6- Hoek-Brown and Generalized Hoek-Brown Material Models

Hoek-Brom failure criterion is the most common failure criterion used for rock masses. Hoek and Brown (1980a, b) introduced their failure criterion in an attempt to provide input data for the analyses required for the design of underground excavations in hard rock. The criterion was derived from the results of research into the brittle failure of intact rock by Hoek (1968) and on model studies of jointed rock mass behavior by Brown (1970). The criterion starts from the properties of intact rock and then by applying reduction factors on the basis of the characteristics of joints in a rock mass is modified to suit the rock mass behavior.

The failure criterion of the Hoek Brown model in terms of principal stresses is

$$F_s = \sigma_1 - \sigma_3 - \sigma_{ci}\left(m_b \frac{-\sigma_1}{\sigma_{ci}} + s\right)^{0.5} = 0 \tag{6.1}$$

where σ_{ci} is the uniaxial compressive strength of the intact material, m is the reduced value of the intact rock parameter m_i, and s is a material constant that can have the maximum value of 1.0 for intact rock.

The mechanical behavior of a material that is modelled with Hoek-Brown model includes features such as:

- Isotropic shear strength (peak and residual) that has cohesive-frictional characteristic, and increases nonlinearly with the level of stress/confinement

- Tensile strength (by using a tension cutoff yield function or the tensile strength that is inherent in the model)

- Dilation (increase in volume) or critical state (constant volume) at failure

- Dependency of shear strength on Lode's angle (observed for most geomaterials)

The model is well suited for evaluation of stability of geotechnical/mining problems in rocks and rock-masses. This includes problems that have wide ranges of stress/confinement, since the dependency of shear strength on the level of stress in nonlinear and more realistic (compared to the Mohr-Coulomb model). Using the Shear Strength Reduction (SSR) method this model can evaluate safety factors equivalent to those calculated based on limit equilibrium approach (Slide), and in some provide better predictions of the failure modes and the safety factors. It can be also used with great success for calculations of load-displacement in simulations that include rocks and rock-masses.

The generalized Hoek-Brown yield surface has an additional parameter a that replaces the 0.5 power term.

$$F_s = \sigma_1 - \sigma_3 - \sigma_{ci}\left(m_b \frac{-\sigma_1}{\sigma_{ci}} + s\right)^{a} = 0 \tag{6.2}$$

To find the strength of the mass rock from intact rock properties, the Geological Strength Index (GSI) was introduced by Hoek, Wood and Shah (1992), Hoek (1994) and Hoek, Kaiser and Bawden (1995).

$$m_b = m_i e^{\left(\frac{GSI-100}{28-14D}\right)} \tag{6.3}$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \tag{6.4}$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right) \tag{6.5}$$

In above D is the disturbance factor due to blast or stress relaxation that varies from 0.0 for undisturbed in situ rock mass to 1.0 for very disturbed rock mass.

In terms of stress invariants the Generalized Hoek-Brown yield surface is

$$F_s = 2\cos\theta\sqrt{J_2} - \sigma_{ci}\left(\frac{m_b}{\sigma_{ci}}\left(\frac{-I_1}{3} + \sqrt{\frac{J_2}{3}}\left(\sin\theta - \sqrt{3}\cos\theta\right)\right) + s\right)^a = 0 \tag{6.6}$$

RS[2] and RS[3] accept peak values and residual values for the all the material properties of these two models. This means that after the initial yielding the strength of the material instantly drops to a lower residual state. The Hoek-Brown and the generalized Hoek-Brown models in RS[2] and RS[3] are elasto-brittle-plastic material model in general. In the case where the residual values are the same as peak values the behavior is elasto-perfect-plastic.

The plastic potential function has the same from as the yield surface

$$Q_s = 2\cos\theta\sqrt{J_2} - \sigma_{ci}\left(\frac{m_\psi}{\sigma_{ci}}\left(\frac{-I_1}{3} + \sqrt{\frac{J_2}{3}}\left(\sin\theta - \sqrt{3}\cos\theta\right)\right)\right)^a = const. \tag{6.7}$$

where m_ψ is the dilation parameter. This parameter should be less than or equal to m_b which makes the flow rule non-associated or associated respectively.

The dialog for defining this constitutive model is shown in Figure 6.1.

