4- Drucker-Prager Material Model

This model is intended to model cohesive geological materials that exhibit pressure-dependent yield, such as soils and rocks. The shear yield surface of this model is (e.g., Owen and Hinton 1980, Simo and Hughes 1998):

$$F_s = \sqrt{J_2} + q_\varphi \frac{I_1}{3} - k = 0 \tag{4.1}$$

q_φ and k are material properties and RS^2 and RS^3 accept peak values and residual values for these two parameters. This means that after the initial yielding the strength of the material instantly drops from its peak state to a lower residual state. The Drucker-Prager model in RS^2 and RS^3 is an elasto-brittle-plastic material model in general. In the case where the residual values are the same as peak values the behavior is elasto-perfect-plastic.

The mechanical behavior of a material that is modelled with Drucker-Prager model includes features such as:

- Isotropic shear strength (peak and residual) that has cohesive-frictional characteristic, and increases linearly with the level of stress/confinement

- Tensile strength (by using a tension cutoff yield function)

- Dilation (increase in volume) or critical state (constant volume) at failure

- Shear strength that is independent of Lode's angle

The plastic potential function has the same from as the yield surface

$$Q_s = \sqrt{J_2} + q_\psi \frac{I_1}{3} = const. \tag{4.2}$$

where q_ψ is the dilation parameter. This parameter should be less than or equal to q_φ which makes the flow rule non-associated or associated respectively.

The dialog for defining this constitutive model is shown in Figure 4.1. Sample stress paths of drained and undrained triaxial compression tests that could be simulated with this model are presented in Figure 4.2 and 4.3. All the tests start form a hydrostatic confinement of $p = p' = 100$ kPa .

