Description

The problem concerns the radial flow towards a pumping well through a confined homogeneous, isotropic aquifer. The problem is axisymmetric. The problem geometry is shown in Figure 15-1.

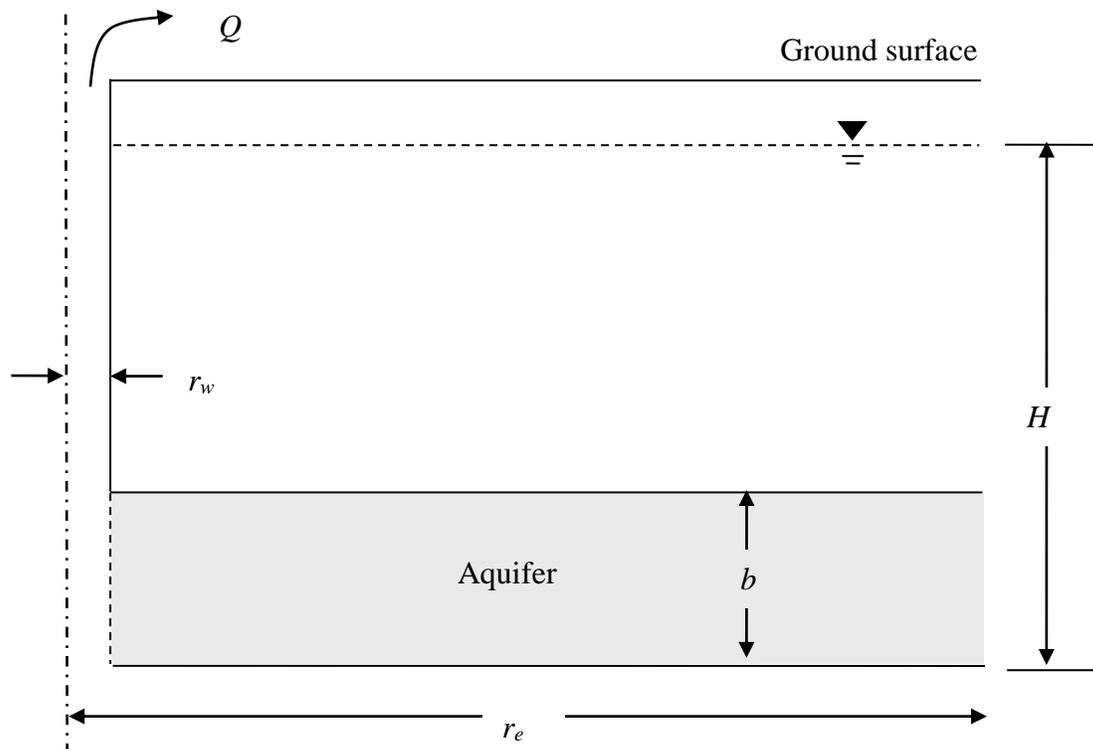


Figure 15-1: Vertical well in confined aquifer

The left side of the figure above is the axis of symmetry and represents the centre line of the well.

The RS2 model used to simulate this problem is shown in Figure 15-2. To ensure highly accurate results, the model mesh was created with 6-noded triangular elements and the discretization density and element density were increased near the well where high pore pressure gradients were expected.

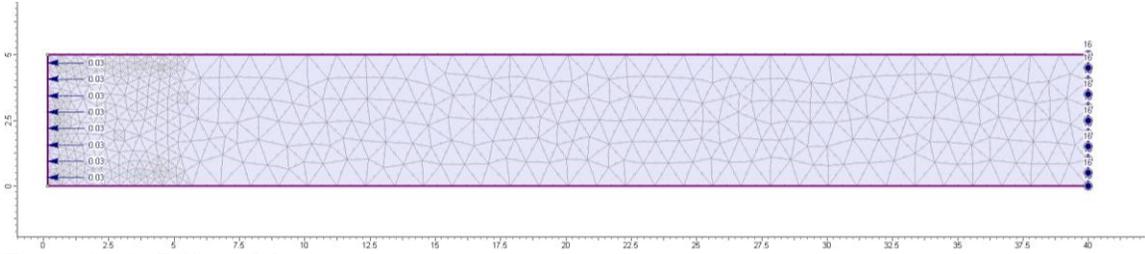


Figure 15-2: RS2 model

Axisymmetry is turned on. The RS2 model uses the following input parameters:

- Well radius $r_w = 0.15$ m
- Boundary radius $r_e = 40$ m
- Aquifer depth $b = 5$ m
- Water table height $H = 16$ m
- Volumetric pumping rate $Q = 0.125$ m³/s
- Soil conductivity $k = 0.002$ m/s

The pumping boundary condition was simulated by applying a negative normal infiltration of q along the length of the well. The magnitude of q was calculated by dividing the volumetric pump rate (Q) by the surface area of the well:

$$q = \frac{Q}{2\pi r_w l} = \frac{0.125}{2\pi(0.15)(5)} = 0.0265 \text{ m/s}$$

where l represents the length of the well. In this case it fully penetrates the reservoir so $l = b$.

15.2 Analytical Solution

According to Davis (1966) [1] the head h at any radius r is given by the analytical solution [1]

$$h = H - \frac{Q}{2\pi k b} \ln\left(\frac{r_e}{r}\right)$$

where H is the head at the far boundary, r_e is the radius of the far boundary, b is the thickness of the aquifer, k , is the permeability in the aquifer and Q is the volumetric pumping rate.

15.3 Results

The steady state solution for total head produced by RS2 is shown in Figure 15-3.

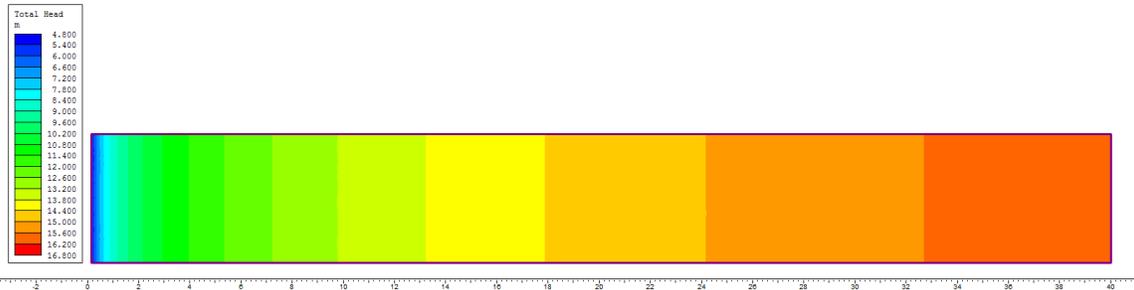


Figure 15-3: Total head contour plot from RS2

Figure 15-4 compares the total head values computed by RS2 with those derived from the analytical solution in [1].

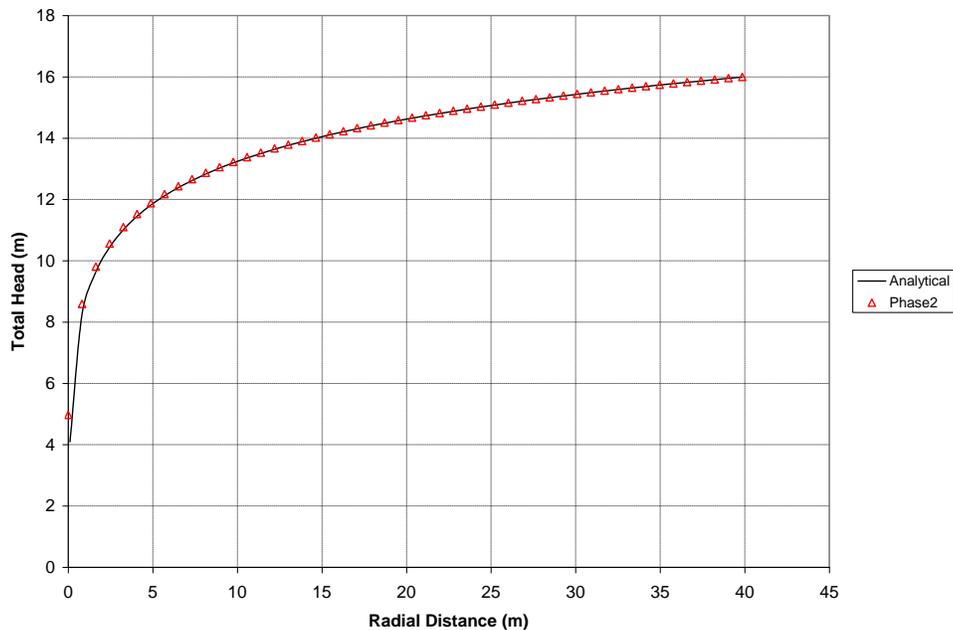


Figure 15-4: Total head distribution with increasing radial distance from well

Clearly, RS2 is in good accordance with the analytical solution. A maximum error of 4.1% was observed at the edge of the well.

15.4 References

1. Davis, S.N. and DeWiest, R.J.M., (1966), *Hydrogeology*, John Wiley & Sons, Inc., New York.

15.5 Data Files

The input data file **groundwater#015.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

16 Radial Flow to a Well in an Unconfined Aquifer

16.1 Problem Description

The problem concerns the radial flow from an aquifer towards a pumping well in a homogeneous, isotropic soil. The aquifer has an impermeable base but is unconfined at the top. The well penetrates the entire aquifer. The problem is axisymmetric. Figure 16-1 shows the problem geometry.

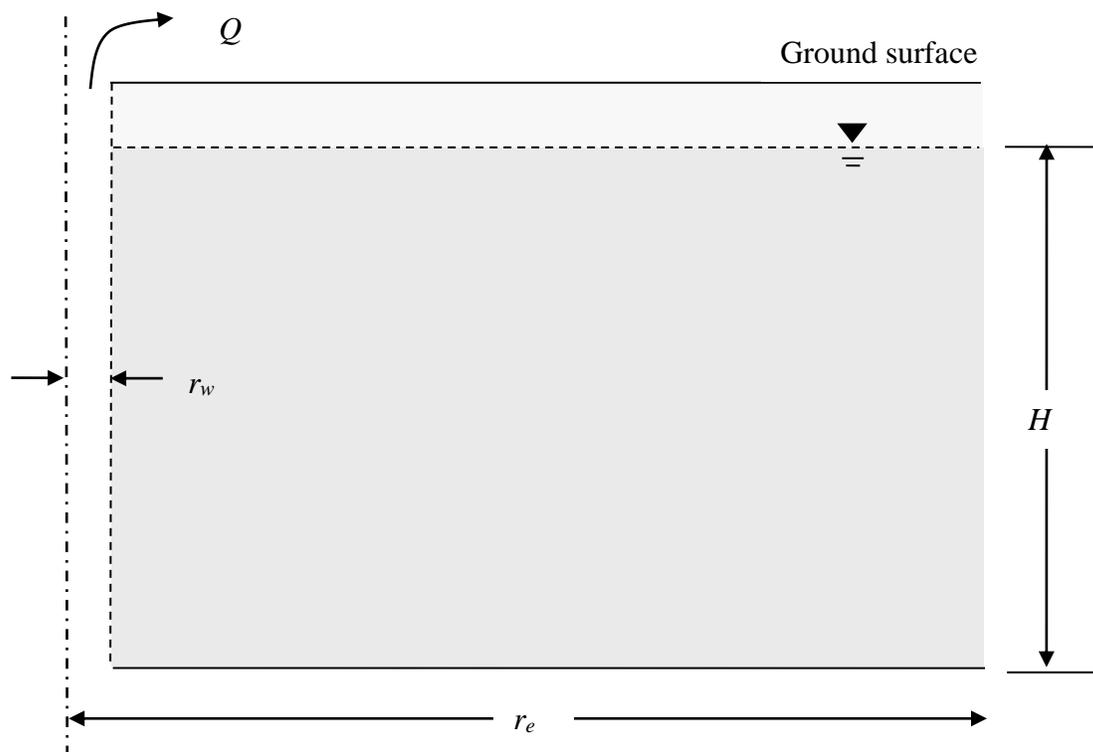


Figure 16-1: Radial flow to a well in an unconfined aquifer

The left side is the axis of symmetry and represents the centre line of the well.

The RS2 model used to simulate this problem is shown in Figure 16-2. To ensure highly accurate results, the model mesh was created with 6-noded triangular elements and the discretization density and element density were increased near the well where high pore pressure gradients were expected.

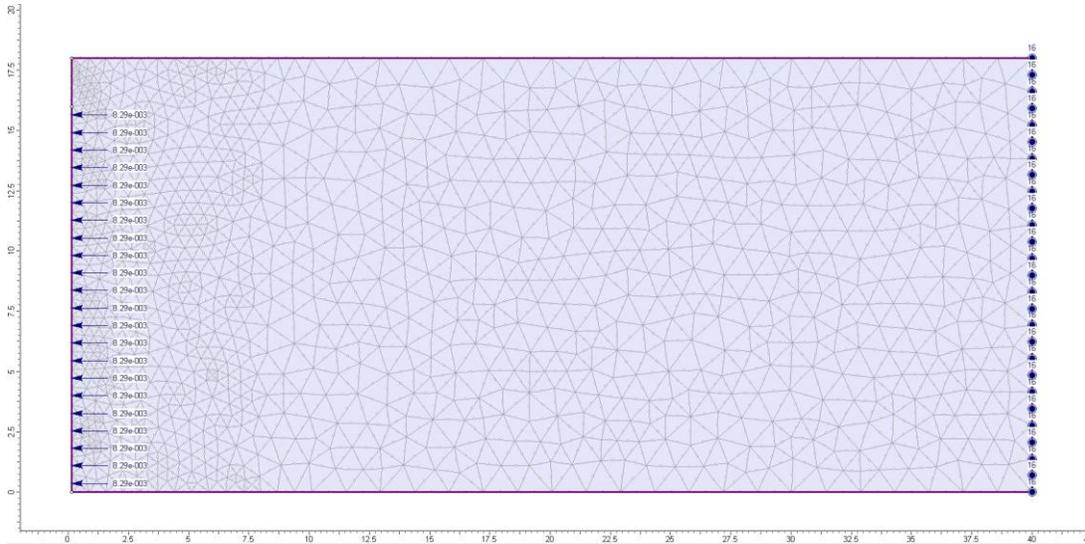


Figure 16-2: Pumping from a well in an unconfined aquifer as modeled in RS2

Axisymmetry is turned on. The RS2 model uses the following input parameters:

- $r_w = 0.15$ m
- $r_e = 40$ m
- $H = 16$ m
- $Q = 0.125$ m³/s
- $k = 0.002$ m/s

The pumping boundary condition was simulated by applying a negative normal infiltration of q along the length of the well. The magnitude of q was calculated by dividing the volumetric pump rate (Q) by the surface area of the well:

$$q = \frac{Q}{2\pi r_w l} = \frac{0.125}{2\pi(0.15)(16)} = 0.00829 \text{ m/s}$$

Where l represents the length of the well. In this case the well fully penetrates the aquifer so $l = 16$ m.

16.2 Analytical Solution

The height of the water table h at any radius r can be obtained from the analytical solution [1]

$$h^2 = H^2 - \frac{Q}{\pi k} \ln\left(\frac{r_c}{r}\right)$$

Where H is the head at the far boundary, r_e is the radius of the far boundary, k , is the permeability in the aquifer and Q is the volumetric pumping rate.

16.3 Results

The steady state solution for pressure head is shown in Figure 16-3.

