3- Elastoplastic Constitutive Equations

In the framework of classical plasticity there are two basic functions or surfaces that are defined in the stress space, i.e. "yield surface" and "plastic potential" (e.g., Owen and Hinton 1980, Simo and Hughes 1998). These two functions are used in the derivation of constitutive equations of the materials.

Based on experimental evidence, plasticity theory postulates that irreversible or plastic strains occur whenever the stress state satisfies the yield criterion

$$F_m(\sigma_{ij}, \kappa) = 0 \tag{3.1}$$

where σ_{ij} is the stress tensor and κ is the material state parameter. Index m indicates that there might exists more than one yield surface in the definition of admissible states of the material. In the framework of plasticity, following the observation on the plastic flow of metals, it is assumed that the plastic strain increments ($\dot{\varepsilon}_{ij}^{p}$) are coaxial with the gradient of plastic potential function Q

$$Q_m(\sigma_{ij}) = const. \quad , \quad \dot{\varepsilon}_{ij}^{p} = \sum_m \dot{\lambda}_m \frac{\partial Q_m}{\partial \sigma_{ij}} \tag{3.2}$$

Each yield surface has its corresponding plastic potential function, $\dot{\lambda}$s are the plastic multipliers and equation (3.2) is referred to as the flow rule. The flow rule is associated if the yield function and plastic potential are the same, this the case for most metals. For geomaterials, however, the flow rule is more likely to be non-associated, meaning that the plastic potential and yield function are different.

In elastoplasticity the constitutive equations can be formed generally by invoking the consistency condition (3.1), Hooke's law (3.3), additivity postulate (3.4), the flow rule (3.2) and hardening rule (3.5) if applicable.

$$\dot{\sigma}_{ij} = D_{ijkl}^{e} \dot{\varepsilon}_{kl}^{e} \tag{3.3}$$

$$\dot{\varepsilon}_{kl} = \dot{\varepsilon}_{kl}^{e} + \dot{\varepsilon}_{kl}^{p} \tag{3.4}$$

$$\dot{\kappa} = f\left(\varepsilon_{kl}^{p}\right) \tag{3.5}$$

Hooke's law relates the increment of stress to the increment of elastic strains using the elastic constitutive equations. In RS^2 and RS^3 elastoplastic models can utilize any of the elastic isotropic models.

The additivity postulate simply sums up the increment of elastic and plastic strain to form the increment of strain. The hardening controls the behavior of material after initial yielding. The expansion/shrinkage of the yield loci is controlled by hardening/softening rules.

In any loading path the Kuhn-Tucker conditions (Luenberger 1984) should be satisfied

$$F_m(\sigma_{ij}, \kappa) \leq 0; \dot{\lambda}_m \geq 0; \dot{\lambda}_m F_m = 0 \tag{3.6}$$

The case of $F(\sigma_{ij}, \kappa) < 0$ yields $\dot{\lambda} = 0$, and corresponds to elastic behavior. For $\dot{\lambda} > 0$ the material undergoes plastic deformation and the stress state remains on the yield surface, i.e. $F(\sigma_{ij}, \boldsymbol{\kappa}) = 0$. During plastic flow the consistency condition is automatically satisfied by $F(\sigma_{ij}, \boldsymbol{\kappa}) = 0$.

The following chapters will introduce individual the elastoplastic models that are included in RS^2 and RS^3.

References

Luenberger D.G. (1984), Linear and Nonlinear Programming, Reading, Massachusetts, Addison-Wesley.

Owen D.R.J. and Hinton E. (1980), Finite Elements in Plasticity- Theory and Practice Pineridge Press, Swansea.

Simo, J. C., & Hughes, T. J. R. (1980). Computational inelasticity. 1998. New York.