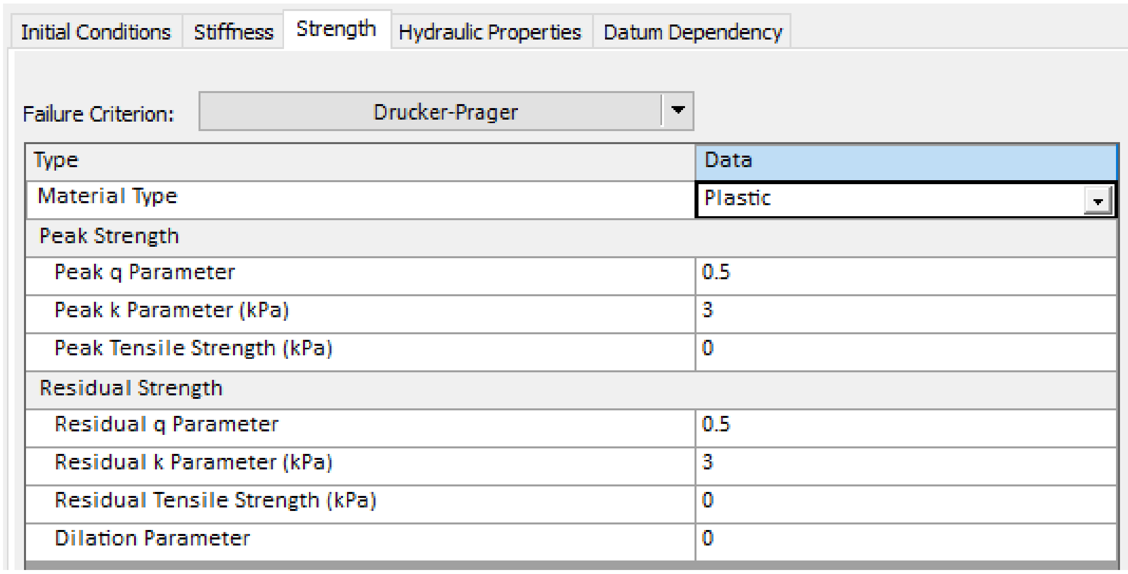

Figure 4.1. Dialog for defining Drucker-Prager model

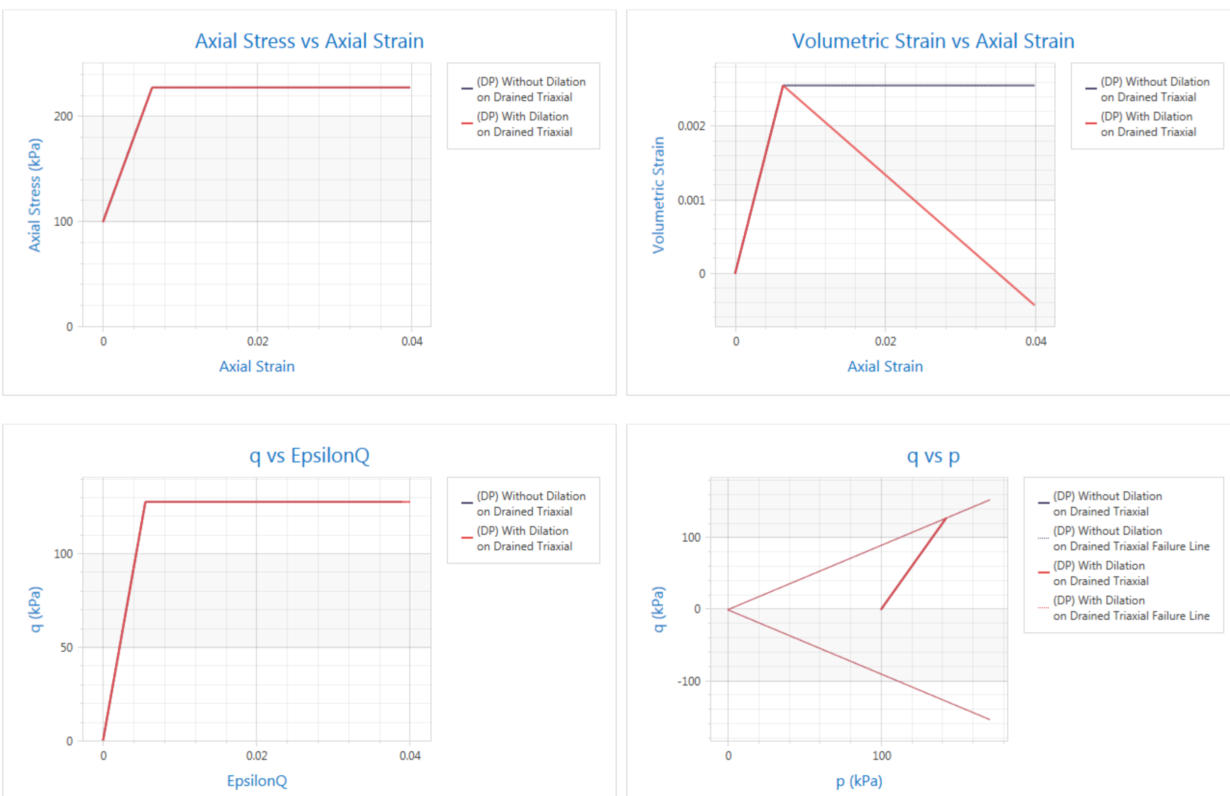

Figure 4.2. Stress paths of drained triaxial tests on materials with Drucker-Prager constitutive model

Stress paths of the drained tests include variations of axial stress and volumetric strain with increasing axial strain, variation of deviatoric stress with deviatoric strain and the stress path in p-q plane. The yield surface is also shown in the p-q plane. The simulated behavior is an elasto-perfect plastic behavior. The dilation effect is illustrated in the variation of volumetric strain with axial strain.

Stress paths of the undrained tests include the variation of axial stress and pore water pressure with increasing axial strain, variation of deviatoric stress with deviatoric strain and the stress path in p-q plane. The yield surface is also shown in the p-q plane. The dilation effect is illustrated in the plot of the stress path in p-q plane that also include the yield surface. The generation of negative pore water pressure in material with dilation leads to the increase in the effective mean stress, as the stress path lays on the yield surface and follows it to higher levels of deviatoric stress.