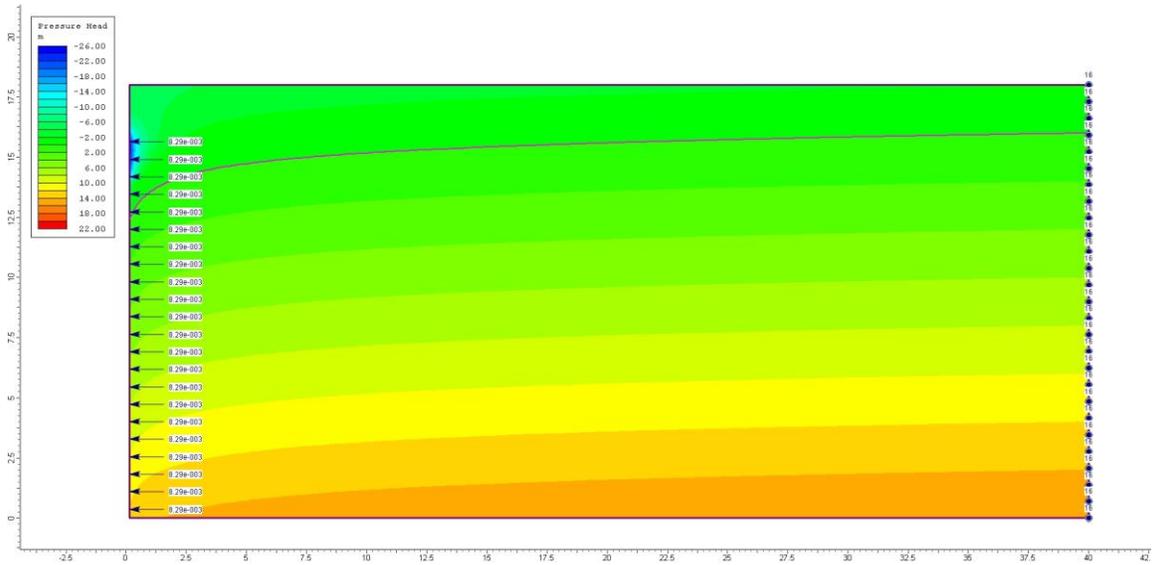


Figure 16-3: Pressure head contours from the RS2 model

The height of the water table compared to the analytical solution is shown in Figure 16-4.

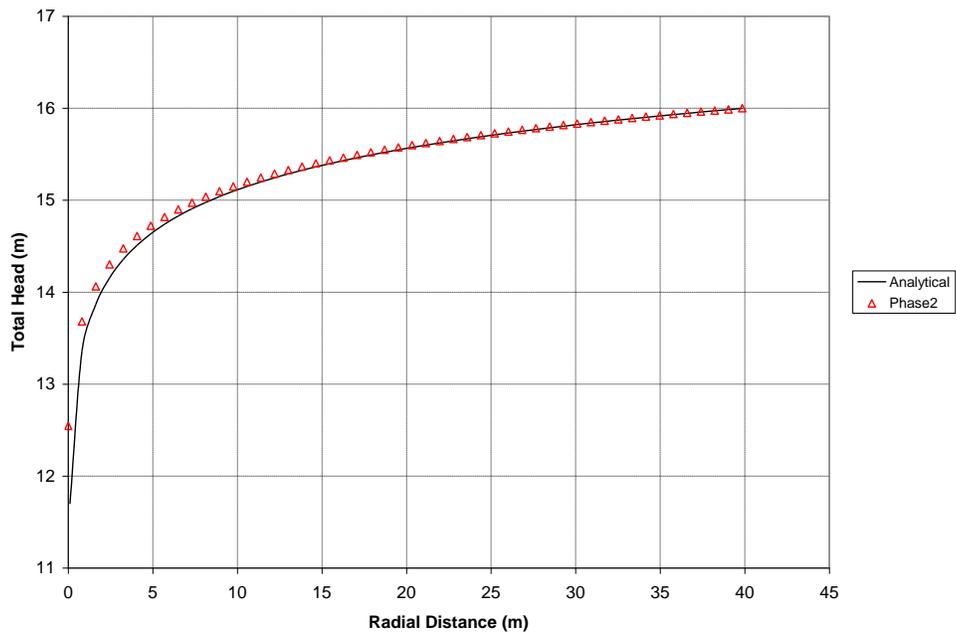


Figure 16-4: Total head in RS2 model compared with analytical results

Clearly, RS2 matches the analytical solution well. The maximum error is 7.3% at the edge of the well.

16.4 References

1. Davis, S.N. and DeWiest, R.J.M., (1966). *Hydrogeology*, John Wiley & Sons, Inc., New York.

16.5 Data Files

The input data file **groundwater#016.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

17 1-D Consolidation with Uniform Initial Excess Pore Pressure

17.1 Problem Description

In this problem, a 1-D soil column with a height of one metre is considered. Two boundary condition cases are considered. The first case allows flow along the top and bottom edges, while the second case only allows flow along the top edge. An initial pressure head of $P = 100$ m is applied uniformly throughout the column. This geometry is shown in Figure 17.1.

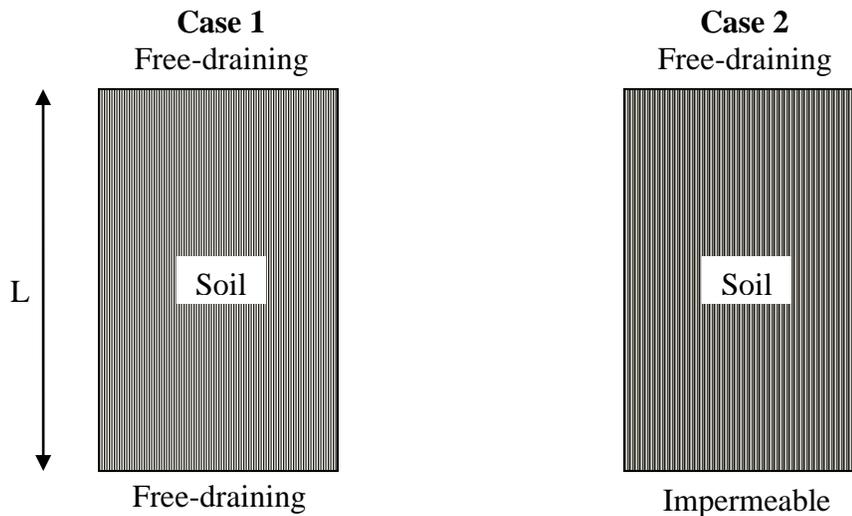


Figure 17-1: Model Geometry

Terzaghi's consolidation equation can be written as

$$\frac{\partial^2 u_e}{\partial Z^2} = \frac{\partial u_e}{\partial T} \quad (17.1)$$

using the dimensionless variables

$$Z = \frac{z}{H} \quad (17.2a)$$

and

$$T = \frac{C_v t}{H^2}, \quad (17.2b)$$

where

- z = depth from the top of the column
- H = maximum drainage path
- C_v = coefficient of consolidation
- t = time
- u_e = excess pore pressure

An initial condition is imposed at $t = 0$:

$$u_e = u_0 \text{ for } 0 \leq Z \leq 1$$

where

$$u_0 = \text{initial excess pore pressure}$$

Along edges where flow is allowed to occur, a boundary condition is imposed for all t :

$$u_e = 0$$

The solution to the consolidation equation is given in Ref [1] as:

$$u_e = \sum_{m=0}^{m=\infty} \frac{2u_0}{M} (\sin MZ) e^{-M^2 T} \quad (17.3)$$

where

$$M = \frac{\pi}{2} (2m + 1)$$

17.2 RS2 Model and Results

Case 1

The RS2 model for Case 1 is shown in Figure 17.2. A uniform initial excess pore pressure of 100 m is set.

The following properties are assumed for the soil:

- $m_w = 0.01$ /kPa
- $C_v = 1.02e-4$ m²/s
- $k = C_v \gamma_w m_w = 1e-5$ m/s

The maximum drainage path is taken as $L/2 = 0.5$ m. The problem is modeled in RS2 with three-noded triangular finite elements. The total number of elements used is 1580 elements.

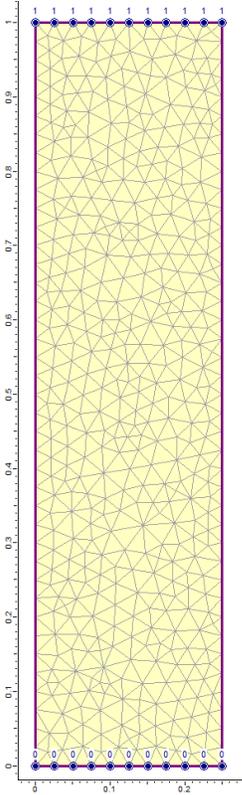


Figure 17-2: RS2 Model for Case 1

Figure 17-3 shows excess pore pressure along the soil column at different times. The single data points represent the RS2 interpretations, while the solid lines represent values calculated using Equation 17.3. The RS2 curves take the same form as published graphs such as in Ref [1]. As seen, the RS2 results are in close agreement with values calculated using Equation 17.3.

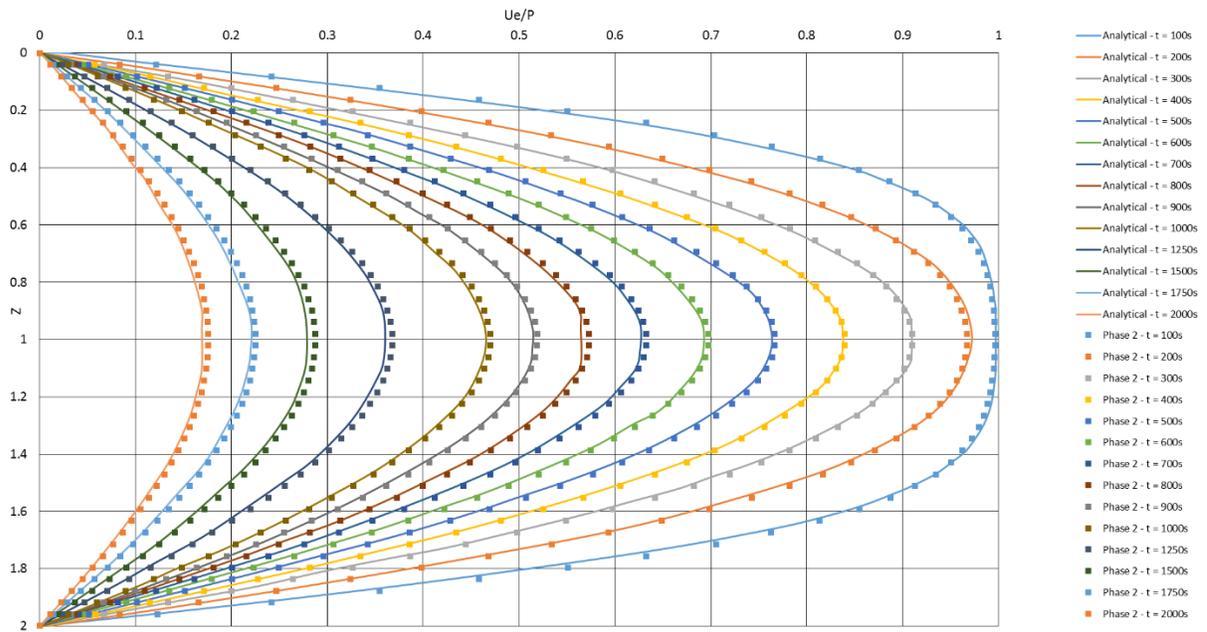


Figure 17-3: Comparison of Pore Pressure Dissipation for Case 1

Case 2

The RS2 model for Case 2, shown in Figure 17-4, uses properties similar to Case 1. The maximum drainage path is taken as $L = 1$ m.

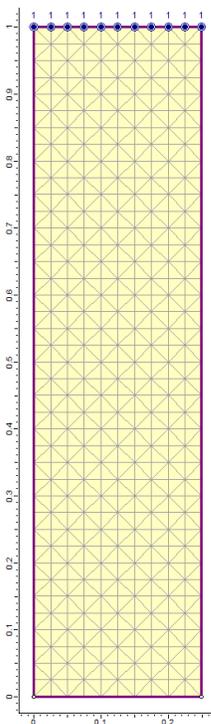


Figure 17-4: RS2 Model for Case 2

The RS2 results for Case 2 shown in Figure 17-5 are again in close agreement with the Terzaghi consolidation equation values.

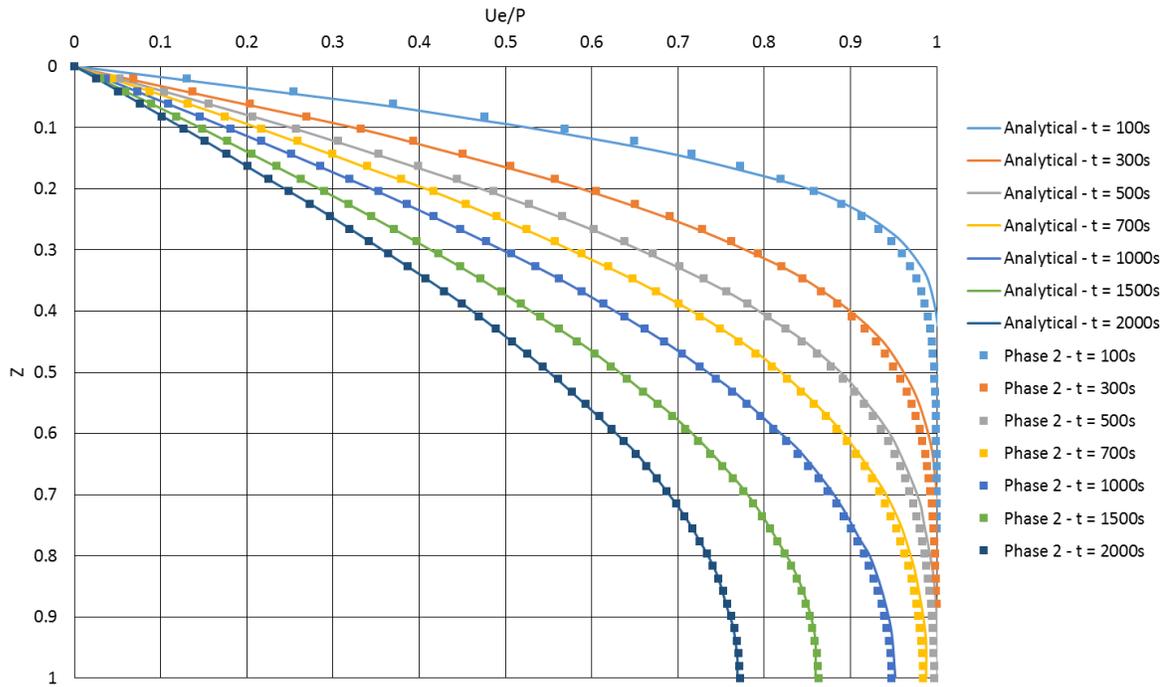


Figure 17-5: Comparison of Pore Pressure Dissipation for Case 2

17.3 References

[1] T.W. Lambe and R.V. Whitman (1979) *Soil Mechanics, SI Version*, New York: John Wiley & Sons.

18 Pore Pressure Dissipation of Stratified Soil

18.1 Problem Description

The problem deals with 1D consolidation of stratified soils. Three cases are considered, which are shown in Figure 18-1. The properties for Soil A and Soil B are shown in Table 18.1. Both the pore fluid specific weight (γ_w) and the height of the soil profiles are assumed to be one unit. An initial pressure head of $P = 1000$ m is applied uniformly throughout the column.

