

## CPillar

# Factor of Safety Calculations – Elastic Analysis (Rectangular Pillar)

Theory Manual

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# 1.Introduction

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This paper documents the calculations used in *CPillar* to determine the shear failure and elastic buckling factors of safety for surface or underground crown pillars, and laminated roof beds. This involves the following series of steps:

1. Determine the crown pillar geometry
2. Determine the horizontal and vertical stress state of the soil/rock
3. Determine the normal stresses on the abutments
4. Compute the driving forces due to surcharge and self-weight
5. Compute the moments and stresses due to bending
6. Compute the resisting forces due to abutment shear strength
7. Calculate the safety factor(s)

## 2.Crown Pillar Geometry

### 2.1. Rectangular Pillar

A rectangular pillar is defined by its length ( $x$ ), width ( $y$ ), and thickness ( $z$ ). A thickness of overburden ( $t_o$ ) can also be added above the pillar. The height of water ( $h_w$ ) can be specified to any height above the base of the pillar.

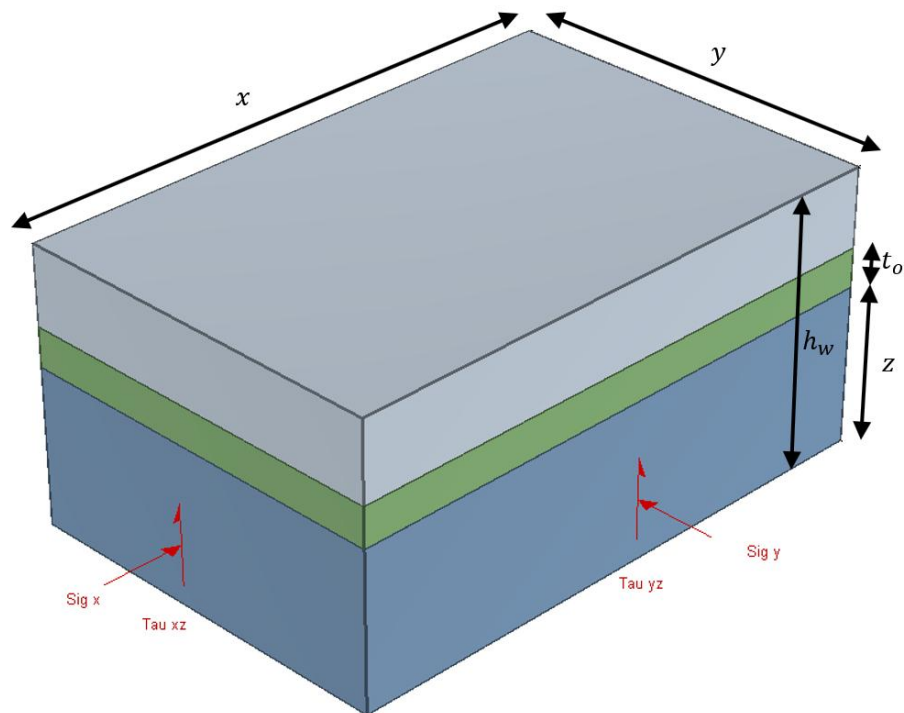


Figure 2.1-1: Rectangular Pillar Geometry

Where:

- $x$  is the pillar length
- $y$  is the pillar width
- $z$  is the pillar height
- $t_o$  is the thickness of overburden above the pillar
- $h_w$  is the height of water from the base of the pillar
- $\gamma_r$  is the rock unit weight
- $\gamma_o$  is the overburden unit weight
- $\gamma_w$  is the water unit weight

## 3. Stresses in the Soil

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### 3.1. Water Pressure

In *CPillar*, the water height is specified from the bottom of the pillar. Water pressure is taken into account if the pillar is specified as **Permeable** (RIGID and ELASTIC options only).

#### 3.1.1. Impermeable Pillar

If the pillar is modelled as impermeable, then any water height less than the combined pillar and overburden thickness will have no impact on the effective vertical and horizontal stress computations. Porewater pressure,  $\mu = 0$ .

If water height is greater than the combined pillar and overburden thickness, then the weight of the free water will have an effect as an extra deadload on the pillar.

#### 3.1.2. Permeable Pillar

If the pillar is modelled as permeable, then the effect of water pressure on vertical and lateral effective stress will be taken into account when computing stresses from GRAVITY.