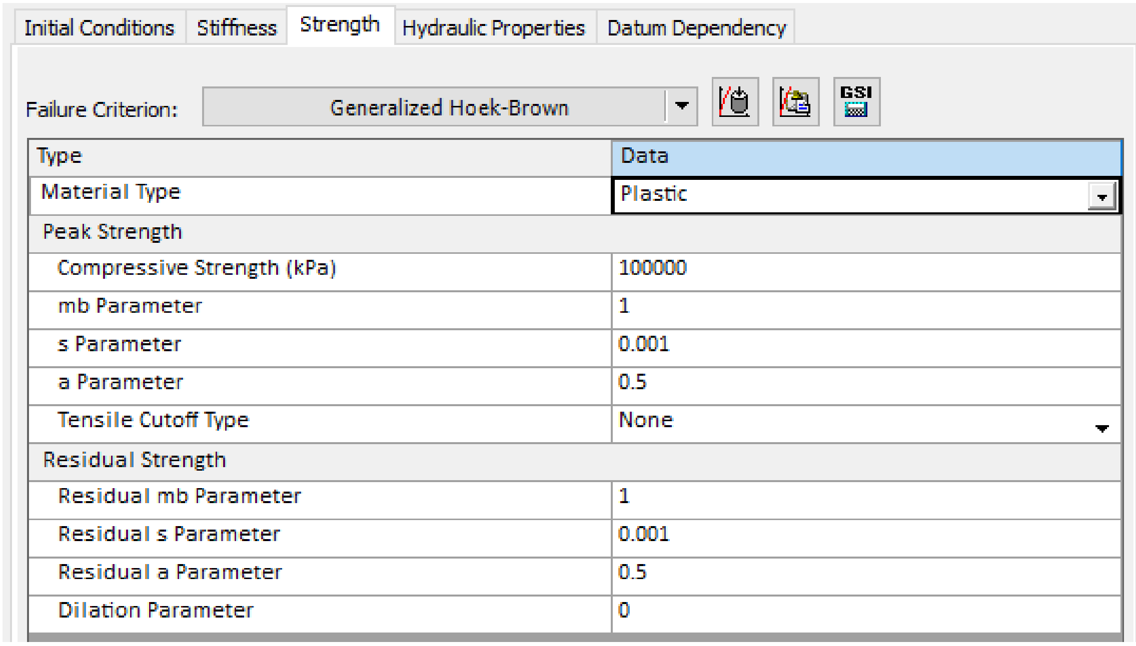

Figure 6.1. Dialog for defining Generalized Hoek-Brown model

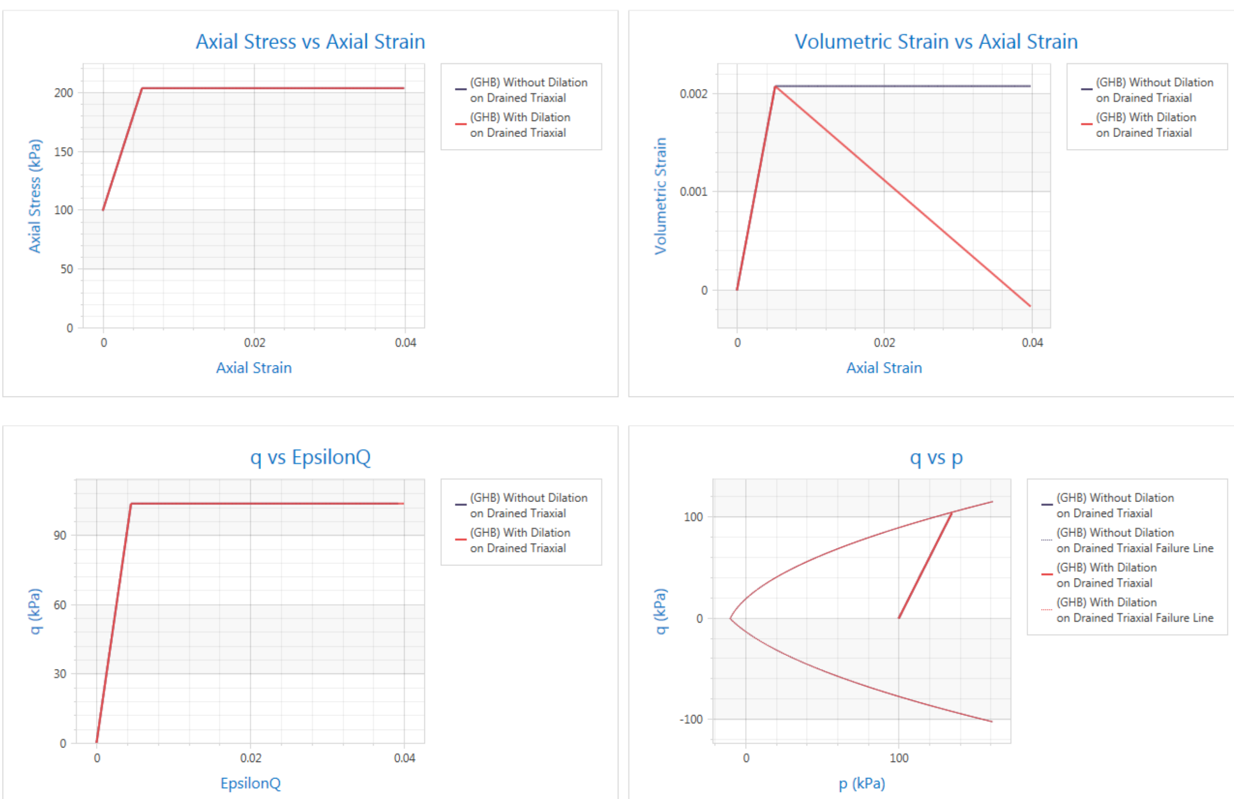

Figure 6.2. Stress paths of drained triaxial