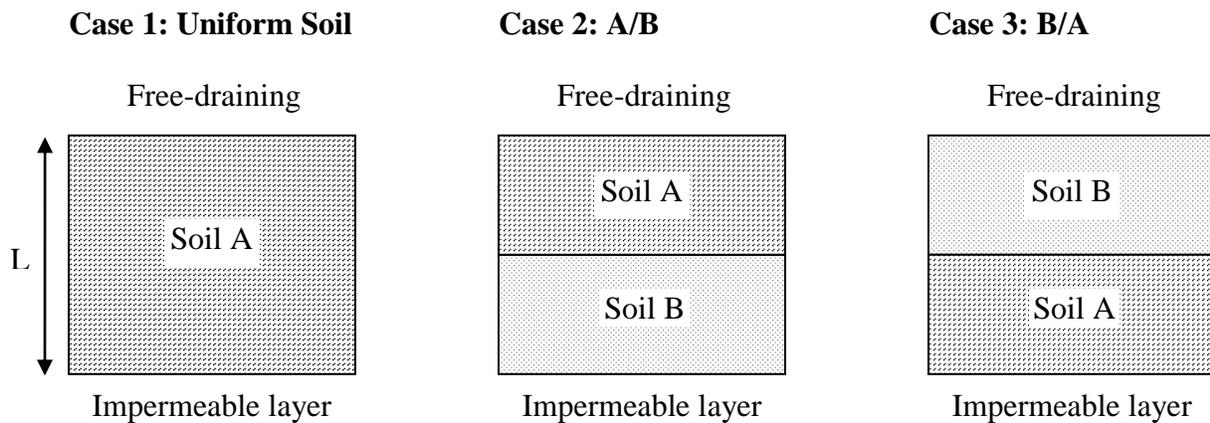


Figure 18-1: Model Geometry

Table 18.1: Soil Properties

	Soil A	Soil B
k	1	10
m_v	1	10
c_v	1	1

18.2 RS2 Model Results

Figure 18-2, Figure 18-3, and Figure 18-4 show comparisons between excess pore pressures in the RS2 model and values from the analytical solution presented in Ref [1]. The single data points represent the RS2 interpretations, while the solid lines represent analytical values from Ref [1]. As shown, the RS2 results are in close agreement with the analytical solutions.

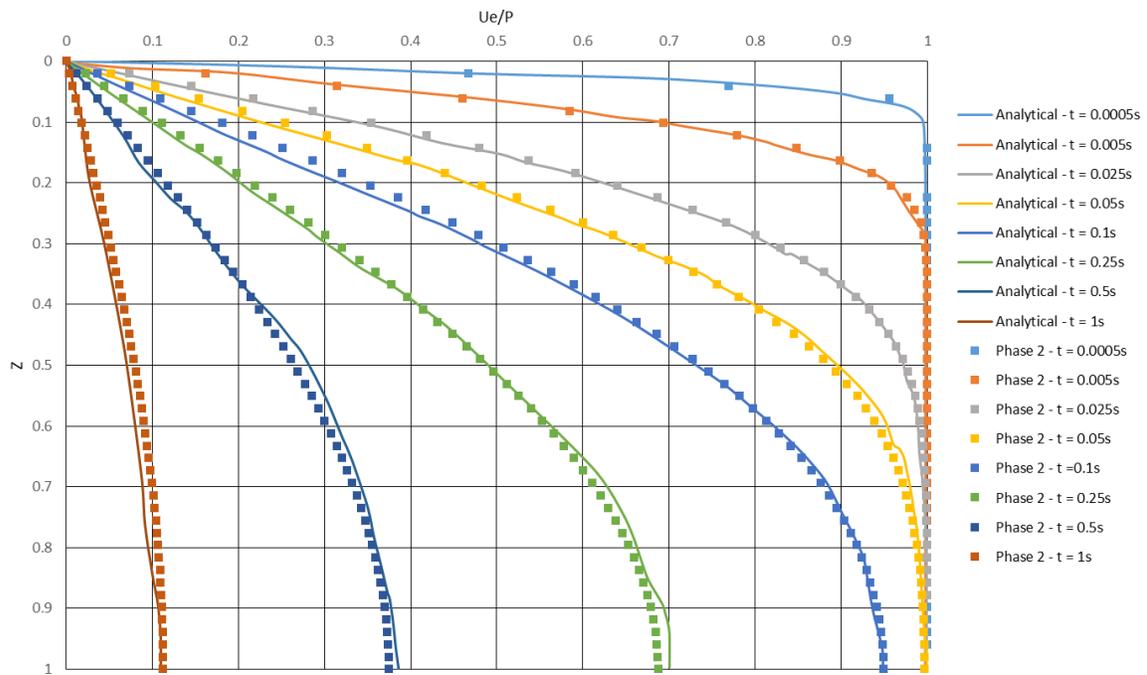


Figure 18-2: Comparison of Excess Pore Pressure for Case 1

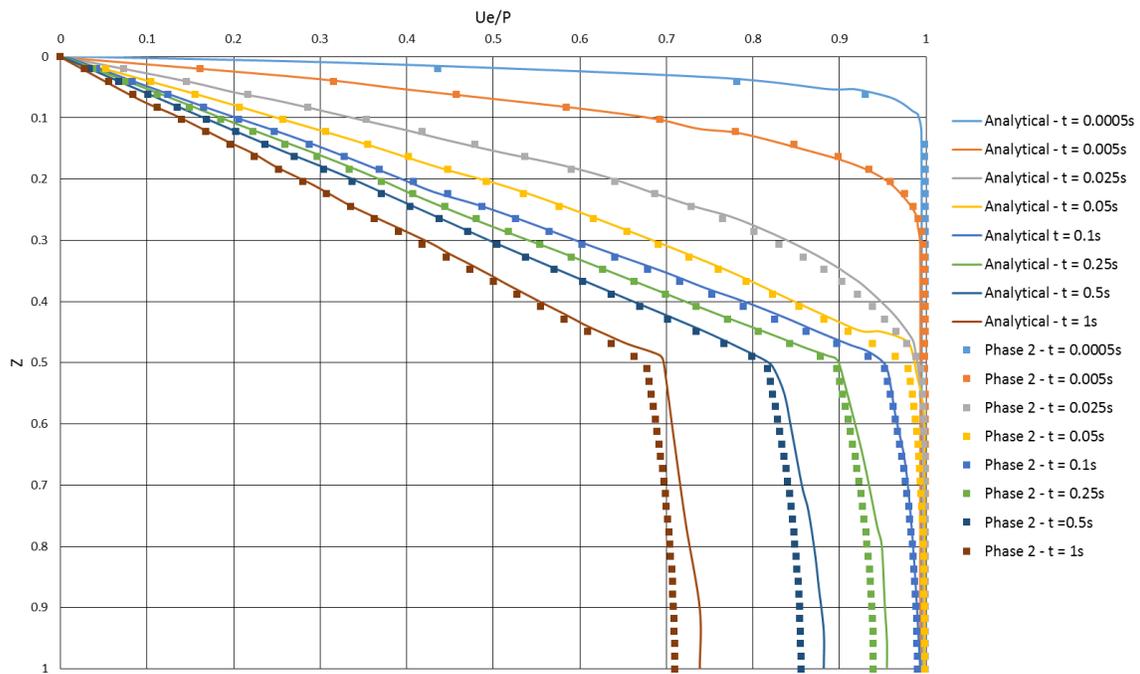


Figure 18-3: Comparison of Excess Pore Pressure for Case 2

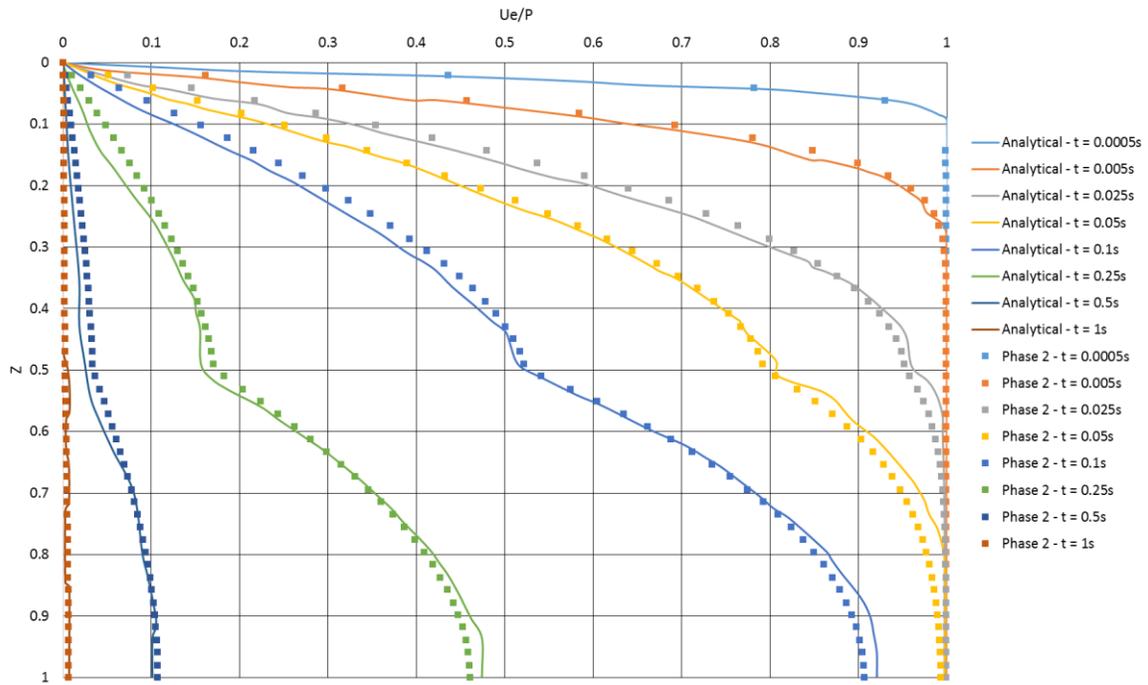


Figure 18-4: Comparison of Excess Pore Pressure for Case 3

18.3 References

[1] Pyrah, I.C. (1996), "One-dimensional consolidation of layered soils", *Géotechnique*, Vol. 46, No. 3, pp. 555-560.

19 Transient Seepage Through an Earth Fill Dam with Toe Drain

19.1 Problem Description

In this problem, an earth fill dam with a reservoir on one side is modeled. The reservoir level is quickly raised, and transient seepage is investigated.

The base of the earth fill dam is 52 m wide and there is a 12 m wide toe drain installed at the downstream side. The initial steady-state reservoir level is 4 m. For transient analysis, the reservoir level is quickly raised to a height of 10 m. Isotropic conditions and a m_v value of 0.003 /kPa are assumed. Figure 19-1 shows the coefficient of permeabilities used for dam material.

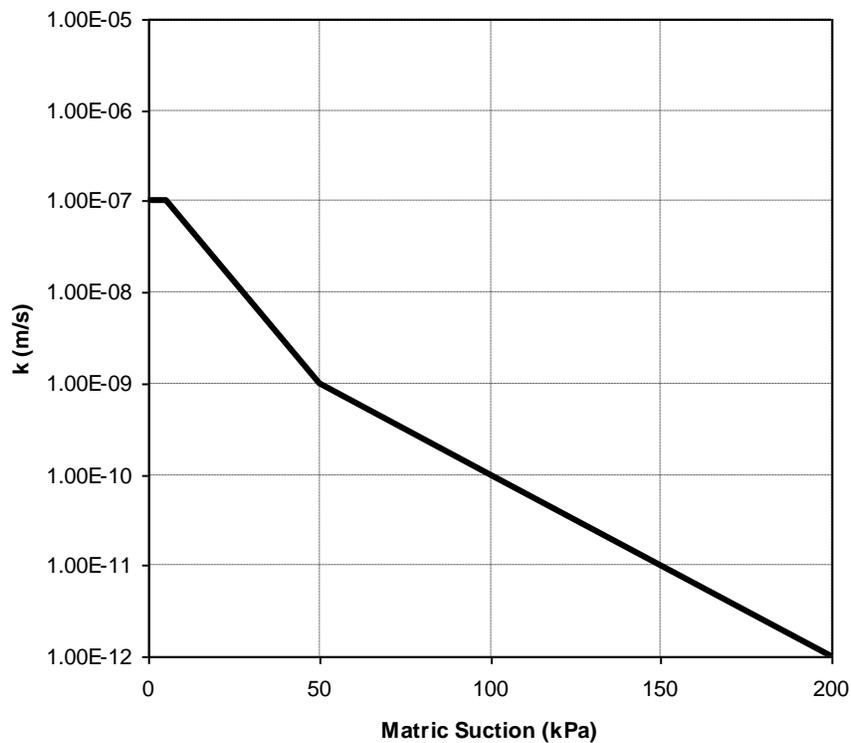


Figure 19-1: Coefficient of Permeability Function

19.2 RS2 Model

The RS2 models for initial steady state and transient analysis are shown in Figure 19-2 and Figure 19-3, respectively. The boundary conditions simulate the rise in the reservoir water level and the installed toe drain.

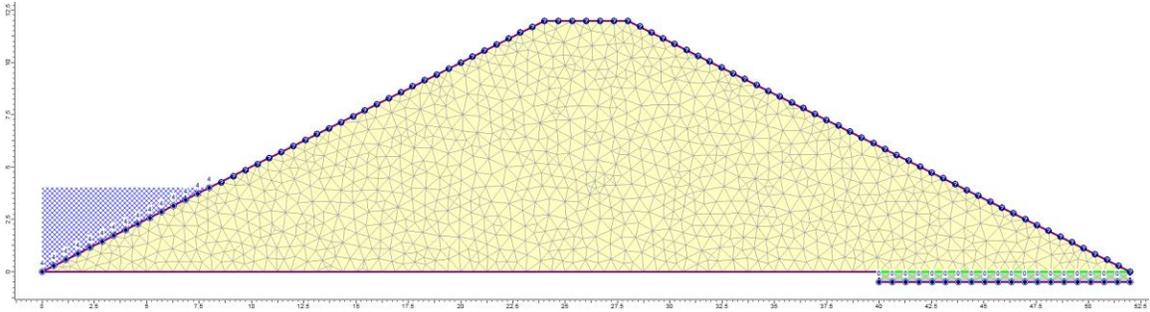


Figure 19-2: RS2 Model – Initial Steady State

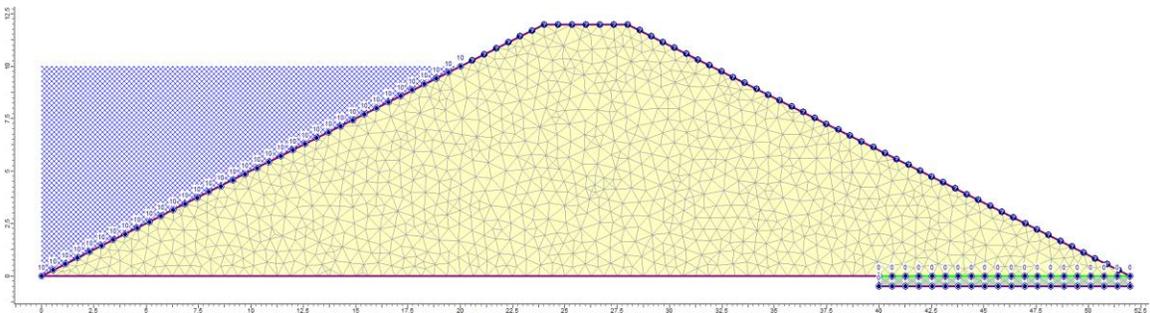


Figure 19-3: RS2 Model - Transient

19.3 Results

The RS2 model results are shown at times 15 hr and 16383 hr in Figure 19-4 and Figure 19-5, respectively. The solid lines represent total head contour results from RS2. The black lines are solutions taken from FlexPDE results in Ref [1], while the pink lines are SEEP/W results from Ref [1].

