Uniaxial compressive strength versus Global strength in the Hoek-Brown criterion

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Definition of uniaxial compressive strength of a rock mass

The Generalized Hoek-Brown criterion is written as

$$\sigma_1^{'} = \sigma_3^{'} + \sigma_{ci}\left(m_b \frac{\sigma_3^{'}}{\sigma_{ci}} + s \right)^a \tag{1}$$

where $\sigma_1^{'}$ and $\sigma_3^{'}$ are the major and minor principal stresses respectively.
σ_{ci} is the uniaxial compressive strength of the intact rock pieces
m_i, s and a are material constants determined from the GSI rating

Substituting $\sigma_3^{'} = 0$ into this equation gives the uniaxial compressive strength of the rock mass as:

$$\sigma_c = \sigma_{ci}.s^a \tag{2}$$

Consider an example in which a rock mass is defined by the following parameters:

> Uniaxial compressive strength of intact rock σ_{ci} = 100 MPa
> Geological Strength Index GSI = 40
> Constant m_i = 10

Using the program RocLab[1], the uniaxial compressive strength of the rock mass is calculated as σ_c = 3.307 MPa and the failure envelope for this rock mass is plotted in Figure 1.

In applying the Generalized Hoek Brown failure criterion in a numerical analysis in which progressive failure of the rock mass is being studied, for example around a tunnel, the uniaxial compressive strength of 3.307 MPa defines the stress at which failure at the boundary of the excavation initiates. An example of such failure initiation in illustrated in Figure 2 which shows spalling in the sidewall of a tunnel excavated in jointed sandstone. This spalling is very shallow and is not dangerous unless the stresses acting on the tunnel increase to the level at which the failure can propagate.

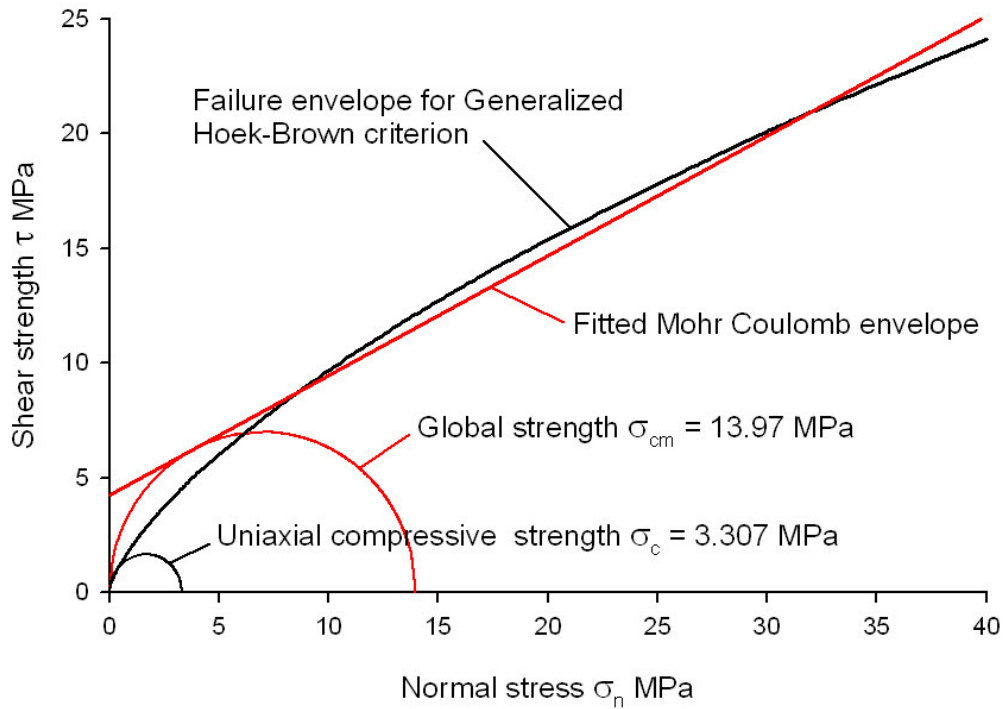[1] Available as a free download from www.rocscience.com

Figure 1: Plot of the failure envelopes for the Generalized Hoek Brown failure criterion and the associated Mohr Coulomb criterion. The uniaxial compressive strength and the global strength of the rock mass are defined in this plot.