The average porewater pressure (located mid-height of the pillar) is computed as follows:

$$\text{If } h_w < z: \quad (1a)$$

$$\mu = \frac{1}{2} \gamma_w h_w \left( \frac{h_w}{z} \right)$$

$$\text{If } h_w \geq z: \quad (1b)$$

$$\mu = \gamma_w \left( h_w - \frac{1}{2} z \right)$$

Where:

$\mu$  is the porewater pressure

$\gamma_w$  is the water unit weight

$h_w$  is the water height

$z$  is the pillar height

### 3.2. Horizontal Soil Pressure

In *CPillar*, lateral stresses can be specified as either CONSTANT or GRAVITY.

### 3.2.1. Constant Stress Type

Constant lateral stresses are entered directly into *CPillar* as  $\sigma_x$  and  $\sigma_y$ . These horizontal stresses act normal to the abutments.

### 3.2.2. Gravity Stress Type

Gravity lateral stresses are computed based on:

- Locked-in stress at the top surface of the pillar,
- Horizontal-to-vertical stress ratio constant,
- Rock unit weight,
- Pillar height, and
- Porewater pressure (if the pillar is permeable).

The average horizontal stresses (located mid-height of the pillar) is computed as follows:

$$\sigma'_x = \sigma_{lockedin} + \frac{1}{2}K_x\gamma_r z - \mu \quad (2a)$$

$$\sigma'_y = \sigma_{lockedin} + \frac{1}{2}K_y\gamma_r z - \mu \quad (2b)$$

Where:

$\sigma'_x$ and $\sigma'_y$	are the lateral effective soil stresses
$\sigma_{lockedin}$	is the constant locked in soil stress due to soil loading history
$K_x$ and $K_y$	are the horizontal-to-vertical stress ratio constants
$\gamma_r$	is the rock unit weight
$z$	is the pillar height
$\mu$	is the porewater pressure

## 4. Dead Load

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The driving forces responsible for the destabilization of the crown pillar are attributed by the deadload of the entire system. The total dead load is computed by summing all the rock, overburden, and water weights.

$$q = W_r + W_o + W_w \quad (3)$$

Where:

$q$  is the total dead load (force per area)

$W_r$  is the weight of rock (i.e. pillar) (force per area)

$W_o$  is the weight of overburden (force per area)

$W_w$  is the weight of free water (force per area)

### 4.1. Self-Weight

The self-weight of the pillar is calculated as follows:

$$W_r = \gamma_r z \quad (4)$$

Where:

$W_r$  is the weight of rock (i.e. pillar)

$\gamma_r$  is the unit weight of rock

$z$  is the height of pillar

### 4.2. Overburden Weight

The weight of the overburden is calculated as follows:

$$W_o = \gamma_o t_o \quad (5)$$

Where:

$W_o$  is the weight of overburden

$\gamma_o$  is the unit weight of overburden

$t_o$  is the thickness of overburden

### 4.3. Water Weight

The weight of the free water is calculated as follows:

$$\text{If } h_w \leq z + t_o: \quad W_w = 0 \quad (6a)$$

$$\text{If } h_w > z + t_o: \quad W_w = \gamma_w [h_w - (z + t_o)] \quad (6b)$$

Where:

$W_w$  is the weight of free water

$\gamma_w$  is the unit weight of water

$h_w$  is the height of water

$z$  is the height of pillar

$t_o$  is the thickness of overburden



## 5. Plate Bending

In *CPillar*, the bending of the pillar is calculated using the equations in “Theory of Plates and Shells” by Timoshenko and Woinowsky-Krieger (1987). The pillar is idealized as a plate of uniform thickness,  $z$ , with dimensions  $a$  and  $b$ . The plate is subject to a uniformly distributed area load,  $q$  (total dead load). The boundary conditions are assumed to be clamped on all edges (Figure 4.3-1).

