Conversion of Non-Linear Strength Envelopes into Generalized Hoek-Brown Envelopes

Introduction

The power curve criterion is commonly used in limit-equilibrium slope stability analysis to define a non-linear strength envelope (relationship between shear stress, τ, and normal stress, σ_n) for soils. In the Rocscience slope stability program *Slide* the criterion has the form:

$$\tau = a\left(\sigma_n + d\right)^b + c + \sigma_n \tan(\theta_w) , \qquad (1)$$

where *a*, *b* and *c* are parameters typically obtained from a least-squares regression fit of data obtained from small-scale shear tests. The *d* parameter represents the tensile strength of a material, while θ_w is known as the "waviness angle."

Another popular strength model used in slope stability analysis is the "shear /normal function." It consists of pairs of shear and normal stress values that define arbitrary, non-linear shear/normal strength envelopes for materials.

Because no flow rules have been derived or defined for the power curve and shear/normal function criteria, it is currently impossible to use them in elasto-plastic finite element analysis. As a result, when such a strength model exists in a *Slide* file that is imported into *Phase²*, it is converted into an equivalent Generalized Hoek-Brown model. The Generalized Hoek-Brown criterion is the most widely used model for characterizing the strength of rock masses, and has a well-defined plastic flow rule.

The next sections will present the equations of the Generalized Hoek-Brown criterion, and will outline the procedures for determining a Generalized Hoek-Brown criterion equivalent to a power curve or shear/normal strength model.

The Generalized Hoek-Brown strength criterion

The non-linear Generalized Hoek-Brown criterion [3] for rock masses defines material strength in terms of major and minor principal stresses as:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a \qquad (2)$$

where σ_{ci} is the uniaxial compressive strength of the intact rock material, while

$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$, $s = \exp\left(\frac{GSI - 100}{9 - 3D}\right)$, and $a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right)$.

m_i is an intact rock material property, *GSI* is known as the geological strength index, while *D* is termed the disturbance factor [1].

Using relationships developed by Balmer [1, 2], a shear-normal stress envelope equivalent to the Generalized Hoek-Brown principal stress envelope can be determined. The shear stress (τ) and normal stress (σ_n) pair corresponding to a point on a principal stress envelope can be determined from the equations

$$\tau = (\sigma_1 - \sigma_3)\frac{\sqrt{d\sigma_1/d\sigma_3}}{d\sigma_1/d\sigma_3 + 1} \qquad (3)$$

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\frac{d\sigma_1/d\sigma_3 - 1}{d\sigma_1/d\sigma_3 + 1}. \qquad (4)$$

For the Generalized Hoek-Brown criterion, the following equations relate σ_n and τ to σ_1 and σ_3:

$$\tau = (\sigma_1 - \sigma_3)\frac{\sqrt{1 + am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}} + s\right)^{a-1}}}{2 + am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}} + s\right)^{a-1}} \qquad (5)$$

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\frac{am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}} + s\right)^{a-1}}{2 + am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}} + s\right)^{a-1}} \qquad (6)$$

For a given set of Generalized Hoek-Brown parameters and a specified σ_3 value, σ_n can be determined from Equation (5) through replacement of σ_1 with the definition of the criterion (Equation (1)).

Figure 2 shows a power curve envelope, τ^{power}, and a new Generalized Hoek-Brown, τ^{GHB}, that approximates the power curve. Both envelopes are drawn in shear-normal space. For any given σ_n value, the square of the error between the reduced and approximated envelopes is defined by the equation:

$$\varepsilon(\sigma_n)^2 = \left(\tau^{power} - \tau^{GHB}\right)^2. \qquad (7)$$

