

## 14- CySoil and Double Yield Material Models – FLAC

This model is the CySoil and Double Yield model presented in FLAC. The model is developed using the user-defined material model option in RS<sup>2</sup> and RS<sup>3</sup>, and the users are provided with the dll file as an advanced example of this extension option. VIP users will have access to the C++ code of this model.

Experimental evidence indicates that the plastic deformation in soils starts from the early stages of loading. To capture such a behavior in a constitutive model the typical elasto-perfect plastic models are not adequate. To simulate such behavior constitutive models that utilize a hardening law after initial yielding are required.

The CySoil model and Double Yield model in RS<sup>2</sup> and RS<sup>3</sup> has been developed to meet the abovementioned need. The hardening rules for these user defined material models are taken from the default functions in FLAC and piece-wise linear tabular functions are not included. Since the model are essentially the same and can be converted to each other by they have been developed as one model, and can be distinguished from each other by a single material parameter.

The models utilize three yield surfaces that includes deviatoric (shear), volumetric (cap) and tension cut off. The yield surfaces and hardening characteristics of this model are illustrated in Figure 14.1. Based on the formulations of this model it is apt to say that this model has three different mechanisms, i.e. deviatoric, volumetric and tension cutoff.

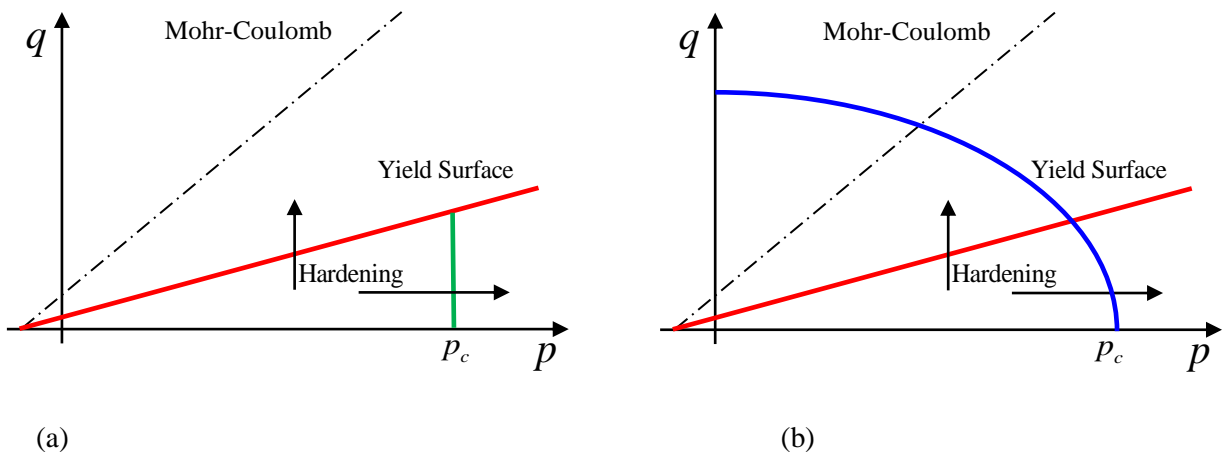


Figure 14.1- a) The yield surfaces of the Double Yield Material Model; Deviatoric yield surface (red) and the vertical cap (green), b) The yield surfaces of the CySoil Material Model; Deviatoric yield surface (red) and the elliptical cap (blue)

The formulations of these three mechanisms, definition of yield surfaces and their corresponding plastic potential and hardening law are presented below.

## 11.1- Deviatoric Hardening Mechanism

The deviatoric mechanism is the core of these models and its yield surface is very similar to the yield surface of the Mohr-Coulomb model. This mechanism is the same for both CySoil and Double Yield models.

The equation for the deviatoric yield surface using the principal stresses ( $p, q, \theta$ ) invariants is given by

$$F_s = q - Mp - Nc = 0 \quad (14.1)$$

where

$$M = \frac{3 \sin \varphi_m}{\sqrt{3} \cos \theta - \sin \theta \sin \varphi_m}, \quad N = \frac{3 \cos \varphi_m}{\sqrt{3} \cos \theta - \sin \theta \sin \varphi_m} \quad (14.2)$$

In above  $\varphi_m$  is the mobilized friction angle and  $c$  is the cohesion. The hardening for this yield surface is considered for the mobilized friction angle and it is attributed to plastic distortion,  $\varepsilon_q^p$

$$\frac{d(\sin \varphi_m)}{d(\varepsilon_q^p)} = \frac{\sqrt{3}}{2} \beta \frac{G_e}{p} \left( 1 - \frac{\sin \varphi_m}{\sin \varphi_f} R_f \right)^2 \quad (14.3)$$

In above  $\varphi_f$  is the ultimate/failure friction angle,  $R_f$  is the failure ratio and one of the material parameters (less than 1.0 with a default value 0.9),  $\beta$  is the calibration factor of the hardening rule and  $G_e$  is defined as

$$G_e = G_e^{ref} \left( \frac{p}{p_{ref}} \right)^m \quad (14.4)$$

Where  $G_e$  is the tangent shear modulus,  $G_e^{ref}$  is the tangent shear modulus at reference pressure,  $p_{ref}$ . Power  $m$ , controls the stress dependency of the elastic modulus and it is within the range of  $0.5 < m < 1.0$ . More detail on the elastic properties of this model will be presented later.