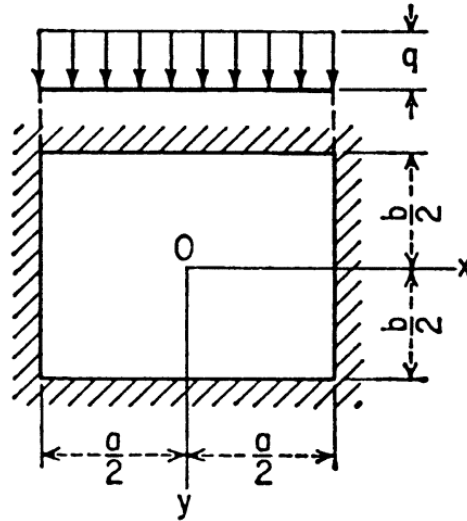


Figure 4.3-1: Rectangular Plate with All Edges Built In, Uniform Load  $q$  (Timoshenko and Woinowsky-Krieger, 1987)

The solution involves the superposition of deflection solutions for a simply supported rectangular plate, and a rectangular plate subject to moments distributed along the edges (Timoshenko and Woinowsky-Krieger, 1987).

### 5.1. Deflection

The deflection of this system is symmetric about each  $x$ -axis and  $y$ -axis of interest. The maximum deflection occurs in the middle of the plate ( $x = 0, y = 0$ ).

The deflection of a simply supported rectangular plate can be represented by the following series expansion:

$$\text{For } -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{b}{2} \leq y \leq \frac{b}{2}: \quad (7)$$

$$w = \frac{4qa^4}{\pi^5 D} \sum_{m=1,3,5,\dots} \frac{(-1)^{(m-1)/2}}{m^5} \cos \frac{m\pi x}{a} \left( 1 - \frac{\alpha_m \tanh \alpha_m + 2}{2 \cosh \alpha_m} \cosh \frac{m\pi y}{a} + \frac{1}{2 \cosh \alpha_m} \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right)$$

with

$$\alpha_m = \frac{m\pi b}{2a}$$

Where:

- $w$  is the vertical deflection
- $q$  is the uniform load magnitude (areal load)
- $D$  is the flexural rigidity  $D = \frac{Eh^3}{12(1-\nu^2)}$
- $a$  is the pillar length
- $b$  is the pillar width

The deflection caused by a moment distributed along the edges of a rectangular plate can be represented by the following series expansions:

For  $-\frac{a}{2} \leq x \leq \frac{a}{2}$  and  $-\frac{b}{2} \leq y \leq \frac{b}{2}$ : (8)

$$w = \frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\dots} \frac{\sin \frac{m\pi x}{a}}{m^2 \cosh \alpha_m} E_m \left( \alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} - \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} \right)$$

with  $\alpha_m = \frac{m\pi b}{2a}$

From Equation (17), (9)

Sub  $x + \frac{a}{2}$  for  $x$ :

$$w_1 = -\frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\dots} E_m \frac{(-1)^{\frac{m-1}{2}}}{m^2 \cosh \alpha_m} \cos \frac{m\pi x}{a} \left( \frac{m\pi y}{a} \sinh \frac{m\pi y}{a} - \alpha_m \tanh \alpha_m \cosh \frac{m\pi y}{a} \right)$$

Sub  $y + \frac{b}{2}$  for  $y$ :

$$w_2 = \frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\dots} \frac{\sin \frac{m\pi x}{a}}{m^2 \cosh \alpha_m} E_m \left( \alpha_m \tanh \alpha_m \cosh \frac{m\pi \left(y + \frac{b}{2}\right)}{a} - \frac{m\pi \left(y + \frac{b}{2}\right)}{a} \sinh \frac{m\pi \left(y + \frac{b}{2}\right)}{a} \right)$$

## 5.2. Rotation

Likewise, the rotation must also take into account the effects from the simple supports and the moment along the edges.

The rotation from the simply supported condition:

From Equation (7), (10)

Rotation at edge  $y = \frac{b}{2}$ :

$$\left(\frac{\partial w}{\partial y}\right)_{y=b/2} = \frac{2qa^3}{\pi^4 D} \sum_{m=1,3,5,\dots} \frac{(-1)^{(m-1)/2}}{m^4} \cos \frac{m\pi x}{a} \left( \frac{\alpha_m}{\cosh^2 \alpha_m} - \tanh \alpha_m \right)$$

Rotation at edge  $x = \frac{a}{2}$ :

$$\left(\frac{\partial w}{\partial x}\right)_{x=a/2} = \frac{2qb^3}{\pi^4 D} \sum_{m=1,3,5,\dots} \frac{(-1)^{(m-1)/2}}{m^4} \cos \frac{m\pi y}{b} \left( \frac{\beta_m}{\cosh^2 \beta_m} - \tanh \beta_m \right)$$