The yield surface of this model is a line in 2D stress space as shown in Figures 4.2 and 4.3 and has a conical shape in 3D stress space as presented in Figure 4.4. The definition of yield surface does not include the Lode’s angle and thus the projection of this yield surface in Π plane, with normal direction being the stress space diagonal, is independent of this stress invariant and is a complete circle.

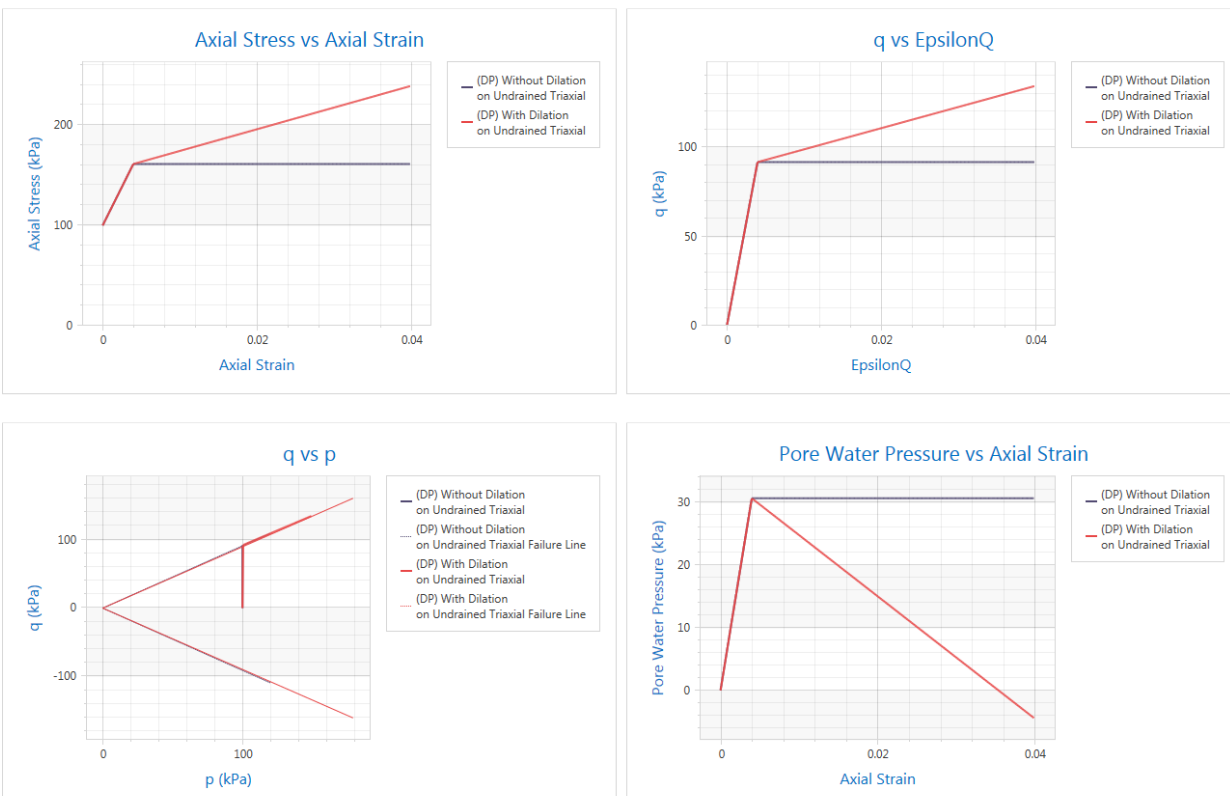

Figure 4.3. Stress paths of undrained triaxial tests on materials with Drucker-Prager constitutive model