There are two option for the flow rule for the deviatoric mechanism of this model. The flow rule is defined in terms of the relationship between the plastic volumetric strain and plastic deviatoric strain in a way that

$$\dot{\varepsilon}_v^p = \sin \psi_m \dot{\varepsilon}_q^p \quad (14.5)$$

where  $\psi_m$  is the mobilized dilation angle.

The first option for the plastic potential uses a constant dilation angle, similar to that of the Mohr-Coulomb model where a dilation angle ( $\psi$ ) is defined and controls the dilation tendency of the material.

The second option is based on the stress-dilatancy theory by Rowe (1962). In this theory the mobilized dilation angle is calculated based on the mobilized friction angle  $\varphi_m$  and critical state friction angle  $\varphi_{cv}$ . The essential concept behind the critical state friction angle is that while under shear the material will undergo compression if  $\varphi_m > \varphi_{cv}$  and will dilate otherwise.

$$\sin \psi_m = \frac{\sin \varphi_m - \sin \varphi_{cv}}{1 - \sin \varphi_m \sin \varphi_{cv}} \quad (14.6)$$

$$\sin \varphi_{cv} = \frac{\sin \varphi - \sin \psi}{1 - \sin \varphi - \sin \psi} \quad (14.7)$$

In above  $\psi$  is the ultimate dilation angle.

## 11.2- Volumetric Hardening Mechanism

The main role of the volumetric mechanism (cap) is to close the elastic domain in space ( $p - q$ ) on the hydrostatic ( $p$ ) axis and simulate the densification/compaction of the material. As shown in Figure 14.1 the cap for the Double Yield model is a vertical line in the ( $p - q$ ) space and is an ellipse for the CySoil model with its apex  $q$  axis.

The yield surface of the vertical cap for the Double Yield surface is defined as

$$F_c = p - p_c \quad (14.8)$$

The formulation for the elliptical cap of the CySoil model is

$$F_c = \left(\frac{q^*}{\alpha}\right)^2 + p^2 - p_c^2 = 0 \quad (14.9)$$

where  $p_c$  is the location of the intersection of this yield surface with the  $p$  axis, and  $\alpha$  is the shape factor for the elliptical shape of the cap. The stress invariant  $q^*$  is defined as

$$q^* = \frac{q}{f(\theta)}, \quad f(\theta) = \frac{3 - \sin(\varphi)}{2(\sqrt{3} \cos(\theta) - \sin(\theta) \sin(\varphi))} \quad (14.10)$$

The hardening for these yield surfaces is considered for  $p_c$  and it is attributed to volumetric plastic strain generated only by the cap yield surface.

In general the soil stiffness is dependent on the level of confinement. In these models it is assumed that this dependency has a nonlinear relationship. Assuming that volumetric behavior is isotropic the following relationship has been proposed between the rates of mean stress and volumetric strain

$$\frac{\dot{p}}{\dot{\varepsilon}_v} = K_{ref}^{iso} \left( \frac{p}{p_{ref}} \right)^m \quad (14.11)$$

where  $K_{ref}^{iso}$  can be obtained from hydrostatic pressure test. A typical results of such a test, in terms of the variation of confining pressure versus volumetric strain, is demonstrated in Figure 14.2 with loading and unloading paths.

It is assumed that the ratio between the rates of elastic and plastic part of the volumetric strain in Figure 14.2 is constant

$$R = \frac{\varepsilon_v^p}{\varepsilon_v^e} \quad (14.12)$$

The parameter  $R$  is one of the model parameters and is also the ratio of the elastic modulus  $K_e$  to the equivalent hardening modulus. Based on this assumption the hardening rule can be derived as

$$\dot{p}_c = \frac{1+R}{R} K_{ref}^{iso} \left( \frac{p}{p_{ref}} \right)^m \dot{\varepsilon}_v \quad (14.13)$$

Similarly the tangent bulk modulus can be calculated as

$$K_e = (1 + R) K_{ref}^{iso} \left( \frac{p}{p_{ref}} \right)^m \quad (14.14)$$

The elastic behavior would be completed by definition of a constant Poisson's ratio.