The rotation from the distributed moment along the edges:

From Equation (9),

(11)

Rotation along edge  $y = \frac{b}{2}$ :

$$\begin{aligned} \left(\frac{\partial w_1}{\partial y}\right)_{y=b/2} &= -\frac{a}{2\pi D} \sum_{m=1,3,5,\dots} E_m \frac{(-1)^{\frac{m-1}{2}}}{m} \cos \frac{m\pi x}{a} \left( \tanh \alpha_m + \frac{\alpha_m}{\cosh^2 \alpha_m} \right) \\ \left(\frac{\partial w_1}{\partial x}\right)_{x=a/2} &= -\frac{1}{4D} \sum_{m=1,3,5,\dots} E_m \frac{1}{\cosh^2 \alpha_m} \left( b \sinh \alpha_m \cosh \frac{m\pi y}{a} - 2y \cosh \alpha_m \sinh \frac{m\pi y}{a} \right) \\ &= -\frac{4b^2}{\pi^2 Da} \sum_{m=1,3,5,\dots} \frac{E_m}{m^3} \sum_{i=1,3,5,\dots} \frac{i(-1)^{(i-1)/2}}{\left(\frac{b^2}{a^2} + \frac{i^2}{m^2}\right)^2} \cos \frac{i\pi y}{b} \end{aligned}$$

Rotation along edge  $x = \frac{a}{2}$ :

$$\begin{aligned} \left(\frac{\partial w_2}{\partial y}\right)_{y=b/2} &= -\frac{4a^2}{\pi^2 Db} \sum_{m=1,3,5,\dots} \frac{F_m}{m^3} \sum_{i=1,3,5,\dots} \frac{i(-1)^{(i-1)/2}}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2}\right)^2} \cos \frac{i\pi x}{a} \\ \left(\frac{\partial w_2}{\partial x}\right)_{x=a/2} &= -\frac{b}{2\pi D} \sum_{m=1,3,5,\dots} F_m \frac{(-1)^{(m-1)/2}}{m} \cos \frac{m\pi y}{b} \left( \tanh \alpha_m + \frac{\beta_m}{\cosh^2 \beta_m} \right) \end{aligned}$$

### 5.3. Boundary Conditions

Since the edges are clamped, the rotation is restricted:

$$\text{Edges } y = \pm \frac{b}{2}: \quad (12a)$$

$$\left(\frac{\partial w}{\partial y}\right)_{y=b/2} + \left(\frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial y}\right)_{y=b/2} = 0 \quad (12b)$$

$$\text{Edges } x = \pm \frac{a}{2}:$$

$$\left(\frac{\partial w}{\partial x}\right)_{x=\frac{a}{2}} + \left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial x}\right)_{x=\frac{a}{2}} = 0$$

### 5.3.1. System of Equations

From Equation (12a),

(13a)

Edges  $y = \pm \frac{b}{2}$ :

$$\begin{aligned} \frac{2qa^3}{\pi^4 D} \sum_{i=1,3,5,\dots} \frac{(-1)^{\frac{i-1}{2}}}{i^4} \cos \frac{i\pi x}{a} \left( \frac{\alpha_i}{\cosh^2 \alpha_i} - \tanh \alpha_i \right) \\ - \frac{a}{2\pi D} \sum_{i=1,3,5,\dots} E_i \frac{(-1)^{\frac{i-1}{2}}}{i} \cos \frac{i\pi x}{a} \left( \tanh \alpha_i + \frac{\alpha_i}{\cosh^2 \alpha_i} \right) \\ - \frac{4a^2}{\pi^2 D b} \sum_{m=1,3,5,\dots} \frac{F_m}{m^3} \sum_{i=1,3,5,\dots} \frac{i(-1)^{\frac{i-1}{2}}}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2}\right)^2} \cos \frac{i\pi x}{a} = 0 \end{aligned}$$

therefore,

$$\frac{E_i}{i} \left( \tanh \alpha_i + \frac{\alpha_i}{\cosh^2 \alpha_i} \right) + \frac{8ai}{\pi b} \sum_{m=1,3,5,\dots} \frac{F_m}{m^3} \frac{1}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2}\right)^2} = \frac{4qa^2}{\pi^3 i^4} \left( \frac{\alpha_i}{\cosh^2 \alpha_i} - \tanh \alpha_i \right)$$