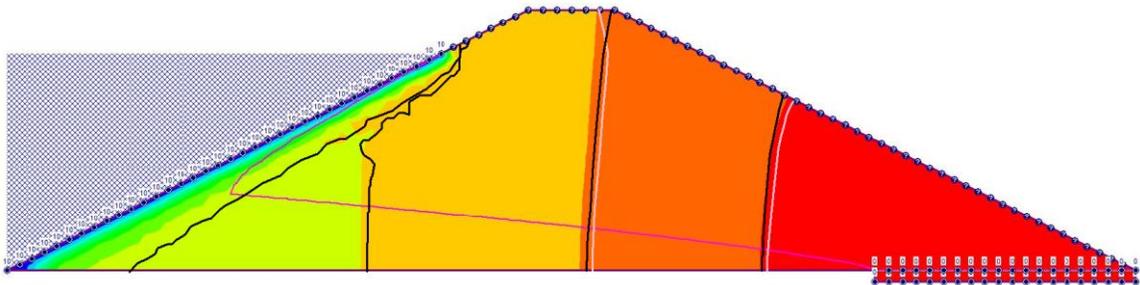


Figure 19-4: Comparison of Total Head Contours for Time 15 hr

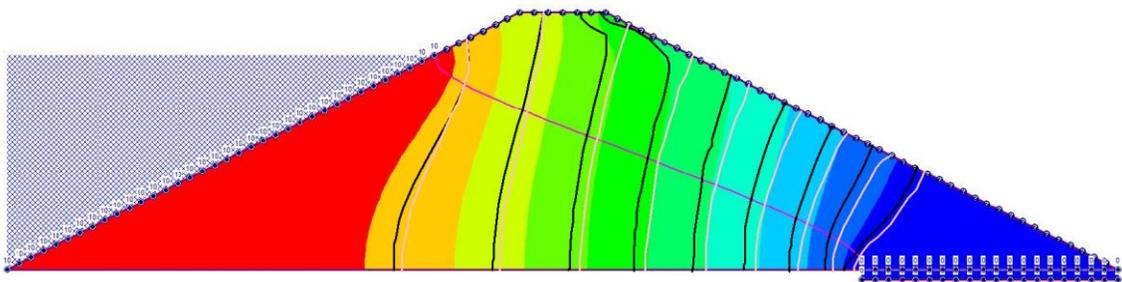


Figure 19-5: Comparison of Total Head Contours for Time 16383 hr

Figure 19-6 and Figure 19-7 show pressure head contours at times 15 hr and 16383 hr, respectively.

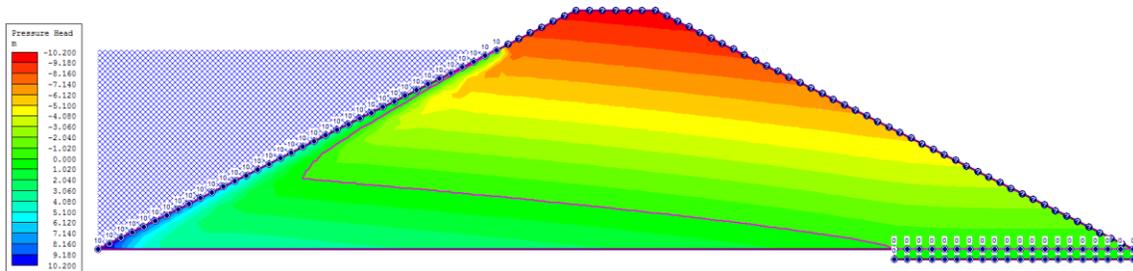


Figure 19-6: Pressure Head Contours for Time 15 hr

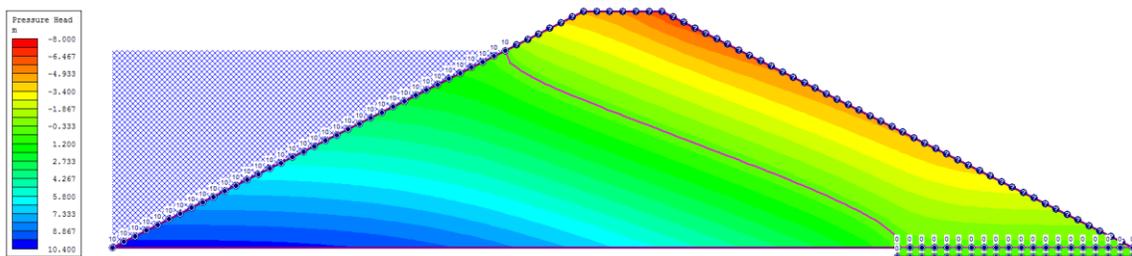


Figure 19-7: Pressure Head Contours for Time 16383 hr

19.4 References

- [1] Pentland, et. al (2001), "Use of a General Partial Differential Equation Solver for Solution of Mass and Heat Transfer Problems in Geotechnical Engineering", 4th Brazilian Symposium on Unsaturated Soil, pp. 29-45.

20 Transient Seepage Through an Earth Fill Dam

20.1 Problem Description

This problem is similar to Verification Example 19.

The base of the earth fill dam is 52 m wide but there is no toe drain. The reservoir level is raised from 4 m to 10 m at the start of analysis time. Isotropic conditions and a m_v value of 0.003 /kPa are assumed for the earth fill. Figure 20.1 shows the coefficient of permeabilities used for the dam material.

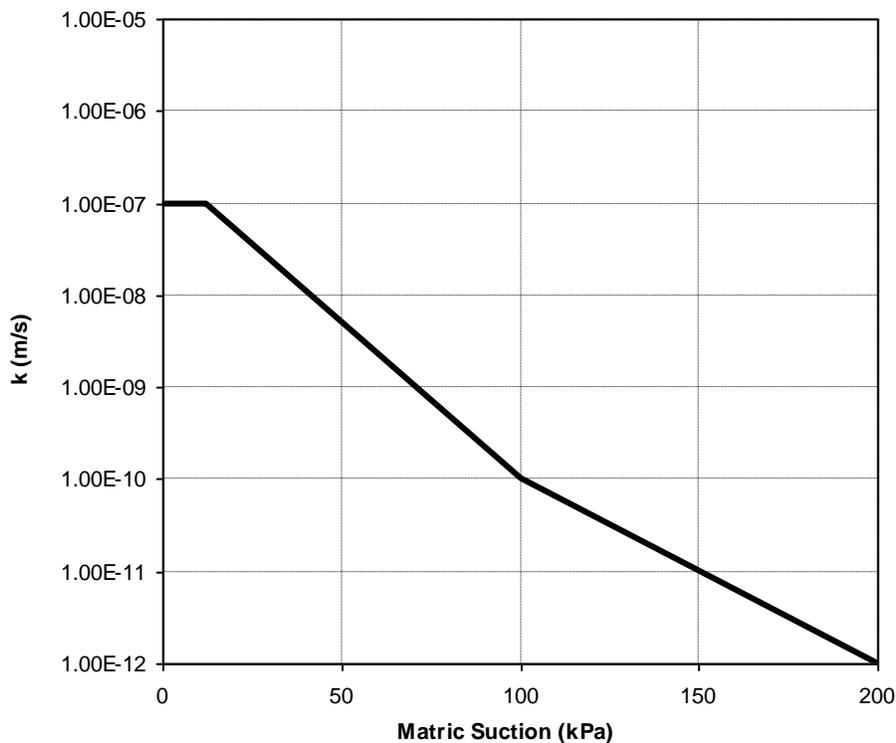


Figure 20.1 – Coefficient of Permeability Function for Dam Material

20.2 RS2 Model

The *Phase 2* models for initial steady state and transient analysis are shown in Figure 20.2 and 20.3, respectively. The boundary conditions simulate the rise in the reservoir water level.

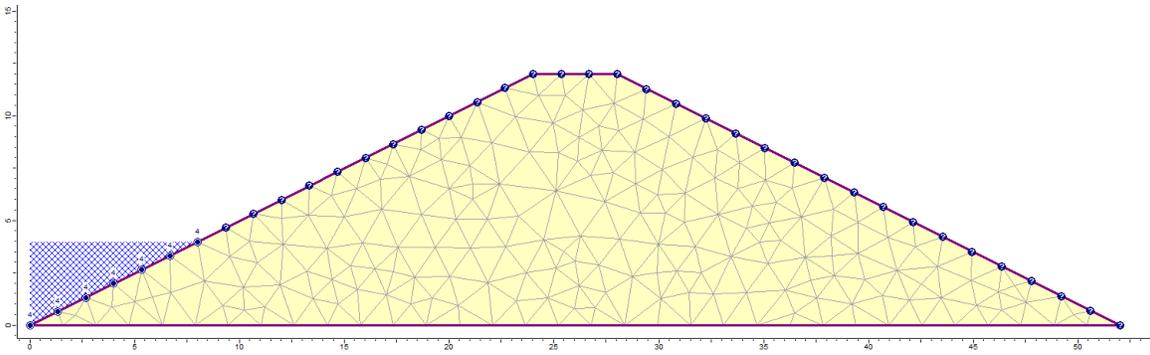


Figure 20.2 – RS2 Model – Initial Steady State

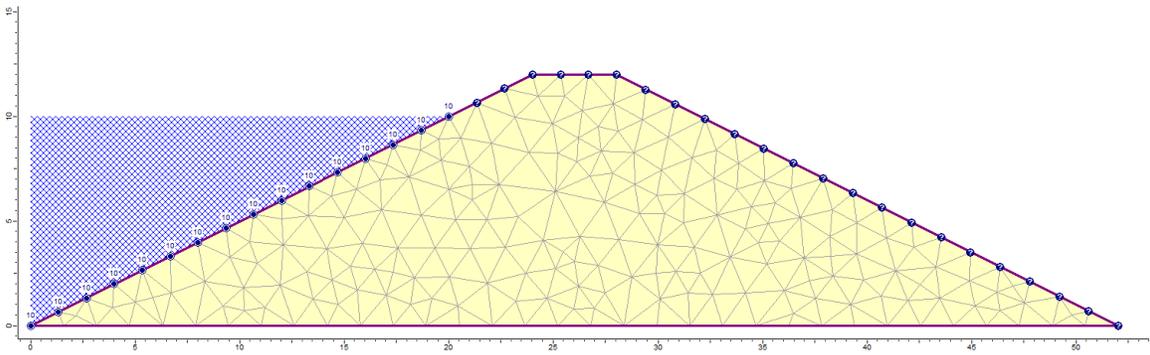


Figure 20.3 – RS2 Model – Transient

20.3 Results

Total head values are sampled along the toe slope as shown in Figure 20.4. These values are compared with values taken from Ref [1] in Figure 20.5. As can be seen, the values are in agreement.

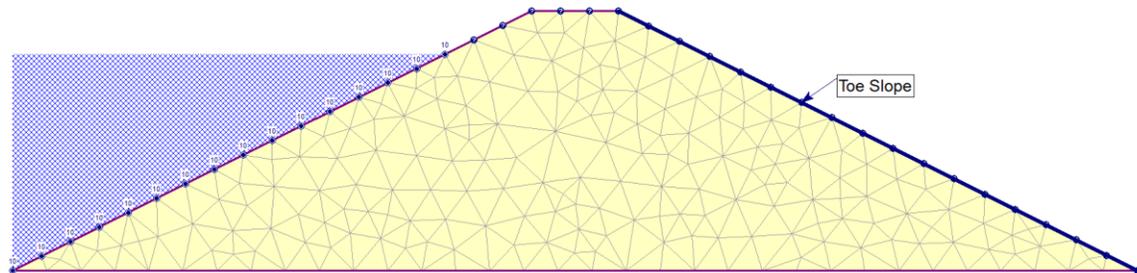


Figure 20.4 – Toe Slope

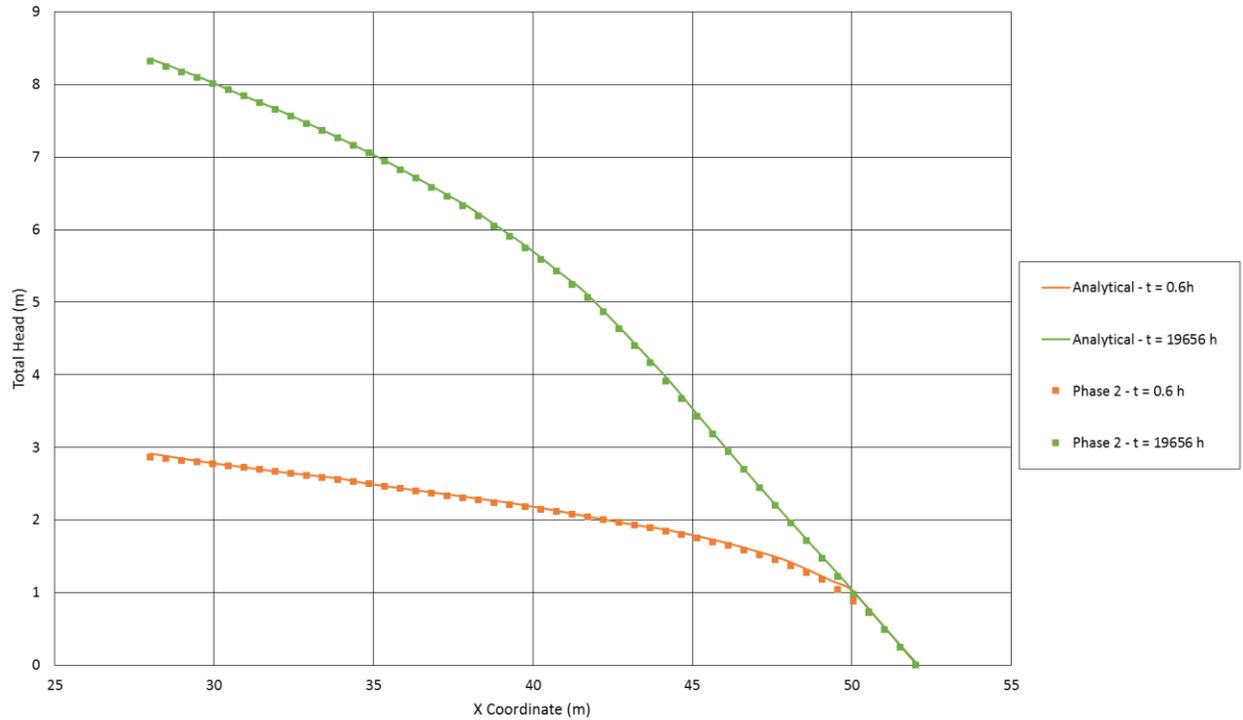


Figure 20.5 – Total Head Comparison

Figures 20.6 and 20.7 show total head contours for times of 0.6 h and 19656 h, respectively. Figures 20.8 and 20.9 show pressure head contours for the same times.

