ROCSCIENCE

CPillar

Factor of Safety Calculations – Rigid Analysis (Polygonal Pillar)

Theory Manual

Table of Contents

1. Introduction

This paper documents the calculations used in *CPillar* to determine the shear failure factor of safety for surface or underground crown pillars, and laminated roof beds. This involves the following series of steps:

1. Determine the crown pillar geometry
2. Determine the horizontal and vertical stress state of the soil/rock
3. Determine the normal stresses on the abutments
4. Compute the driving forces due to surcharge and self-weight
5. Compute the resisting forces due to abutment shear strength
6. Calculate the safety factor

2. Crown Pillar Geometry

2.1. Polygonal Pillar

A polygonal pillar is defined by a set of at least three unique, non-intersecting (x_i, y_i) coordinates, and thickness (z). A thickness of overburden (t_o) can also be added above the pillar. The height of water (h_w) can be specified to any height above the base of the pillar. The abutments of the pillar can be slanted in any direction by a trend (measured 0 to 360 degrees clockwise from north) and plunge (measured 0 to 90 degrees down from the horizontal).

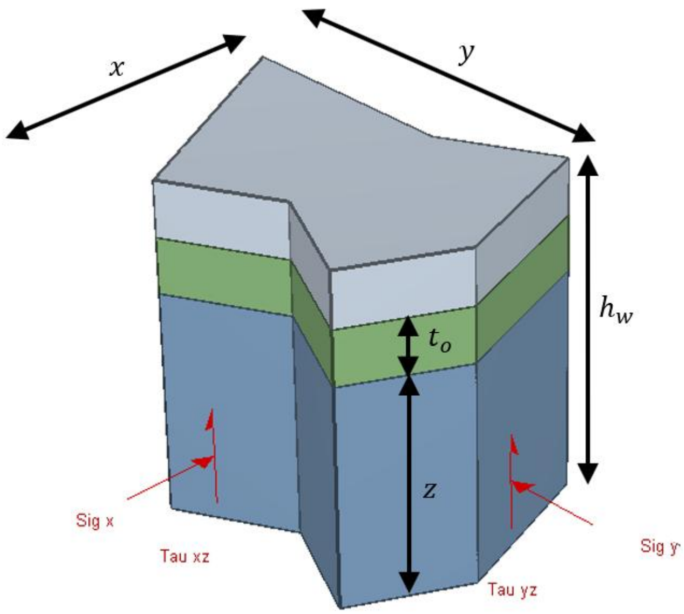

Figure 2.1.1: Polygonal Pillar Geometry

Where:

x	is the overall pillar length in the x-direction
y	is the overall pillar width in the y-direction
z	is the pillar height
t_o	is the thickness of overburden above the pillar
h_w	is the height of water from the base of the pillar
γ_r	is the rock unit weight
γ_o	is the overburden unit weight
γ_w	is the water unit weight

3. Stresses in the Soil

3.1. Water Pressure

In *CPillar*, the water height is specified from the bottom of the pillar. Water pressure is taken into account if the pillar is specified as **Permeable** (RIGID and ELASTIC options only).

3.1.1. Impermeable Pillar

If the pillar is modelled as impermeable, then any water height less than the combined pillar and overburden thickness will have no impact on the effective vertical and horizontal stress computations. Porewater pressure, μ = 0.

If water height is greater than the combined pillar and overburden thickness, then the weight of the free water will have an effect as an extra deadload on the pillar (See Section 0).

3.1.2. Permeable Pillar

If the pillar is modelled as permeable, then the effect of water pressure on vertical and lateral effective stress will be taken into account when computing stresses from GRAVITY.

The average porewater pressure (located mid-height of the pillar) is computed as follows:

If $h_w < z$: (1)

$$\mu = \frac{1}{2}\gamma_w h_w \left(\frac{h_w}{z}\right)$$

If $h_w \geq z$:

$$\mu = \gamma_w \left(h_w - \frac{1}{2}z\right)$$

Where:

μ is the porewater pressure

γ_w is the water unit weight

h_w is the water height

z is the pillar height

3.2. Horizontal Soil Pressure

In *CPillar*, lateral stresses can be specified as either CONSTANT or GRAVITY.

3.2.1. Constant Stress Type

Constant principal lateral stresses are entered directly into *CPillar* as σ_1 and σ_3, whereby σ_1 and σ_3 are oriented horizontal and perpendicular to one another. The orientation of σ_3 is specified by the trend (measured 0 to 180 degrees clockwise from north). These horizontal stresses do not always act normal to the abutments. See Section 3.2.4 for 2D stress transformation and assembly of 3D stress tensor.

3.2.2. Gravity Stress Type

Gravity lateral stresses are computed based on:

- Locked-in stress at the top surface of the pillar,
- Horizontal-to-vertical stress ratio constant,
- Rock unit weight,
- Pillar height, and
- Porewater pressure (if the pillar is permeable).