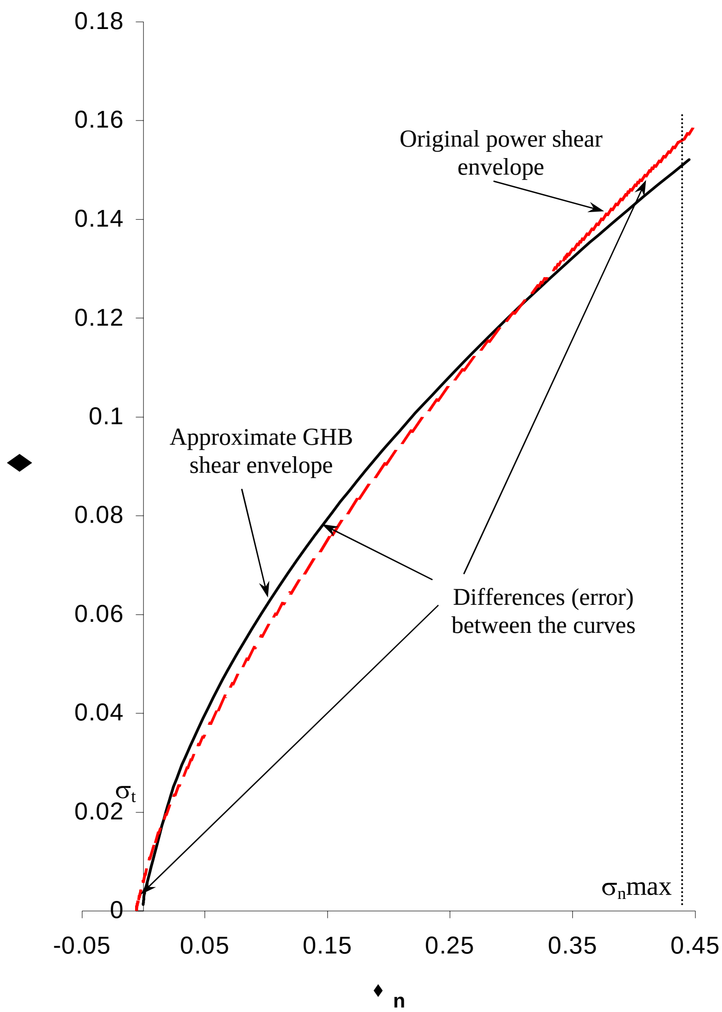

Figure 2. Approximation of a power curve with an equivalent Generalized Hoek-Brown envelope in shear-normal space. Notice the regions of error or differences between the two curves.

The total error of the fit of τ^{GHB} to τ^{power} can be obtained through integration of the squared error function:

$$Total\ error = \int_{\sigma_t}^{\sigma_n \max} \varepsilon\left(\sigma_n\right)^2 d\sigma_n \qquad (8)$$

over the range σ_t (the tensile strength) to a maximum normal stress value, $\sigma_n \max$. Because the squared error function does not explicitly relate σ_n to τ, the integration is best performed using a numerical approach such as gaussian quadrature.

The parameters of the best-fit Generalized Hoek-Brown envelope to the power curve strength envelope can be obtained through minimization of the total squared error. *Phase²* does this minimization the Simplex technique, which does not require derivatives of the function being minimized.

Procedure for computing equivalent Generalized Hoek-Brown parameters

To reduce the number of parameters to be determined, the curve-fitting procedure assumes the disturbance parameter $D = 0$, and estimates best-fit values for the three parameters σ_{ci}, m_i and *GSI*. This is because, as seen from the equation that define the Generalized Hoek-Brown criterion, the parameters m_b, s, and a can be calculated using m_i and *GSI*. Assuming $D = 0$ simplifies calculations substantially with practically no penalty to the accuracy of the curve-fitting procedure.

The steps for estimating the Generalized Hoek-Brown parameters equivalent to a power curve envelope are then as follows:

(i) Establish the range of minor principal stresses acting in a slope. Since the minimum stress is taken to be the tensile strength, σ_t, it is only necessary to determine the maximum σ_3 value in the slope.

(ii) Determine the corresponding value of normal stress, $\sigma_n \max$, using Equation (5).

(iii) Minimize the squared error function over the range $[\sigma_t, \sigma_n \max]$ using a technique such as the Simplex method. (The integration in the squared error function is performed using the numerical gaussian quadrature method.) The variables of the function are σ_{ci}, m_i and *GSI*. *D* is assumed to have a fixed value of zero.

(iv) Use m_i and *GSI* to calculate the parameters m_b, s, and a.

Determination of equivalent Generalized Hoek-Brown curve for Shear-Normal function

The procedures for determining a Generalized Hoek-Brown curve that best fits a shear-normal function is very similar to those described above for the power curve model. The primary difference lies in the squared error function. Since the shear-normal function is defined by a discrete number *m* of data points, the squared error function instead of having an integral uses the summation:

$$Total\ error = \sum_{i=1}^{m} \varepsilon\left(\sigma_{n,i}\right)^2 . \qquad (9)$$

REFERENCES

1. Hoek E., C. Carranza-Torres, and B. Corkum. 2002. Hoek-Brown criterion – 2002 edition. In *Proceedings of the 5th North American Rock Mechanics Symposium and the 17th Tunnelling Association of Canada: NARMS-TAC 2002, Toronto, Canada*, eds. R.E. Hammah et al, Vol. 1, pp. 267-273.
2. Balmer G. 1952. A general analytical solution for Mohr's envelope. *American Society for Testing and Materials*, vol. 52, pp. 1260-1271.