tests on materials with Generalized Hoek-Brown model

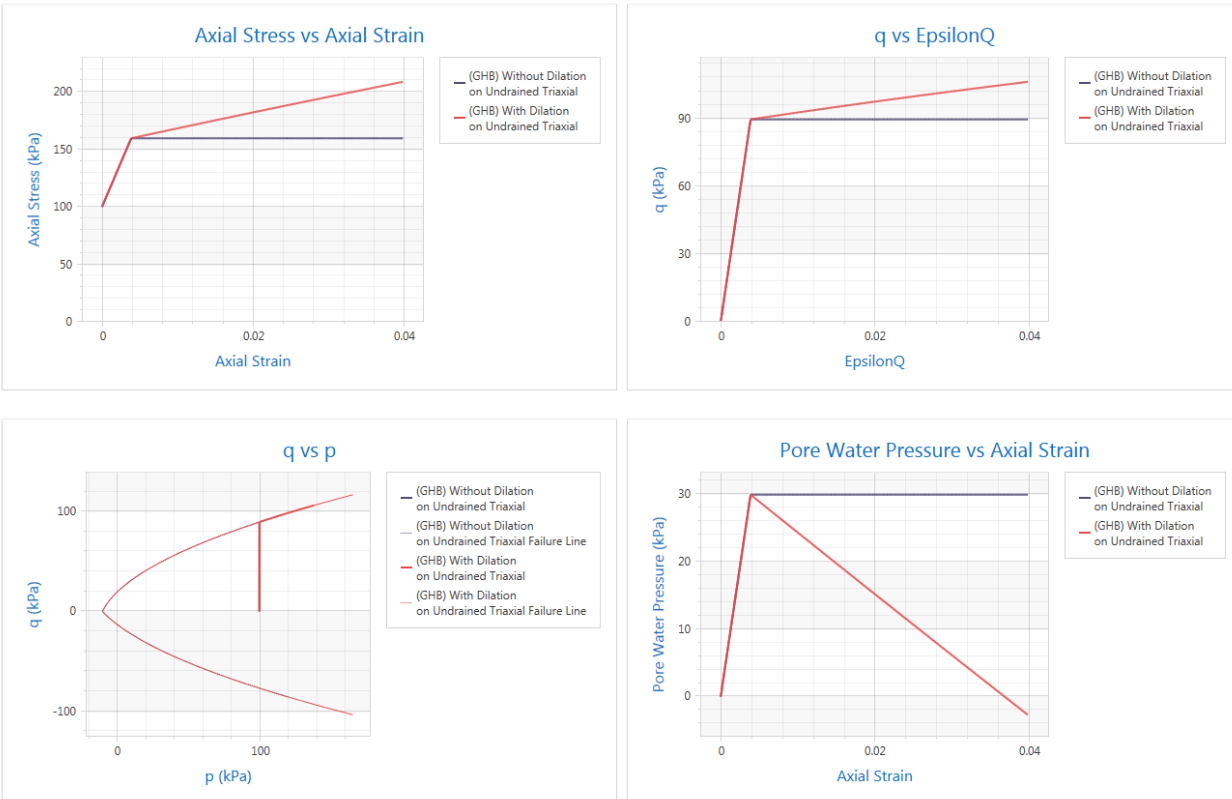

Figure 6.3. Stress paths of undrained triaxial tests on materials with Generalized Hoek-Brown model