Edges  $x = \pm \frac{a}{2}$ :

(13b)

$$\begin{aligned} \frac{2qb^3}{\pi^4 D} \sum_{m=1,3,5,\dots} \frac{(-1)^{(m-1)/2}}{m^4} \cos \frac{m\pi y}{b} \left( \frac{\beta_m}{\cosh^2 \beta_m} - \tanh \beta_m \right) \\ - \frac{4b^2}{\pi^2 D a} \sum_{m=1,3,5,\dots} \frac{E_m}{m^3} \sum_{i=1,3,5,\dots} \frac{i(-1)^{\frac{i-1}{2}}}{\left(\frac{b^2}{a^2} + \frac{i^2}{m^2}\right)^2} \cos \frac{i\pi y}{b} \\ - \frac{b}{2\pi D} \sum_{i=1,3,5,\dots} F_i \frac{(-1)^{\frac{i-1}{2}}}{i} \cos \frac{i\pi y}{b} \left( \tanh \alpha_i + \frac{\beta_i}{\cosh^2 \beta_i} \right) = 0 \end{aligned}$$

therefore,

$$\frac{F_i}{i} \left( \tanh \beta_i + \frac{\beta_i}{\cosh^2 \beta_i} \right) + \frac{8bi}{\pi a} \sum_{m=1,3,5,\dots} \frac{E_m}{m^3} \frac{1}{\left(\frac{b^2}{a^2} + \frac{i^2}{m^2}\right)^2} = \frac{4qb^2}{\pi^3 i^4} \left( \frac{\beta_i}{\cosh^2 \beta_i} - \tanh \beta_i \right)$$

Where:

$E_i, E_m, F_i$  and  $F_m$  are coefficients (unknown)

Expanding to the  $n^{\text{th}}$  term of the series results in a  $2n$  by  $2n$  system of equations.

### 5.3.2. Bending Moment

Moment along edges  $y = \pm \frac{b}{2}$ : (14a)

$$(M_y)_{y=\pm b/2} = \sum_{m=1,3,5,\dots} (-1)^{\frac{m-1}{2}} E_m \cos \frac{m\pi x}{a}$$

Moment along edges  $x = \pm \frac{a}{2}$ : (14b)

$$(M_x)_{x=\pm a/2} = \sum_{m=1,3,5,\dots} (-1)^{\frac{m-1}{2}} F_m \cos \frac{m\pi y}{b}$$

Where:

$M_y$  is the bending moment about the y-axis, per unit perpendicular to edge

$M_x$  is the bending moment about the x-axis, per unit perpendicular to edge

To be conservative, the moments are taken at the mid-point along the edge (maximum edge moments).

Note: In *CPillar*, the series expansion is taken to 20 terms. 20 terms are sufficient since the functions series solutions converge rapidly.

## 5.4. Axial Stress Due to Bending

The tensile stresses due to the bending moments are computed as follows:

For a rectangular cross-section, (15a)

Tensile (axial in x-direction) stress due to bending about the y-axis:

$$\sigma_{b_x} = \frac{6M_y}{z^2}$$

(15b)

Tensile (axial in y-direction) stress due to bending about the x-axis:

$$\sigma_{b_y} = \frac{6M_x}{z^2}$$

Where:

$\sigma_{b_x}$  and  $\sigma_{b_y}$  are the tensile stresses due to bending, per unit perpendicular to edge

$z$  is the pillar height

## 6. Shear Strength

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The resisting forces are provided by the shear strength along the abutments of the pillar.

### 6.1. Normal Stress

The normal stresses to the abutments are equal to the lateral (principal) effective soil stresses.

$$\begin{aligned}N_x &= \sigma'_x \\N_y &= \sigma'_y\end{aligned}\tag{16}$$

Where:

$N_x$  and  $N_y$  are the normal stresses along the abutments

$\sigma_x$  and  $\sigma_y$  are the lateral soil stresses

### 6.2. Shear Strength

The following shear strength criteria are available in *CPillar* for defining the strength of the rock:

1. Mohr-Coulomb
2. Hoek-Brown
3. Generalized Hoek-Brown

#### 6.2.1. Mohr Coulomb

$$\tau = c + N \cdot \tan \phi\tag{17}$$

Where:

$c$  is the cohesion

$N$  is the normal stress along the abutments

$\phi$  is the friction angle

#### 6.2.2. Hoek-Brown

Note that this is a special case of the **Generalized Hoek-Brown** criterion, with the constant  $a = 0.5$ .