The average horizontal stresses (located mid-height of the pillar) is computed as follows:

$$\sigma_1' = \sigma_{lockedin} + \frac{1}{2}K_1\gamma_r z - \mu \tag{2}$$

$$\sigma_3' = \sigma_{lockedin} + \frac{1}{2}K_3\gamma_r z - \mu$$

Where:

σ_1' and σ_3'	are the lateral effective soil stresses
$\sigma_{lockedin}$	is the constant locked in soil stress due to soil loading history
K_x and K_y	are the horizontal-to-vertical stress ratio constants
γ_r	is the rock unit weight
z	is the pillar height
μ	is the porewater pressure

3.2.3. Vertical Soil Stress

In the case of non-vertical abutments, vertical soil stresses must be computed to account for the component of normal stresses generated by the weight of the pillar, overburden, and water.

Vertical (gravity) soil stress is computed based:

- Rock unit weight,
- Pillar height,
- Overburden unit weight,
- Overburden height,
- Water height, and
- Porewater pressure (if the pillar is permeable).

The average vertical stresses (located mid-height of the pillar) is computed as follows:

$$\sigma_z' = \frac{1}{2}W_r + W_o + W_w - \mu \tag{3}$$

Where:

σ_z'	is the vertical effective soil stress
W_r	is the weight of rock (i.e. pillar)
W_o	is the weight of overburden
W_w	is the weight of free water
μ	is the porewater pressure

See Section 0 for the calculation of weights.

3.2.4. Stress Transformation

In order to transform σ_1 and σ_3 back to global coordinates (σ_x and σ_y), a 2D rotation is applied.

$$\begin{Bmatrix} \sigma_x' \\ \sigma_y' \\ \sigma_{xy}' \end{Bmatrix} = \begin{bmatrix} \cos^2\theta_{t3} & \sin^2\theta_{t3} & 2\sin\theta_{t3}\cos\theta_{t3} \\ \sin^2\theta_{t3} & \cos^2\theta_{t3} & -2\sin\theta_{t3}\cos\theta_{t3} \\ -\sin\theta_{t3}\cos\theta_{t3} & \sin\theta_{t3}\cos\theta_{t3} & \cos^2\theta_{t3} - \sin^2\theta_{t3} \end{bmatrix} \begin{Bmatrix} \sigma_1' \\ \sigma_3' \\ 0 \end{Bmatrix} \tag{4}$$

Where:

σ_x' and σ_y'	are the lateral effective soil stresses in the global coordinate system
σ_{xy}'	is the lateral effective shear soil stress in the global coordinate system
σ_1' and σ_3'	are the lateral effective soil stresses in the local coordinate system
θ_{t3}	is the trend angle of σ_3

Note: $\sigma_{13}' = 0$ since σ_1 and σ_3 are principal stresses.

The 3D stress tensor is:

$$\boldsymbol{\sigma}_{\mathbf{ji}} = \begin{bmatrix} \sigma'_x & \sigma'_{xy} & 0 \\ \sigma'_{xy} & \sigma'_y & 0 \\ 0 & 0 & \sigma'_z \end{bmatrix} \tag{5}$$

Where:

$\boldsymbol{\sigma}_{\mathbf{ji}}$	is the stress tensor matrix
σ'_x and σ'_y	are the lateral effective soil stresses in the global coordinate system
σ'_{xy}	is the lateral effective shear soil stress in the global coordinate system
σ'_z	is the vertical effective soil stress in the global coordinate system

4. Dead Load

The driving forces responsible for the destabilization of the crown pillar is attributed by the deadload of the entire system. The total deadload is computed by summing all the rock, overburden, and water weights.

$$q = W_r + W_o + W_w \quad (6)$$

Where:

q is the total dead load (force per area)

W_r is the weight of rock (i.e. pillar) (force per area)

W_o is the weight of overburden (force per area)

W_w is the weight of free water (force per area)

4.1. Self-Weight

The self-weight of the pillar is calculated as follows:

$$W_r = \gamma_r z \quad (7)$$

Where:

W_r is the weight of rock (i.e. pillar)

γ_r is the unit weight of rock

z is the height of pillar

4.2. Overburden Weight

The weight of the overburden is calculated as follows:

$$W_o = \gamma_o t_o \quad (8)$$

Where:

W_o is the weight of overburden

γ_o is the unit weight of overburden

t_o is the thickness of overburden

4.3. Water Weight

The weight of the free water is calculated as follows:

If $h_w \leq z + t_o$: $$W_w = 0 \quad (9a)$$

If $h_w > z + t_o$: $$W_w = \gamma_w[h_w - (z + t_o)] \quad (9b)$$

Where:

W_w is the weight of free water

γ_w is the unit weight of water

h_w is the height of water

z is the height of pillar

t_o is the thickness of overburden

5. Shear Strength

The resisting forces are provided by the shear strength along the abutments of the pillar.