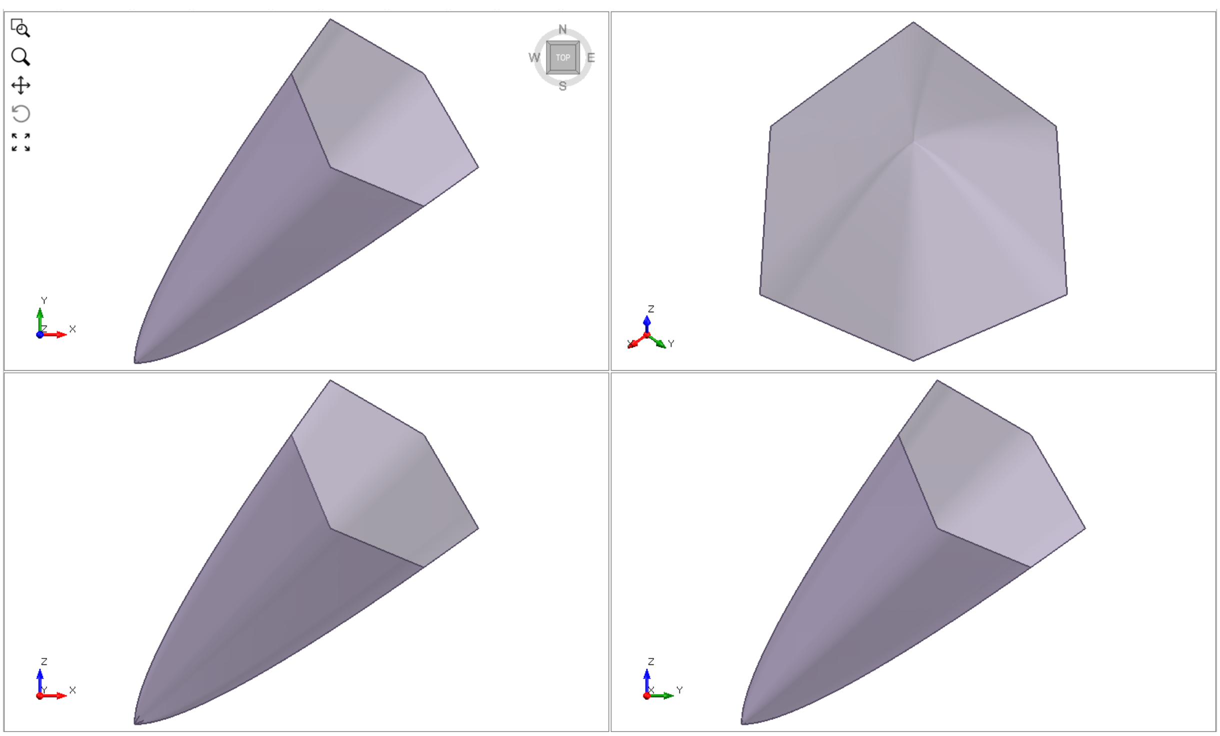

Figure 6.4. Yield surface of Generalized Hoek-Brown model in 3D stress space

Sample stress paths of drained and undrained triaxial compression tests that could be simulated with this model are presented in Figure 6.2 and 6.3. All the tests start form a hydrostatic confinement of $p = p' = 100$ kPa.

Stress paths of the drained tests include variations of axial stress and volumetric strain with increasing axial strain, variation of deviatoric stress with deviatoric strain and the stress path in p-q plane. The yield surface is also shown in the p-q plane. The simulated behavior is an elasto-perfect plastic behavior. The dilation effect is illustrated in the variation of volumetric strain with axial strain.

Stress paths of the undrained tests include the variation of axial stress and pore water pressure with increasing axial strain, variation of deviatoric stress with deviatoric strain and the stress path in p-q plane. The yield surface is also shown in the p-q plane. The dilation effect is illustrated in the plot of the stress path in p-q plane that also include the yield surface. The generation of negative pore water pressure in material with dilation leads to the increase in the effective mean stress, as the stress path lays on the yield surface and follows it to higher levels of deviatoric stress.