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^{0.5} \quad (18)$$

(Hoek and Bray, 1981)

If  $s = 0$ :

$$\tau = 0$$

If  $s \neq 0$ :

$$\tau = \left( \frac{1}{\tan \phi_i} - \cos \phi_i \right) \frac{m_b \sigma_{ci}}{8}$$

with

$$\phi_i = \tan^{-1} \left( \frac{1}{\sqrt{4h \cos^2 \theta - 1}} \right)$$

$$h = 1 + \frac{16(m_b * N + s * \sigma_{ci})}{3m_b^2 \sigma_{ci}}$$

$$\theta = \frac{1}{3} \left( \frac{\pi}{2} + \tan^{-1} \frac{1}{\sqrt{h^3 - 1}} \right)$$

Where:

$m_b$  is a reduced value (for the rock mass) of the material constant  $m_i$  (for the intact rock)

$s$  is a constant which depends upon the characteristics of the rock mass

$\sigma_{ci}$  is the uniaxial compressive strength (UCS) of the intact rock pieces

$\sigma'_1$  is the axial effective principal stress

$\sigma'_3$  is the confining effective principal stress

### 6.2.3. Generalized Hoek-Brown

Generalized Hoek-Brown ( $m_b$ ,  $s$ ,  $a$ ):

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left( m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (19)$$

(Hoek and Bray, 1981)

Check for tensile strength:

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b}$$

If  $N < \sigma_t$ :

Generalized Hoek-Brown ( $GSI$ ,  $m_i$ ,  $D$ ):

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (20)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (21)$$

$$a = \frac{1}{2} + \frac{1}{6} \left[ \exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right) \right] \quad (22)$$

Where:

$m_b$  is a reduced value (for the rock mass) of the material constant  $m_i$  (for the intact rock)

$s$ ,  $a$  are constants which depend upon the characteristics of the rock mass

$\sigma_{ci}$  is the uniaxial compressive strength (UCS) of the intact rock pieces

$\sigma'_1$  is the axial effective principal stress

$\sigma'_3$  is the confining effective principal stress

$GSI$  is the Geological Strength Index

$m_i$  is a material constant for the intact rock

$D$  is a "disturbance factor" which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation (varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses)



# 7. Factor of Safety

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## 7.1. Elastic Analysis

For an ELASTIC analysis, two failure modes are considered:

1. Shear, and
2. Elastic Buckling.

The pillar is treated as a clamped beam, with span and breadth equal to the shorter and longer of the  $x$  and  $y$  dimensions, respectively.

## 7.2. Shear Factor of Safety

Similar to RIGID analysis, the factor of safety of the pillar against vertical downward sliding is given by the ratio of the sum of the shear forces acting on the four sides of the pillar, to the total weight of the pillar, including overburden and free water.

In an ELASTIC analysis, the area on which the shear stresses act is lowered if bending stresses are high. If confining stresses are very low, the shear factor of safety will be about half that calculated from a RIGID analysis. As confining stress is increased, the correction factor for bending approaches 1. Therefore, at high confining stress, the shear factor of safety calculated from either a RIGID or ELASTIC analysis will be the same.

If the abutments are under tension, then apply a correction factor:

$$\text{If } \sigma_x + \sigma_{tx} < 0: \quad (23a)$$

$$C_x = \frac{(\sigma_{bx} - \sigma_x)}{2\sigma_{tx}}$$

$$z_1 = C_x z$$

$$\text{If } \sigma_y + \sigma_{ty} < 0: \quad (23b)$$

$$C_y = \frac{(\sigma_{by} - \sigma_y)}{2\sigma_{ty}}$$

$$z_2 = C_y z$$

Where:

$C_x$  and  $C_y$  are the correction factors for bending in the  $x$  and  $y$  directions (between 0.5 to 1.0)