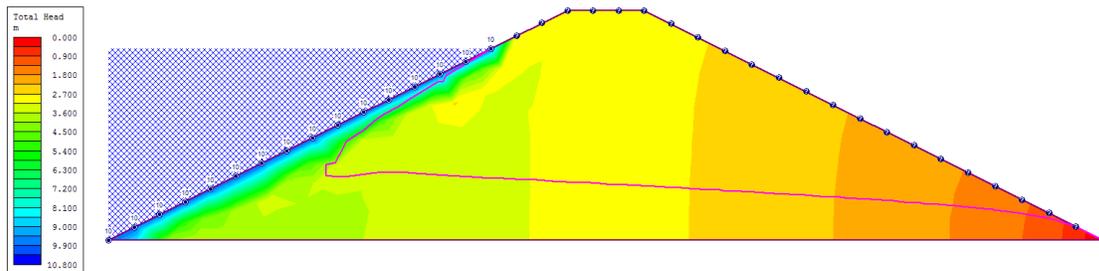


Figure 20.6 – Total Head Contours at 0.6 h

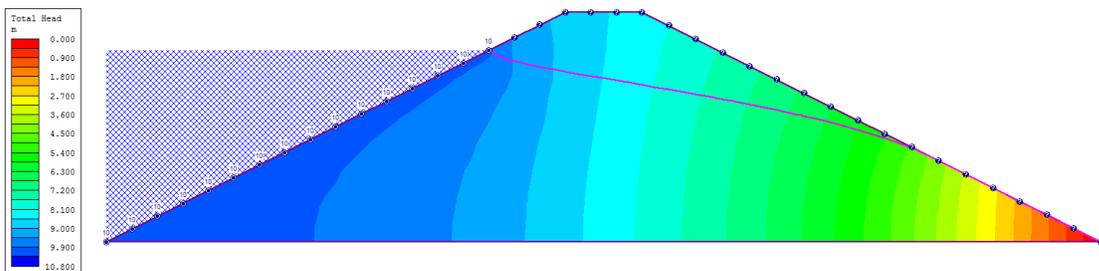


Figure 20.7 – Total Head Contours at 19656 h

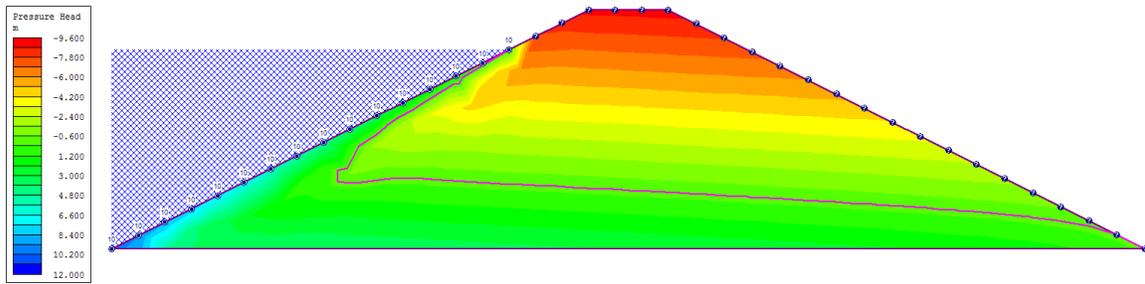


Figure 20.8 – Pressure Head Contours at 0.6 h

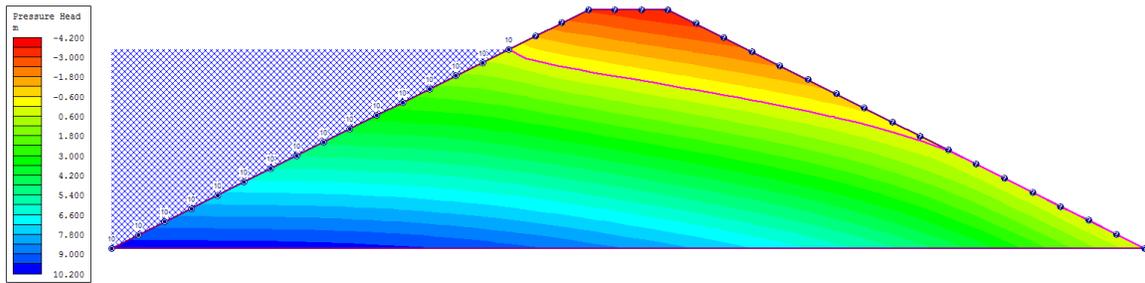


Figure 20.9 – Pressure Head Contours at 19656 h

20.4 References

- [1] Fredlund, D.G. and Rahardjo, H. (1993), *Soil Mechanics for Unsaturated Soils*, New York: John Wiley & Sons.

21 Seepage Below a Lagoon

21.1 Problem Description

This example deals with transient seepage below a lagoon. One half of the model geometry is considered since it is symmetrical. The section of the lagoon considered is 2 m wide. A 1 m deep soil liner is directly under the lagoon and the soil is assumed to extend 9 m below the soil liner before an impermeable boundary is encountered. An initial steady-state water table at a depth of 5 m from the ground surface is assumed. At analysis time zero, the water level in the lagoon is instantaneously raised to a height of 1 m. The model geometry for transient analysis at time zero is shown in Figure 21.1.

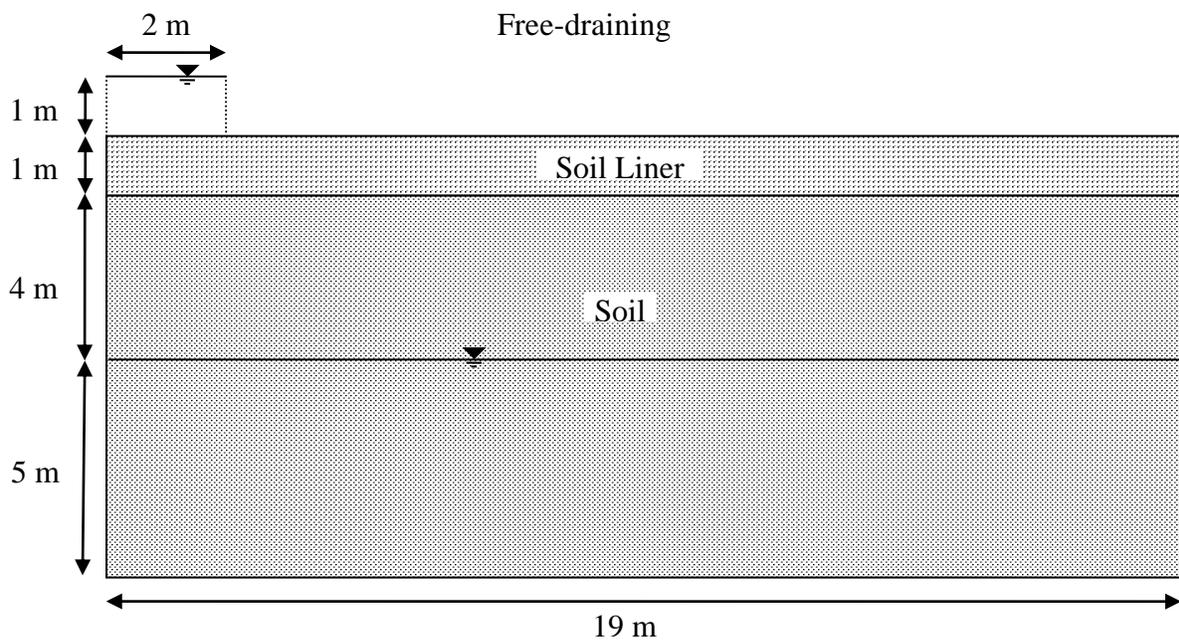


Figure 21.1 – Model Geometry

An m_v value of 0.002 /kPa was assumed for both the soil and the liner. The permeability functions for the sands are shown in Figure 21.2

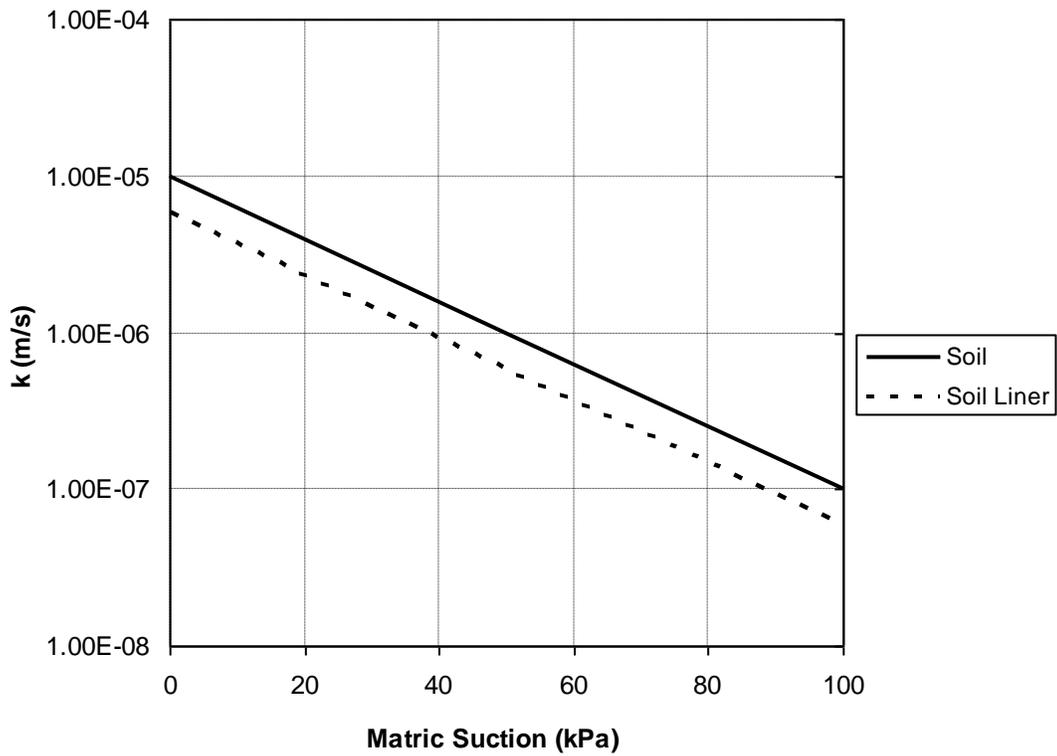


Figure 21.2 – Coefficient of Permeability Functions

21.2 RS2 Model

The RS2 models for initial steady state and transient analysis are shown in Figures 21.3 and 21.4, respectively. The boundary conditions model the rise in water level in the lagoon. No flow is assumed across the lagoon centerline.

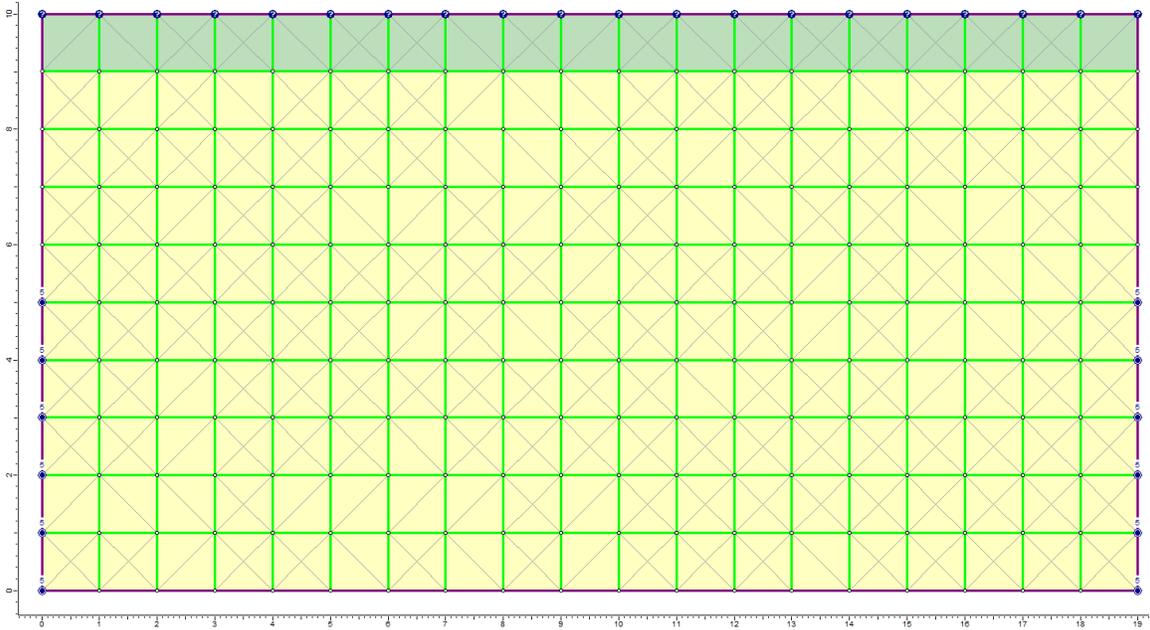


Figure 21.3 – RS2 Model – Initial Steady State

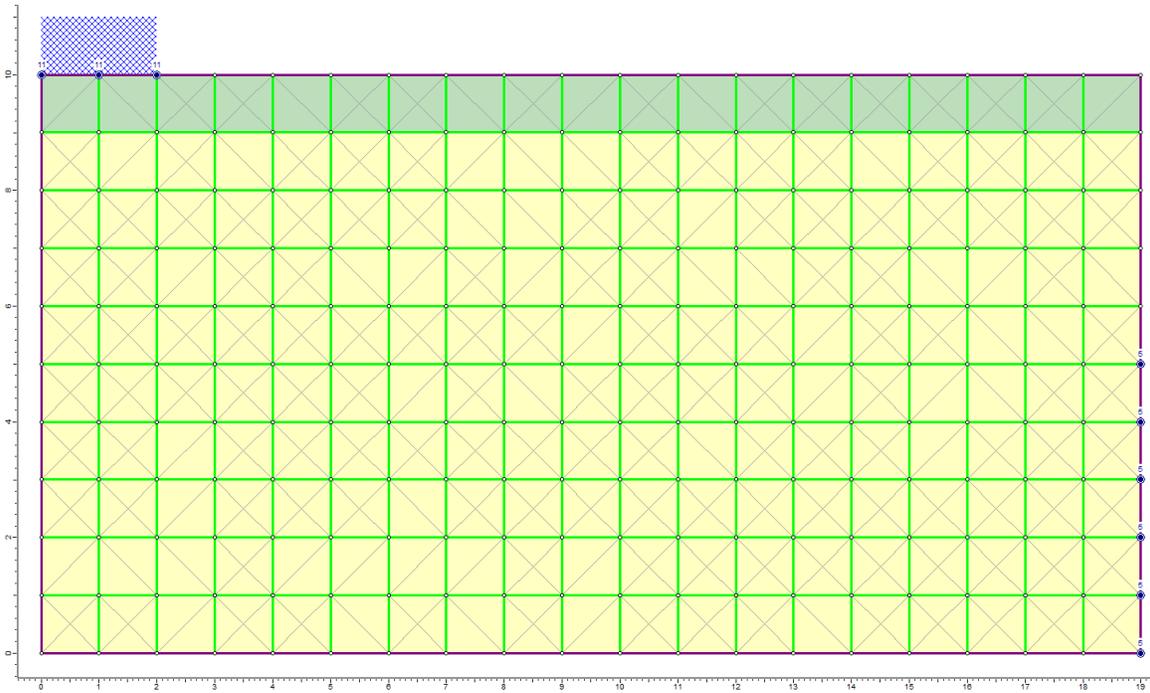


Figure 21.3 – RS2 Model – Transient

21.3 Results

Figures 21.4 to 21.7 show pressure head contours for different transient analysis times.