The yield surface of this model is a curved line in 2D stress space as shown in Figures 6.2 and 6.3 and has an irregular hexagonal pyramid shape in 3D stress space as presented in Figures 5.4. The definition of yield surface includes the Lode's angle and thus the projection of this yield surface in Π plane, with normal direction being the stress space diagonal, deviates from the circular shape of Drucker-Prager model and has a shape similar to the Mohr-Coulomb failure envelope.

The model also accept a tension cutoff. The yield surface of the tension cut off is

$$F_T = \sigma_1 - T = 0 \tag{6.8}$$

In above T is the tensile strength of the material. The flow rule for tensile failure is associated. There couple of options for the tensile strength of Hoek-Brown model. The maximum value of the tensile strength from can be calculated from the definition of the yield surface in equation 6.1 or 6.2.

$$T_{max} = \frac{s\sigma_{ci}}{m} \tag{6.9}$$

If the tensile strength is set to a higher value than T_{max} the program will ignore that value and use T_{max} instead. Hoek and Martin (2014) has proposed this alternative relationship for the tensile resistance

$$T = \frac{\sigma_{ci}}{8.62+0.7m_i} \tag{6.10}$$

The user defined option for tensile strength is also available to the users.

In slope stability analysis using the Finite Element Method with Shear Strength Reduction, the SRF can be applied in two different ways on the Generalized Hoek Brown shear strength criteria. Due to nonlinearity of the formulation of this model the application of SRF is not as straight forward as in what was presented for linear criteria such as Mohr Coulomb.

The first method is based on the approach presented by Hammah et. al. (2005). In this method, using relationships developed by Balmer (1952), a shear-normal stress envelope equivalent to the Generalized Hoek-Brown principal stress envelope, equation (6.2), can be determined. The shear and normal stress pair corresponding to a point on a principal stress envelope can be determined from the following equations:

$$\tau = (\sigma_1 - \sigma_3)\frac{\sqrt{1+am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}}+s\right)^{a-1}}}{2+am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}}+s\right)^{a-1}} \tag{6.11}$$

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\frac{am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}}+s\right)^{a-1}}{2+am_b\left(m_b\frac{\sigma_3}{\sigma_{ci}}+s\right)^{a-1}} \tag{6.12}$$

The SRF can be applied to the shear strength in equation 6.11. Calculating the strength parameters for the reduced shear strength criterion is not as straightforward as in the case for Mohr Coulomb model. Instead, the factored shear strength vs normal stress combination is considered to obtain a new set of parameters by a curve fitting algorithm. For detail of the curve fitting algorithm refer to Hammah et. al. (2005).

To apply SRF to the dilation parameter the ratio of the factored dilation parameter to factored m_b is assumed to be the same as the ratio of the original values.

$$\left(\frac{m_\psi}{m_b}\right)_{SRF} = \frac{m_\psi}{m_b} \tag{6.11}$$

The factored maximum tensile strength, equation 6.9, is calculated based on the new factored generalized Hoek Brown parameters. In case no tensile strength is defined for the original material this new maximum tensile strength is considered as the tensile strength.

The second method is based on the formulation presented by Benz et. al. (2008), in which an intrinsic strength factor, η , is included in the shear strength criteria in equation 6.2.

$$F_s = \sigma_1 - \sigma_3 - \frac{\sigma_{ci}}{\eta}\left(m_b\frac{-\sigma_1}{\sigma_{ci}} + s\right)^a = 0 \tag{6.14}$$

η is related to the SRF and is calculated based on the instantaneous friction angel that can be calculated by locally fitting a tangential Mohr Colomb criterion to the generalized Hoek Brown criterion:

$$\eta = \frac{1}{2}\left(SRF(2+\acute{f})\sqrt{1+\frac{\left(\frac{1}{SRF^2-1}\right)\acute{f}^2}{(2+\acute{f})^2}} - \acute{f} \right) \tag{6.15}$$

$$\acute{f} = a m_b \left(m_b \frac{-\sigma_1}{\sigma_{ci}} + s \right)^{a-1} \tag{6.16}$$

For detail of the formulation refer to Benz et. al. (2008).

To apply SRF on flow rule, the dilation parameter will remain the same, but the same intrinsic factor is applied on the plastic potential function.

The factored maximum tensile strength in this approach, is the same as in the original material. The intersection of yield criterion with minor principal axis and first invariant of stress tensor will remain the same.

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