$\sigma_{bx}$  and  $\sigma_{by}$  are the tensile axial stresses due to bending

$\sigma_x$  and  $\sigma_y$  are the axial stresses due to lateral soil pressure

$z_1$  and  $z_2$  are the corrected pillar heights

The correction factor effectively reduces the shear area in the factor of safety computations.

$$FS_{shear} = \frac{\text{shear resistance}}{\text{dead load}} = \frac{2 \left( \frac{\tau_{xz} z_1}{x} + \frac{\tau_{yz} z_2}{y} \right)}{q} \quad (24)$$

Where:

$FS_{shear}$  is the shear factor of safety

$\tau_{xz}$  and  $\tau_{yz}$  are the shear strengths

$z_1$  and  $z_2$  are the corrected pillar heights

$x$  is the pillar length

$y$  is the pillar width

$q$  is the total dead load

### 7.2.1. Elastic Buckling Factor of Safety

The horizontal confining stress is set equal to the stress along the breadth of the beam (the longer dimension).

Buckling or bending is assumed to occur in the longer dimension. If the  $x$  and  $y$  dimensions are equal, then bending direction is dictated by the principal horizontal stress magnitudes.

If  $x = y$ : (25a)

$$\sigma_c = \max(\sigma_x, \sigma_y)$$

$$L_{span} = x = y$$

If  $x > y$ : (25b)

$$\sigma_c = \sigma_y$$

$$L_{span} = y$$

If  $x < y$ : (25c)

$$\sigma_c = \sigma_x$$

$$L_{span} = x$$

Where:

$\sigma_{ci}$  is the horizontal confining stress

$L_{span}$  is the span of the pillar

$x$  is the pillar length

$y$  is the pillar width

$\sigma_x$  and  $\sigma_y$  are the principal horizontal stresses

If the lateral stresses are very low, the elastic buckling safety factor will be very high.

$$FS_{buckling} = \frac{\text{Euler buckling stress}}{\text{horizontal confining stress}} = \frac{\left( \frac{\pi^2 E_{rm} z^2}{3 L_{span}^2} \right)}{\sigma_c} \quad (26)$$

Where:

$FS_{shear}$  is the shear factor of safety  
 $E_{rm}$  is the rock modulus  
 $z$  is the pillar height  
 $L$  is the span of the pillar  
 $\sigma_c$  is the horizontal confining stress

There is a high degree of uncertainty in computing elastic buckling factor of safety for rock material. Therefore, span-to-depth ratios more than 3 are not recommended for an ELASTIC analysis.

## 8. References

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- Beer, G. and Meek, J.L. Design curves for roofs and hanging-walls in bedded rock based on "voussoir" beam and plate solutions. Trans. Instn. Min. Metall. 91, 1982, pp A18-A22.
- Brady, B.H.G. and Brown, E.T. Rock Mechanics for Underground Mining (2nd Edition) (Chapter 8), London: Allen and Unwin, 1993, 571 pages.
- Duncan, J.M. (2000). Factors of safety and reliability in geotechnical engineering. J. Geotechnical & Geoenvironmental Engineering, April, pp. 307-316.
- Evans, W.H. The strength of undermined strata. Trans. Instn. Min. Metall., 50, 1941, 475-532.
- Hoek, E. (1989) A Limit Equilibrium Analysis of Surface Crown Pillar Stability. Proc. Int. Conf. on Surface Crown Pillars Active & Abandoned Metal Mines Timmins, pp. 3-13.
- Hoek, E. Lessons from Case Histories in Rock Engineering. Course notes, Dept. of Civil Engineering, University of Toronto, Toronto, Ontario, Canada, 1992, pp 71-79.
- Hoek, E. and Brown, E.T. The Hoek-Brown Failure Criterion - a 1988 Update. 15th Canadian Rock Mechanics Symposium, 1988, pp 31-38.
- Hutchinson, D.J. and Diederichs, M. (1996). Cablebolting in Underground Mines, Vancouver: Bitech. 400 pages.
- Law, A.M. and Kelton, D.W. (1991). Simulation Modeling and Analysis (2nd edition), McGraw-Hill, Inc., New York.
- Rosenbleuth, E. Point estimates for probability moments. Proc. Nat. Acad. Sci. 10, 1972, pp 3812-3814.
- Timoshenko, S. and Woinowsky-Kreiger, S. (1987). Theory of Plates and Shells (2nd ed.). McGraw-Hill Book Company, Inc.