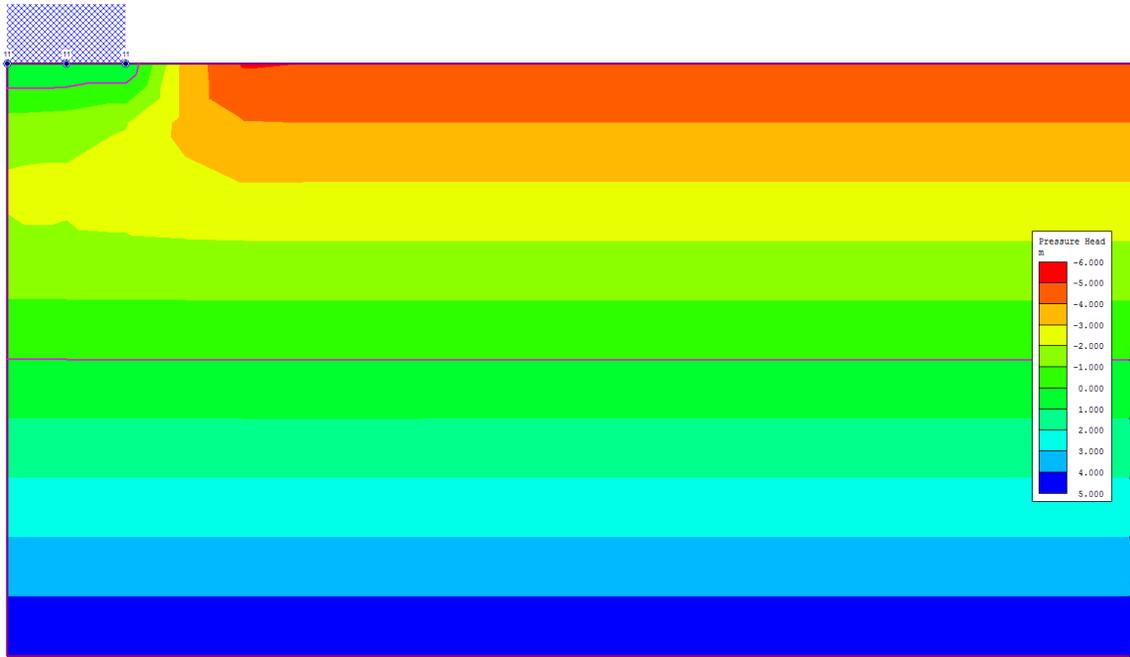


Figure 21.4 – Pressure Head Contours at 73 minutes

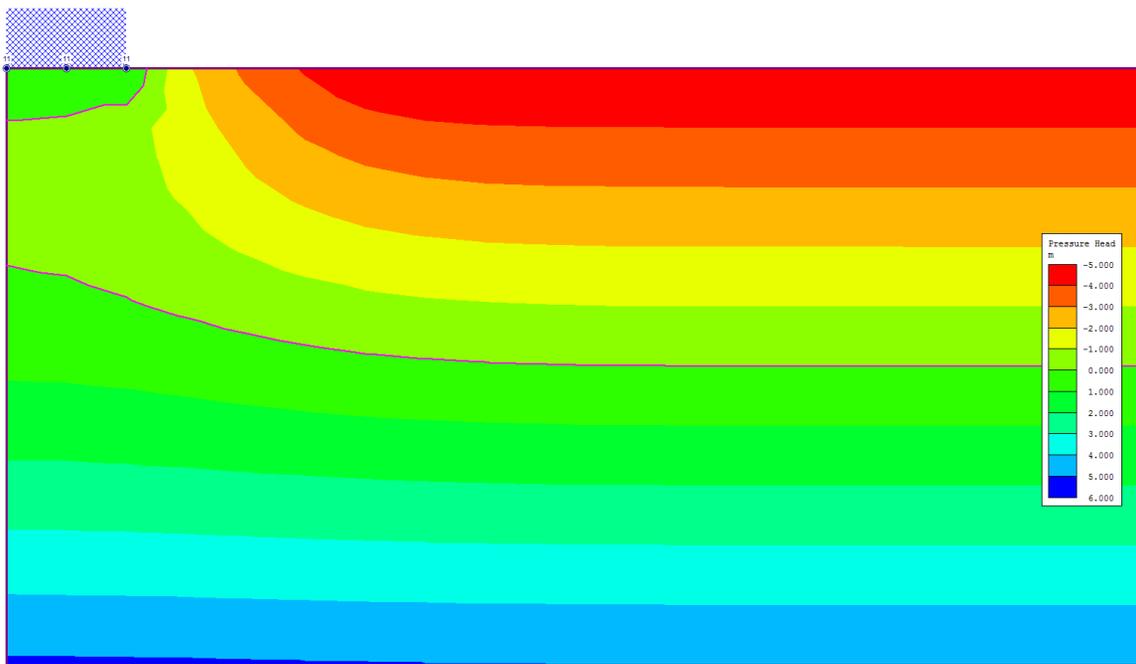


Figure 21.5 – Pressure Head Contours at 416 minutes

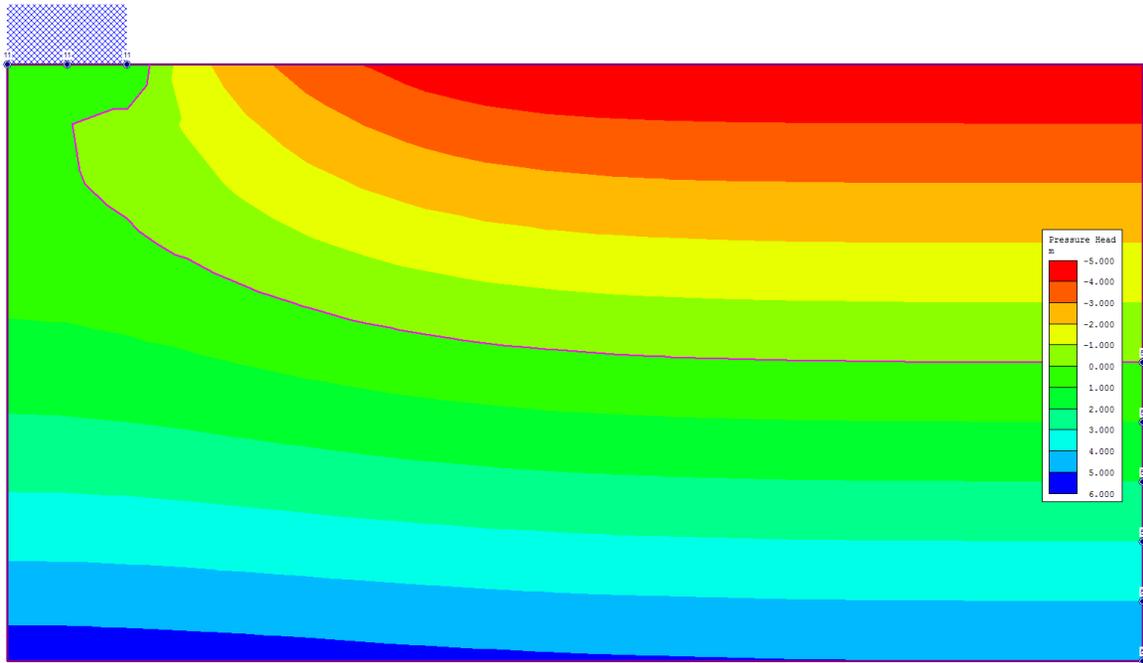


Figure 21.6 – Pressure Head Contours at 792 minutes

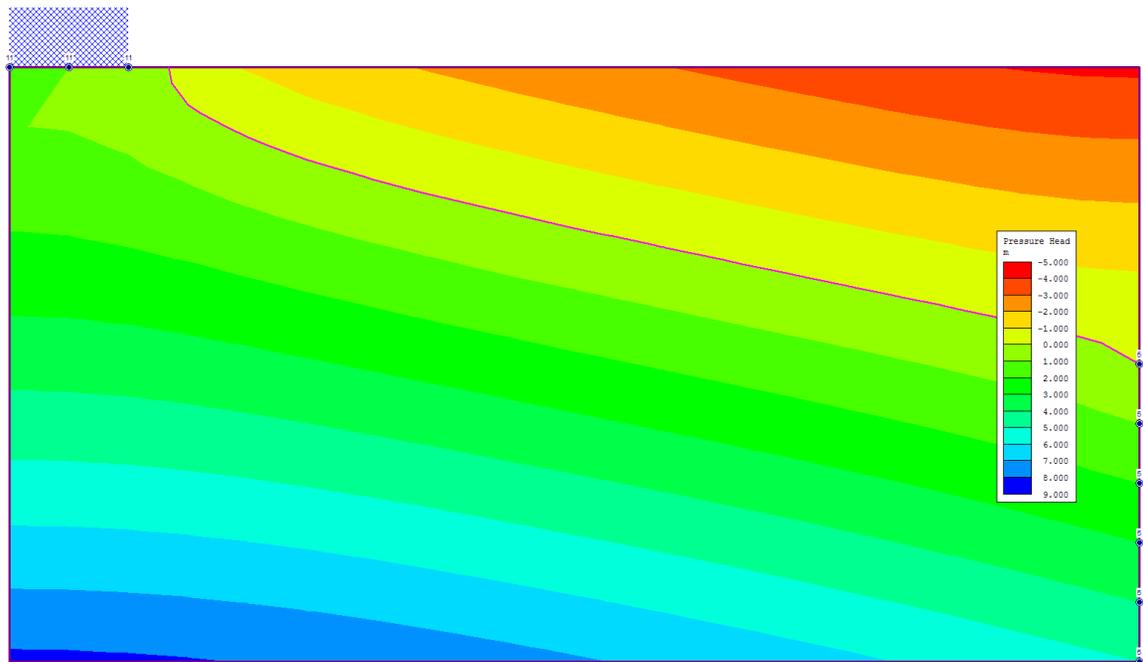


Figure 21.7 – Pressure Head Contours at 11340 minutes

Pressure head values are sampled along the top boundary as shown in Figure 21.8. These values from RS2 are plotted in comparison to values from Ref [1] in Figure 21.9.

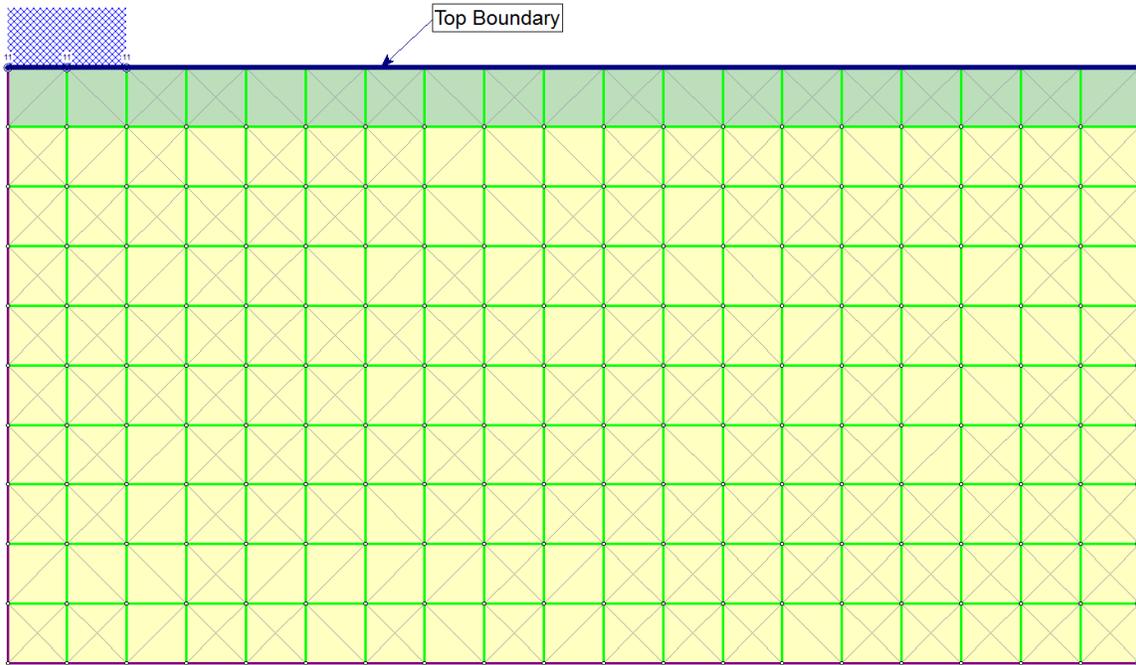


Figure 21.8 – Query Line

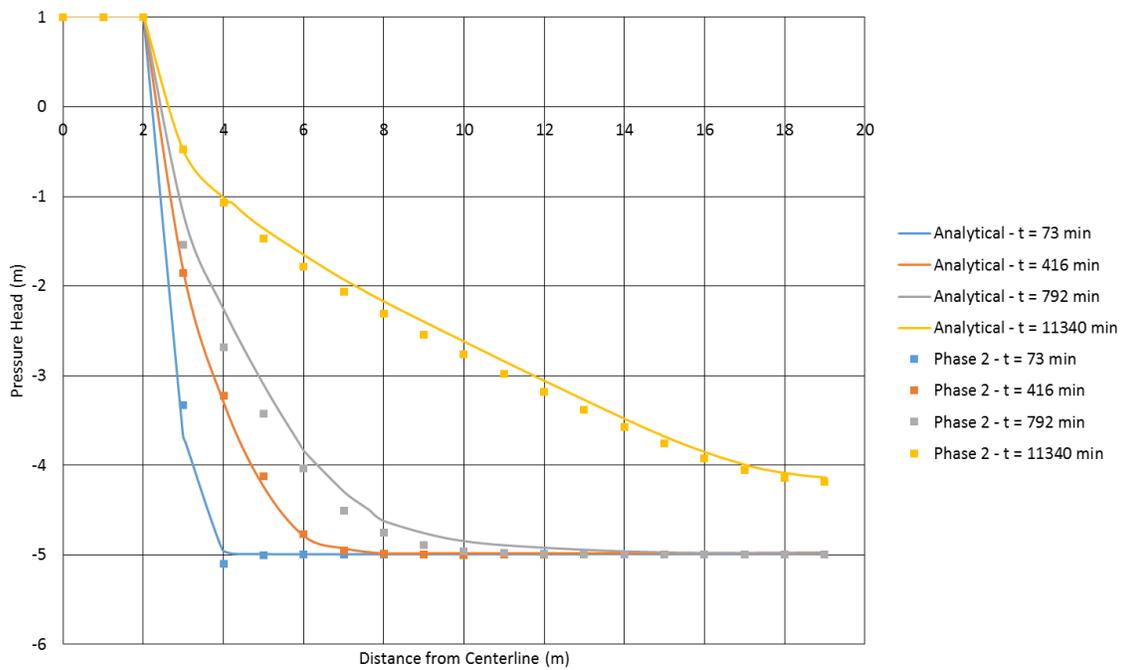


Figure 21.9 – Comparison of Pressure Head Values along Top Boundary

21.4 References

- [1] Fredlund, D.G. and Rahardjo, H. (1993), *Soil Mechanics for Unsaturated Soils*, New York: John Wiley & Sons.

22 Seepage in a Layered Slope

22.1 Problem Description

This problem deals with transient seepage in a layered slope. The slope consists of medium sand with a horizontal fine sand layer. At initial steady-state conditions, the water table is located at a height of 0.1 m from the toe of the slope. A constant infiltration of 2.1×10^{-4} m/s is applied at the top of the slope at time zero. An m_v value of 0.002 /kPa is assumed for both materials, and the permeability functions for the sands are shown in Figure 22.1.