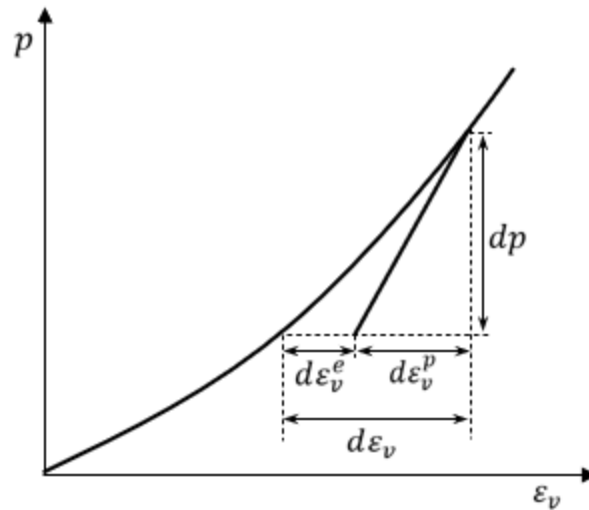


Figure 14.2- Isotropic compression test with loading and unloading paths; variation of confining pressure with volumetric strain

### 14.3- Tension Cut off

This mechanism is to incorporate the tensile strength of the material to this model. In this mechanism the minor principal stress is limited to the tensile strength of the material. The flow rule is associated and the mechanism has no hardening.

$$F_T = \sigma_1 - T = 0 \quad (14.15)$$

In above  $T$  is the tensile strength of the material.

### 14.4- Examples

Figure 14.3 shows the numerical results of drained triaxial tests on dense, medium dense and loose sand samples. A comparison is made between the results obtained by CySoil model in FLAC and simulation results of the same model in Rocscience products. The model parameters are presented in Table 14.1.

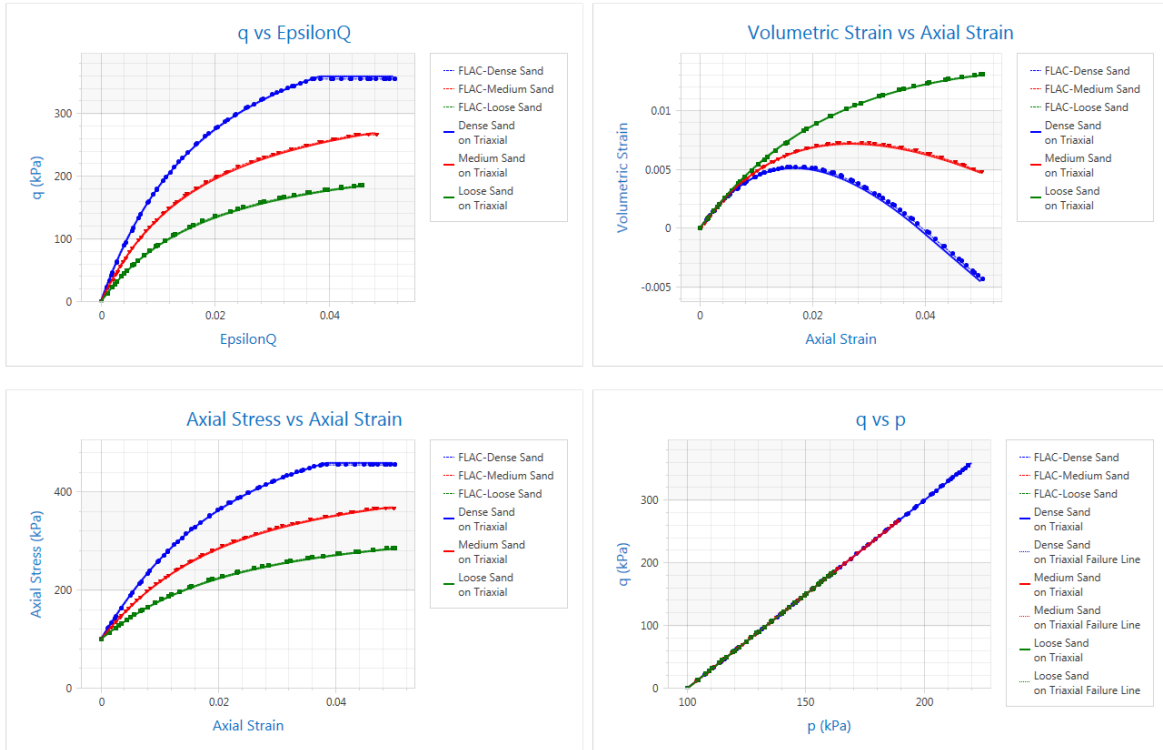


Figure 14.3. Stress paths of drained triaxial tests on dense, medium dense and loose sand

| Parameter               | Dense Sand | Medium-Dense Sand | Loose Sand |
|-------------------------|------------|-------------------|------------|
| $G_e^{ref}$ (MPa)       | 50         | 37.5              | 25         |
| $p_{ref}$ (kPa)         | 100        | 100               | 100        |
| $m$                     | 1.0        | 1.0               | 1.0        |
| $\nu$ (Poisson's ratio) | 0.2        | 0.2               | 0.2        |
| $\phi_f$ (degrees)      | 40         | 35                | 30         |
| $\psi_f$ (degrees)      | 10         | 5                 | 0          |
| $c$ (kPa)               | 0.0        | 0.0               | 0.0        |
| $R_f$                   | 0.9        | 0.9               | 0.9        |
| $\beta$                 | 0.35       | 0.35              | 0.35       |

Table 14.1. CySoil model parameters for dense, medium-dense and loose sand (PLAXIS 2011)

## References

FLAC, "User's manual." (2011).