Figure 2: Spalling in the sidewall of a tunnel in jointed sandstone indicating the initiation of failure. This spalling is very shallow and is not dangerous unless the stresses acting on the tunnel increase to a level at which the failure can propagate into the rock mass surrounding the tunnel.

Rock mass failure

Figure 3 shows the results of a rockburst resulting from the collapse of a pillar in an underground mine. This collapse has occurred as a result of propagation of failure through the rock mass and, in designing against such failures, the mining engineer is more interested in the average strength of the pillar than in the detailed mechanism of failure propagation. Under such circumstances the uniaxial compressive strength of the rock mass, defined by equation 2 and plotted in Figure 1, is of limited use since it defines the strength at the pillar boundaries only and not in the interior of the pillar. The question that arises from consideration of such problems is how does one define the average or "global" rock mass strength that can be used by designers who do not wish to or do not have the resources to carry out detailed numerical analyses of the entire failure process?

Figure 3: Results of a rockburst resulting from the collapse of a pillar in an underground mine.

The average strength of a rock mass surrounding a tunnel, within a pillar or in the rock mass into which a slope has been excavated is dependent upon the degree of confinement provided by the mass. Because of the highly curvilinear nature of the Hoek-Brown failure envelope at low stress levels, a very small amount of confinement results in a large increase in the strength of the rock mass. Unless a very detailed analysis is carried out, it is very difficult to calculate the average strength in a structure in which the confining stresses can vary by significant amounts. Consequently, some compromise solution must be found for this problem.

During the 1980s I worked on the problem of pillar design in underground mines and, in those days, practically all software was written in terms of the Mohr Coulomb failure criterion defined by the equation:

$$\tau = c' + \sigma'_n \tan \phi \tag{3}$$

where c' is the effective cohesive strength and ϕ is the friction angle.

The uniaxial compressive strength of a rock mass which fails in accordance with the Mohr Coulomb failure criterion is defined by:

$$\sigma'_{cm} = \frac{2c' \cos \phi'}{1 - \sin \phi'} \tag{4}$$

Because of the linear nature of the Mohr Coulomb criterion this compressive strength is much less sensitive to confinement than the equivalent strength for the curvilinear Hoek Brown criterion. It turned out that the compressive strength defined by equation 4 gave a reasonable estimate of the average or "global" rock mass strength.

The question is then, can one fit an equivalent Mohr failure envelope to an envelope defined by the Hoek Brown criterion such that the global strength defined by equation 4 is meaningful in terms of actual rock mass behaviour?

Equivalent Mohr Coulomb and Hoek Brown failure envelopes

The problem of fitting an equivalent Mohr failure envelope to the failure envelope defined by the Hoek Brown criterion is not a trivial one. There is no direct theoretical link between the two criteria and any fitting process must therefore be one of trial and error. The actual process of curve fitting has been discussed in detail in Hoek, Carranza-Torres and Corkum (2002)[2] and the following relationship between the global rock mass strength σ_{cm} and the Hoek Brown parameters is given by:

$$\sigma_{cm} = \sigma_{ci} \cdot \frac{(m_b + 4s - a(m_b - 8s))(m_b/4 + s)^{a-1}}{2(1+a)(2+a)} \tag{5}$$

for the confining stress range

$$0 < \sigma_3 < \frac{\sigma_{ci}}{4} \tag{6}$$

[2] Hoek E., Caranza-Torres C.T. and Corcum B., 2002. Hoek-Brown failure criterion-2002 edition. In: Bawden, H.R.W., Curran, J., Telsenicki M., (Eds). *Proc. North American Rock Mechanics Society (NARMS-TAC 2002).* Mining Innovation and Technology, 267-273, Toronto, Canada. This paper can be downloaded with RocLab from www.rocscience.com.