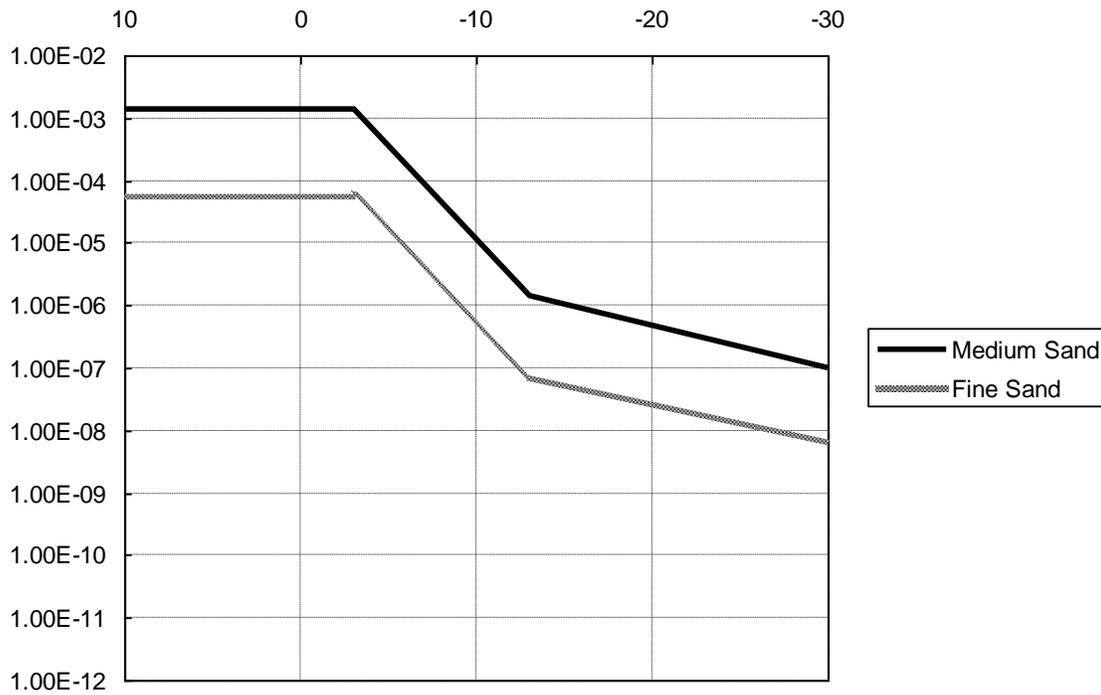


Figure 22.1 – Coefficient of Permeability Functions

22.2 RS2 Model

Figure 22.2 shows the RS2 model used to perform transient analysis with constant infiltration.

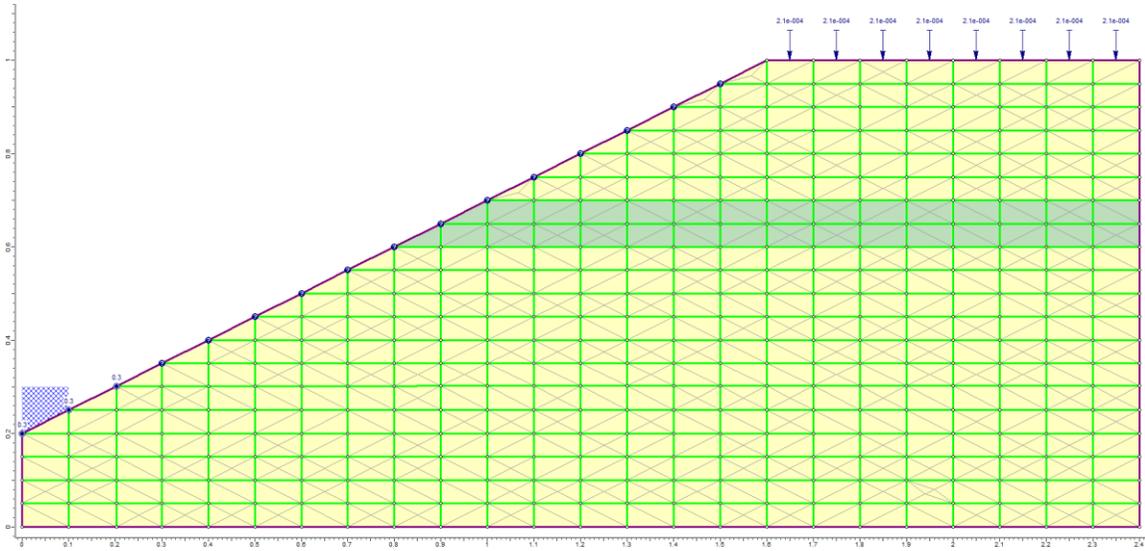


Figure 22.2 – RS2 Model

22.3 Results

Figures 22.3 to 22.5 show the total head contour results from RS2.

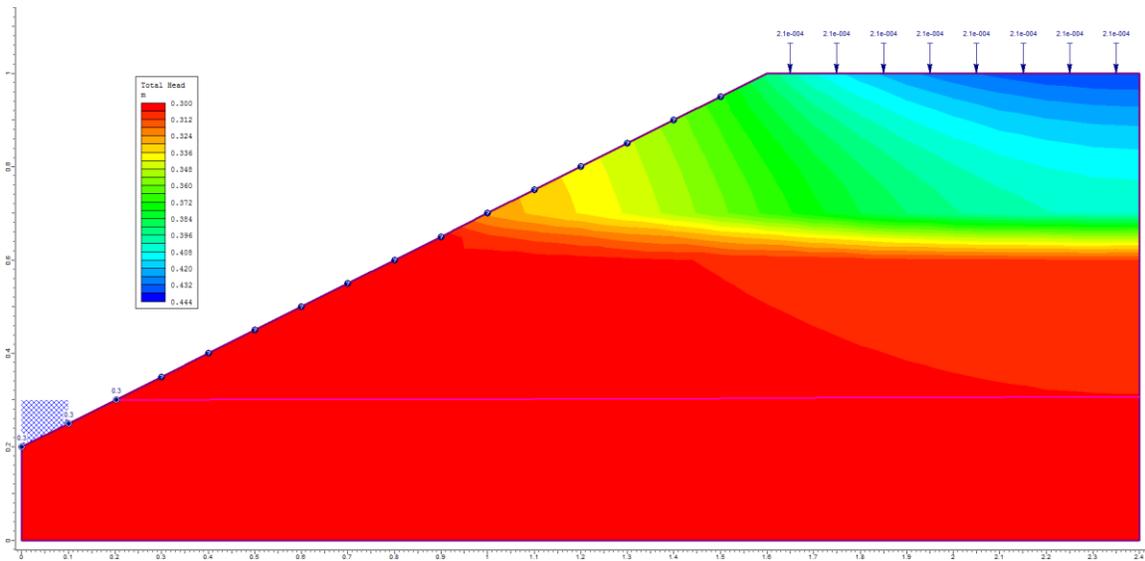


Figure 22.3 – Total Head Contours for 4.6 seconds

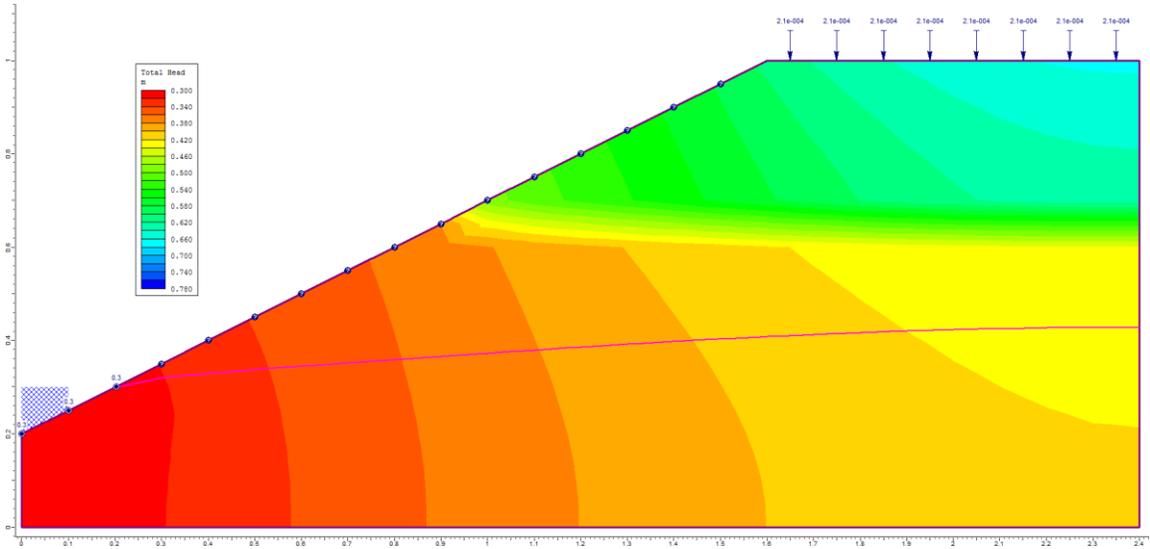


Figure 22.4 – Total Head Contours for 31 seconds

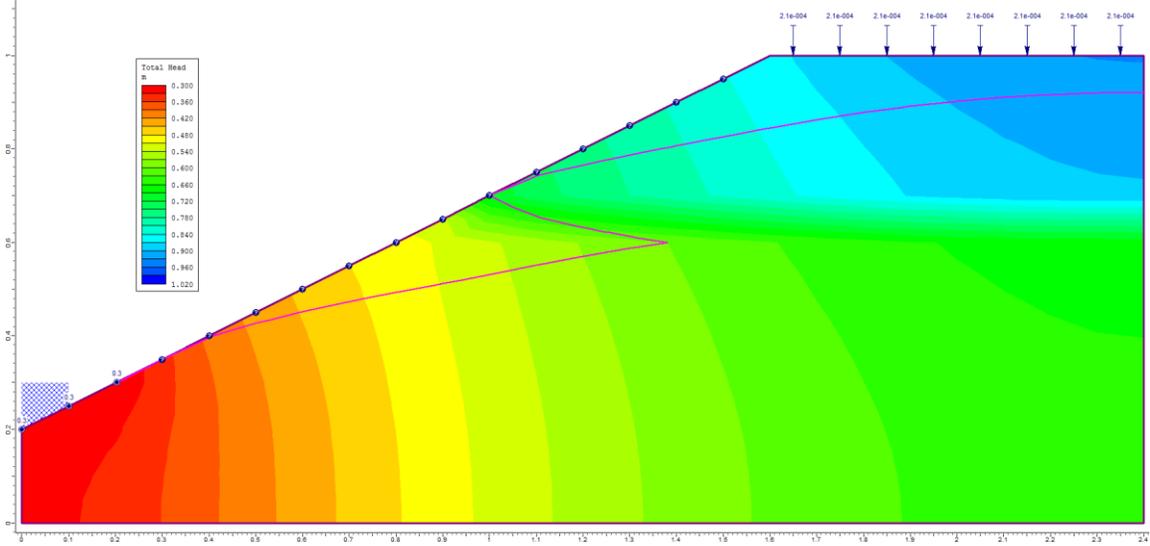


Figure 22.5 – Total Head Contours for 208 seconds

Values of total head are taken along the query line shown in Figure 22.6. Figure 22.7 compares RS2 results with those taken from Ref [1].

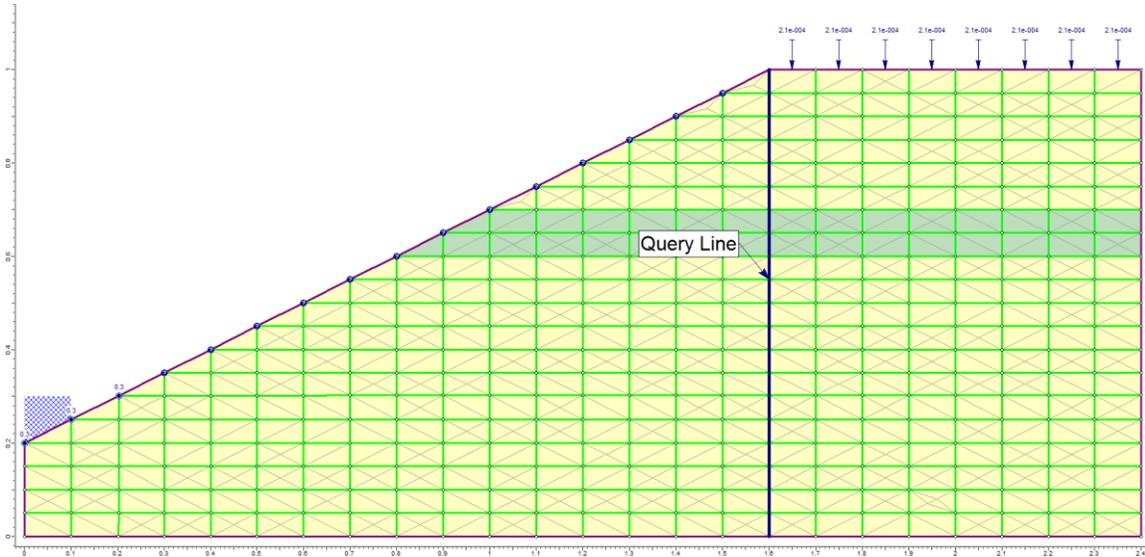


Figure 22.6 – Query Line

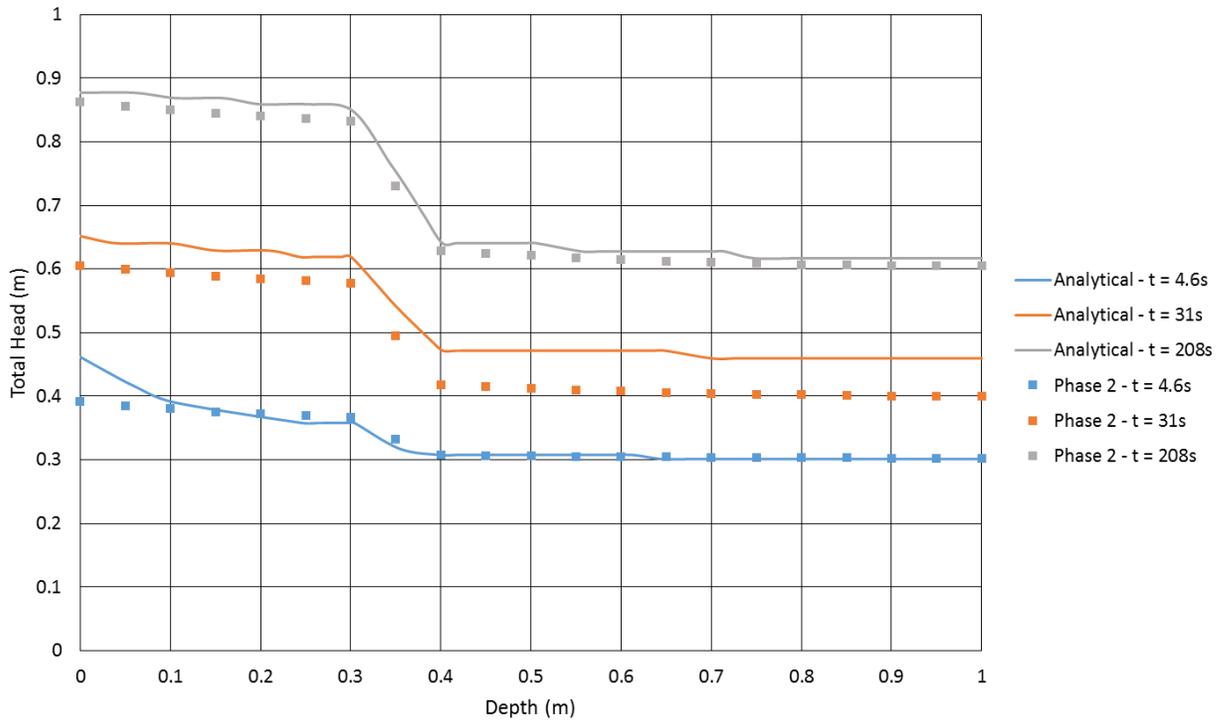


Figure 22.7 – Comparison of Total Head Values

22.4 References

- [1] Fredlund, D.G. and Rahardjo, H. (1993), *Soil Mechanics for Unsaturated Soils*, New York: John Wiley & Sons.