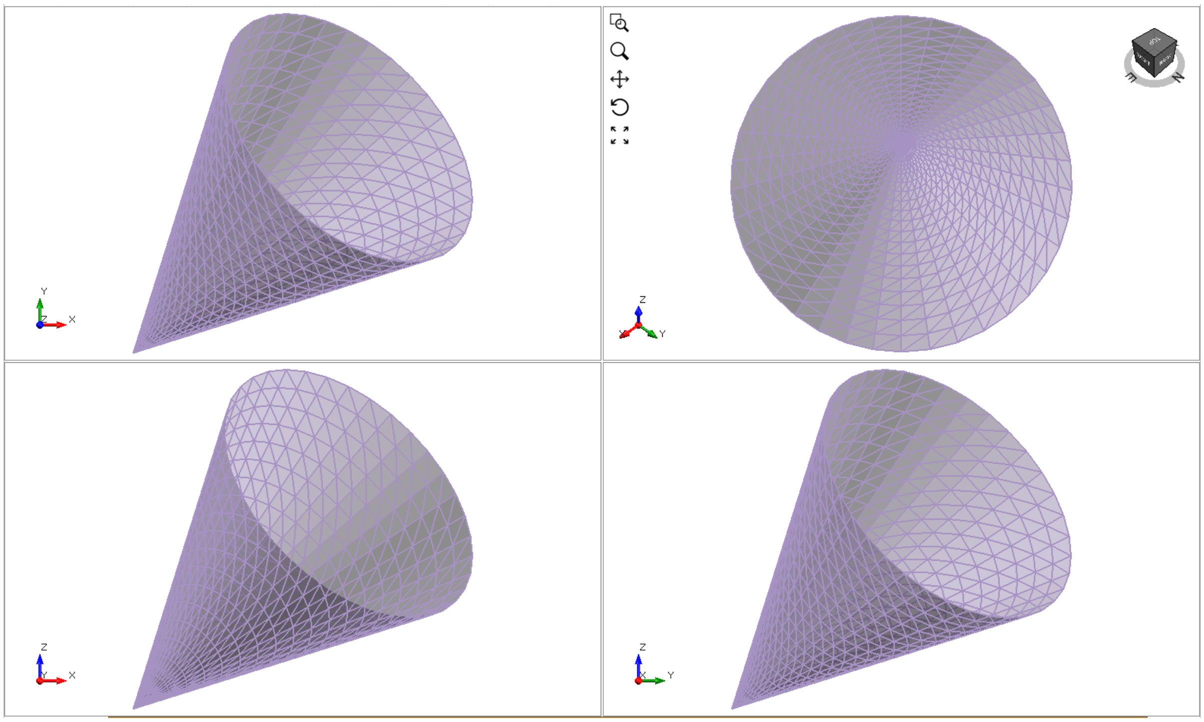

Figure 4.4. Yield surface of Drucker-Prager model in 3D stress space

The model also accepts a tension cutoff. The yield surface of the tension cut off is

$$F_T = \sigma_1 - T = 0 \tag{4.3}$$

In above T is the tensile strength of the material. The flow rule for tensile failure is associated.

In slope stability analysis using the Finite Element Method with Shear Strength Reduction, the factored shear strength can be calculated by applying the Strength Reduction Factor to the shear failure criteria defined in equation (4.1). Shear strength is represented by square root of the second invariant of deviatoric stress tensor, $\sqrt{J_2}$.

$$\frac{\sqrt{J_2}}{SRF} = \frac{-q_\varphi \frac{I_1}{3} + k}{SRF} \tag{4.4}$$

The factored Drucker-Prager properties after the application of SRF are

$$\left(q_{\varphi}\right)_{SRF} = \frac{q_{\varphi}}{SRF} \ , \quad k_{SRF} = \frac{k}{SRF} \tag{5.5}$$

The SRF is applied to the dilation parameter, q_{ψ}, in the same way that is applied to q_{φ}.

References

Griffiths, D.V. and Lane, P.A. (1999), Slope stability analysis by finite elements, Geotechnique, vol. 49, no. 3, pp. 387-403.

Owen D.R.J. and Hinton E. (1980), Finite Elements in Plasticity- Theory and Practice Pineridge Press, Swansea.

Hughes, T. J. R., & Simo, J. C. (1998). Computational inelasticity. New York.