This stress range was chosen on the basis of experience and was found to work well for a wide variety of practical situations.

In using the program RocLab for the Generalized Hoek Brown criterion the value of "sigcm" (the Global strength of the rock mass) is that calculated using equation 5 for the stress range defined by equation 6.

Example of application of global strength concept

Hoek and Marinos (2000)[3] showed that the percentage strain in a tunnel could be predicted with reasonable accuracy from the plot of strain versus the ratio of global rock mass strength to in situ stress σ_{cm} / p_o given in Figure 4. This plot was derived from a Monte Carlo analysis using two different closed form solutions for tunnel deformation.
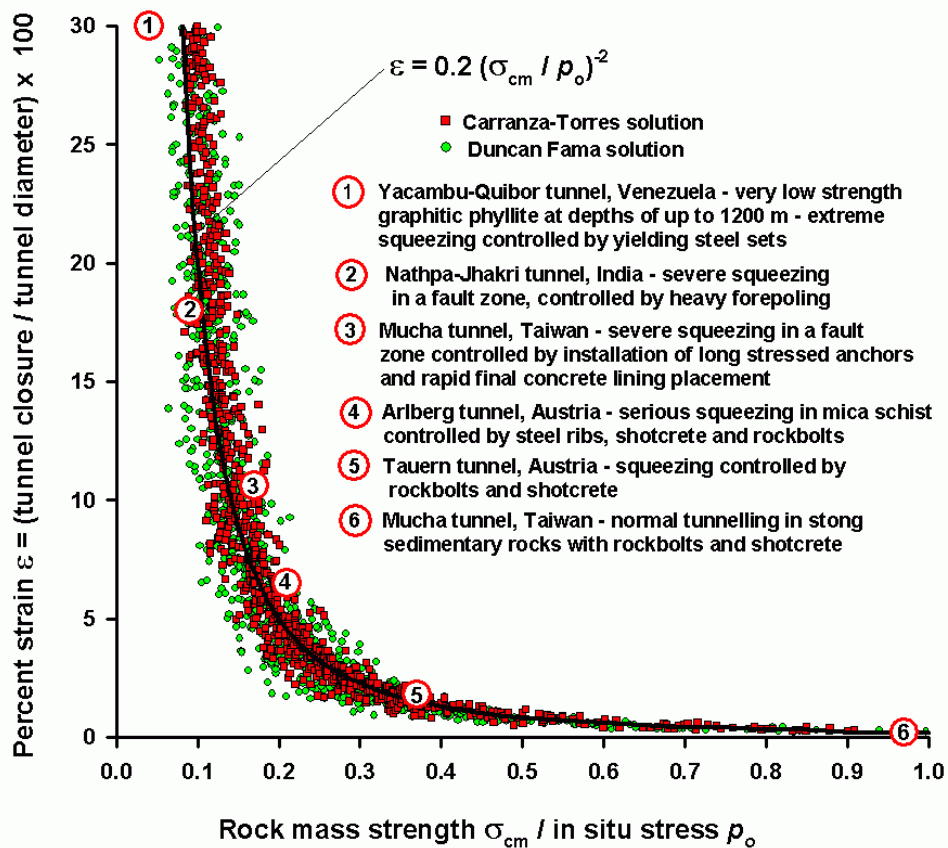

Figure 4: Predicted tunnel strain for different ratios of global rock mass strength to in situ stress. A number of case histories are also included in this plot.

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[3] Hoek, E. and Marinos, P. 2000. Predicting Tunnel Squeezing. *Tunnels and Tunnelling International.* Part 1 – November 2000, Part 2 – December, 2000