23 Transient Seepage Through a Fully Confined Aquifer

23.1 Problem Description

This problem deals with transient seepage through a fully confined aquifer. Two head conditions are examined. In both cases, the aquifer has an initial pore-water distribution that is changed through the introduction of five feet of hydraulic head to the left side of the aquifer. Seepage is then examined in the x -direction with time. The aquifer is 100 feet long and five feet thick. An illustration of the problem is presented in Figure 23.1.

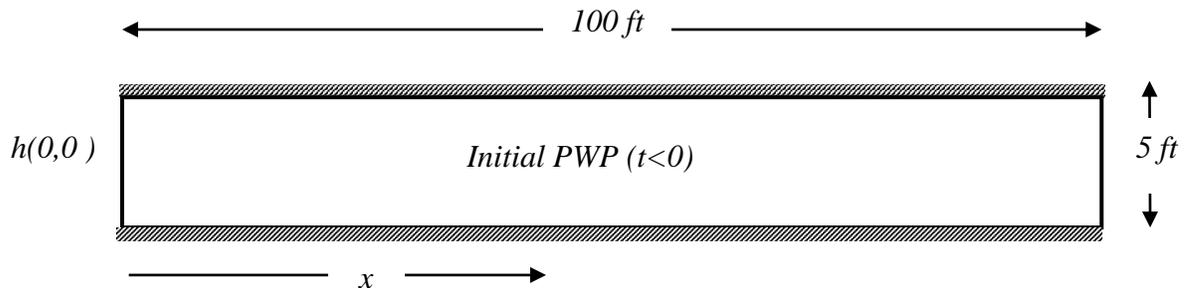


Figure 23.1 Model geometry

The soil has a hydraulic conductivity of 4 ft/hr and an m_v of 0.1. The hydraulic property is assumed to be fully saturated.

The equation for transient seepage through a fully confined aquifer can be expressed through the J.G. Ferris Formula [1] as,

$$h(x,t) = h(x,0) + \Delta H \cdot \operatorname{erfc}\left(\frac{x}{\sqrt{4t(T/S)}}\right)$$
$$T/S = \frac{k}{\gamma_w \cdot m_v}$$

Where $h(x,t)$ is the hydraulic head at position x at time t ; ΔH is the head difference between the initial pore-water distribution and the introduced hydraulic head; and erfc is the complimentary error function.

23.2 RS2 Model

1. No initial pore-water distribution

Figure 23.2 shows the RS2 model used to perform a transient analysis with 0 feet of initial pore-water pressure.

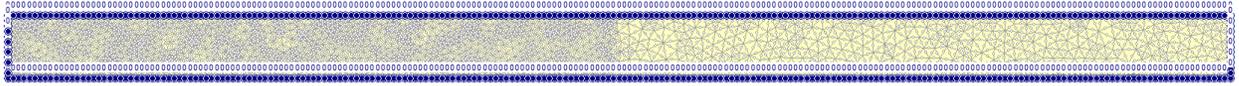


Figure 23.2 RS2 Model – 0 feet of Initial PWP

2. Initial pore-water distribution of 5 feet

Figure 23.3 shows the *Phase 2* model used to perform a transient analysis with 5 feet of initial head (assigned by setting the steady state boundary condition of the problem to 5 feet of head). Note that the boundary condition on the left face is set to 10 feet (5 feet of initial PWP plus 5 feet of introduced hydraulic head).

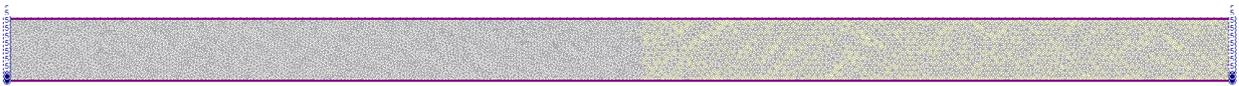


Figure 23.3 RS2 Model – 5 feet of Initial PWP

23.3 Results

Figures 23.4 and 23.5 show the total head contour results from RS2 at 600 hours.

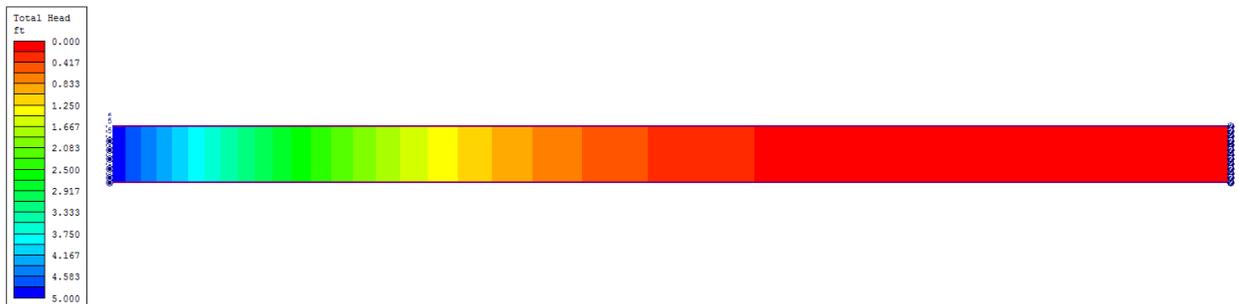


Figure 23.4 – Total Head Contours, 600 hours, no initial PWP

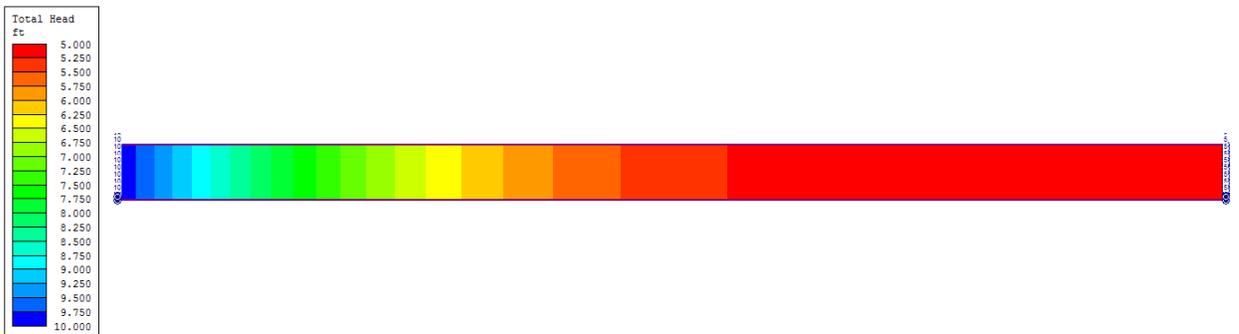


Figure 23.5 – Total Head Contours, 600 hours, 5 feet of initial PWP

A comparison of the RS2 results and the analytical solution for Case 1 is presented in Figure 23.6.

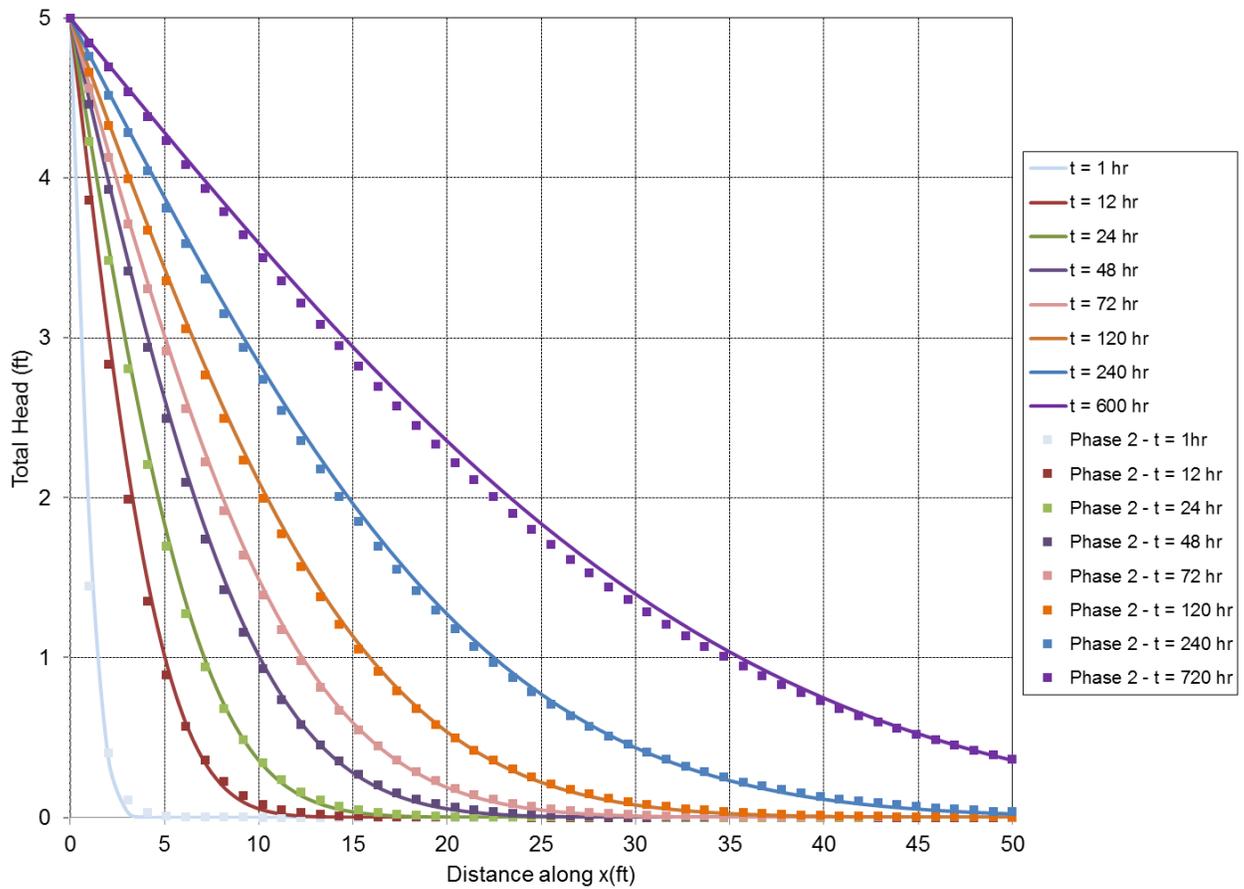


Figure 23.6 – Comparison of RS Results and Analytical Solution - Case 1

A comparison of the RS2 results and the analytical solution for Case 2 is presented in Figure 23.7.

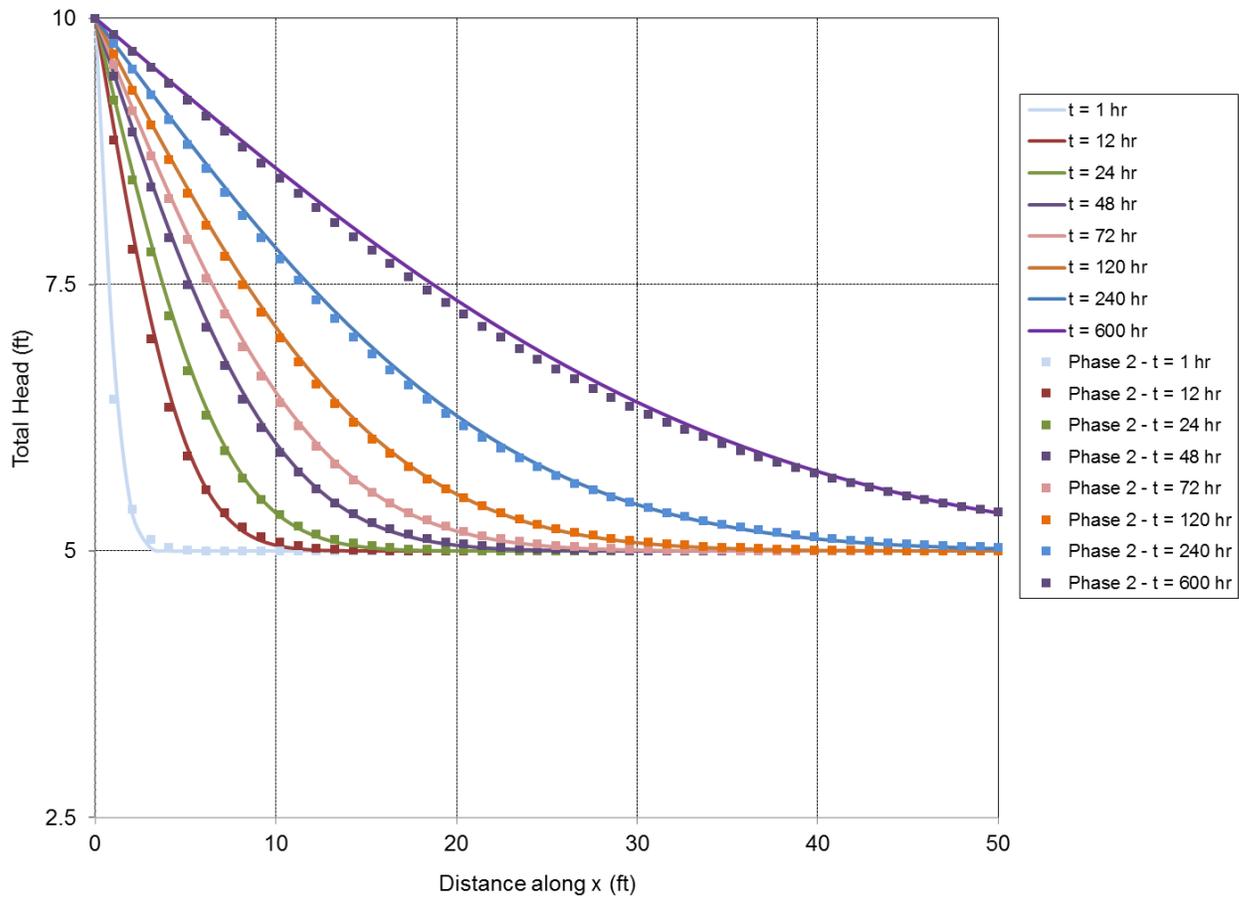


Figure 23.7 – Comparison of RS2 results and Analytical Solution - Case 2

23.4 References

[1] Tao, Y. and Xi, D. (2006), “Rule of Transient Phreatic Flow Subjected to Vertical and Horizontal Seepage:” *Applied Mathematics and Mechanics*. (27), 59-65.