5.1. Normal Stress

Unlike the rectangular shaped pillars in *CPillar*, the lateral stresses are not always specified to be normal to the abutments.

The normal stresses to the abutments are calculated from the stress tensor and normals of the abutments.

$$N = \boldsymbol{\sigma}_{\mathbf{ji}}\mathbf{n}_{\mathbf{j}}\mathbf{n}_{\mathbf{i}} = \begin{bmatrix} \sigma'_x & \sigma'_{xy} & 0 \\ \sigma'_{xy} & \sigma'_y & 0 \\ 0 & 0 & \sigma'_z \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix} \{n_x \quad n_y \quad n_z\} \tag{10}$$

Where:

N	are the normal stresses along the abutments
$\boldsymbol{\sigma}_{\mathbf{ji}}$	is the stress tensor matrix
$\mathbf{n}_{\mathbf{j}}$ or $\mathbf{n}_{\mathbf{i}}$	are the normal vectors to the abutments

5.2. Shear Strength

The following shear strength criteria are available in *CPillar* for defining the strength of the rock:

1. Mohr-Coulomb
2. Hoek-Brown
3. Generalized Hoek-Brown

5.2.1. Mohr Coulomb

$$\tau = c + N \cdot \tan\phi \tag{11}$$

Where:

c	is the cohesion
N	is the normal stress along the abutments
ϕ	is the friction angle

5.2.2. Hoek-Brown

Note that this is a special case of the **Generalized Hoek-Brown** criterion, with the constant a = 0.5.

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^{0.5} \tag{12}$$

(Hoek and Bray, 1981)

If $s = 0$:

$$\tau = 0$$

If $s \neq 0$:

$$\tau = \left(\frac{1}{\tan\phi_i} - \cos\phi_i\right)\frac{m_b\sigma_{ci}}{8}$$

with

$$\phi_i = \tan^{-1}\left(\frac{1}{\sqrt{4h\cos^2\theta - 1}}\right)$$

$$h = 1 + \frac{16(m_b * N + s * \sigma_{ci})}{3{m_b}^2\sigma_{ci}}$$

$$\theta = \frac{1}{3}\left(\frac{\pi}{2} + \tan^{-1}\frac{1}{\sqrt{h^3 - 1}}\right)$$

Where:

m_b	is a reduced value (for the rock mass) of the material constant *mi* (for the intact rock)
s	is a constant which depends upon the characteristics of the rock mass
σ_{ci}	is the uniaxial compressive strength (UCS) of the intact rock pieces
σ_1'	is the axial effective principal stress
σ_3'	is the confining effective principal stress

5.2.3. Generalized Hoek-Brown

Generalized Hoek-Brown (m_b, s, a):

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s\right)^{a} \tag{13}$$

(Hoek and Bray, 1981)

Check for tensile strength:

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b}$$

If $N < \sigma_t$:

Generalized Hoek-Brown (GSI, mi, D):

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \tag{14}$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \tag{15}$$

$$a = \frac{1}{2} + \frac{1}{6}\left[\exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right)\right] \tag{16}$$

Where:

m_b is a reduced value (for the rock mass) of the material constant *mi* (for the intact rock)

s, a are constants which depend upon the characteristics of the rock mass

σ_{ci} is the uniaxial compressive strength (UCS) of the intact rock pieces

σ_1' is the axial effective principal stress

σ_3' is the confining effective principal stress

GSI is the Geological Strength Index

m_i is a material constant for the intact rock

D is a "disturbance factor" which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation (varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses)

6. Factor of Safety

6.1. Rigid Analysis

For a RIGID analysis, the only failure mode considered is shear. The crown pillar is idealized as a rigid block which fails by plug-sliding along the abutments. The factor of safety of the pillar against vertical downward sliding is given by the ratio of the sum of the shear forces acting on the four sides of the pillar, to the total weight of the pillar, including overburden and free water, if any.

6.1.1. Shear Factor of Safety

The shear factor of safety of obtained by summing the shear resistance forces along each abutment (i.e. polygon segment) and dividing by the component weight of the deadloads in the shearing direction.

$$FS_{shear} = \frac{shear\ resistance}{dead\ load} = \frac{\sum \tau A_a}{q A_c \sin \theta_{a_p}} \tag{17}$$

Where:

FS_{shear}	is the shear factor of safety
τ	are the shear strengths along the abutments
A_a	are the areas of the abutments
A_c	is the area of the pillar (i.e. polygon)
q	is the total dead load (force per area)
θ_{a_p}	is the abutment plunge

7. References

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