1976

Reprinted from the
Proceedings of the Specialty Conference on
Rock Engineering for Foundations and Slopes
ASCE/Boulder, Colorado/August 15-18, 1976

TOPPLING OF ROCK SLOPES

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ABSTRACT

Toppling is a mode of failure of slopes cut in rock masses with regularly spaced layers or foliation. It occurs under gravity alone when the layers are inclined into the hill but can occur even when the layers dip towards the excavation if load is transferred from potentially sliding blocks above; the latter is a case of "secondary toppling." Toppling is common in slates and schists in open pits and in natural slopes, but it also occurs in steeply dipping thin-bedded sediments, in columnar-jointed volcanics, and in regularly-jointed granitics. A number of examples of topples of different types are discussed in this paper, and a limit equilibrium analysis is examined for the special case of block toppling on a stepped base; the product of this analysis is the required support force at the toe of the slope to achieve a specified factor of safety. A simple kinematic test on the stereographic projection is also suggested.

INTRODUCTION

Toppling

"Toppling" is a failure mode of slopes involving overturning of interacting columns. In rock, such columns are formed by regular bedding planes, cleavage, or joints which strike parallel to the slope crest and dip into the rock mass; this contrasts with the structure of slides in which the controlling discontinuities dip into the open space. Toppling mechanisms also operate in soft rocks and

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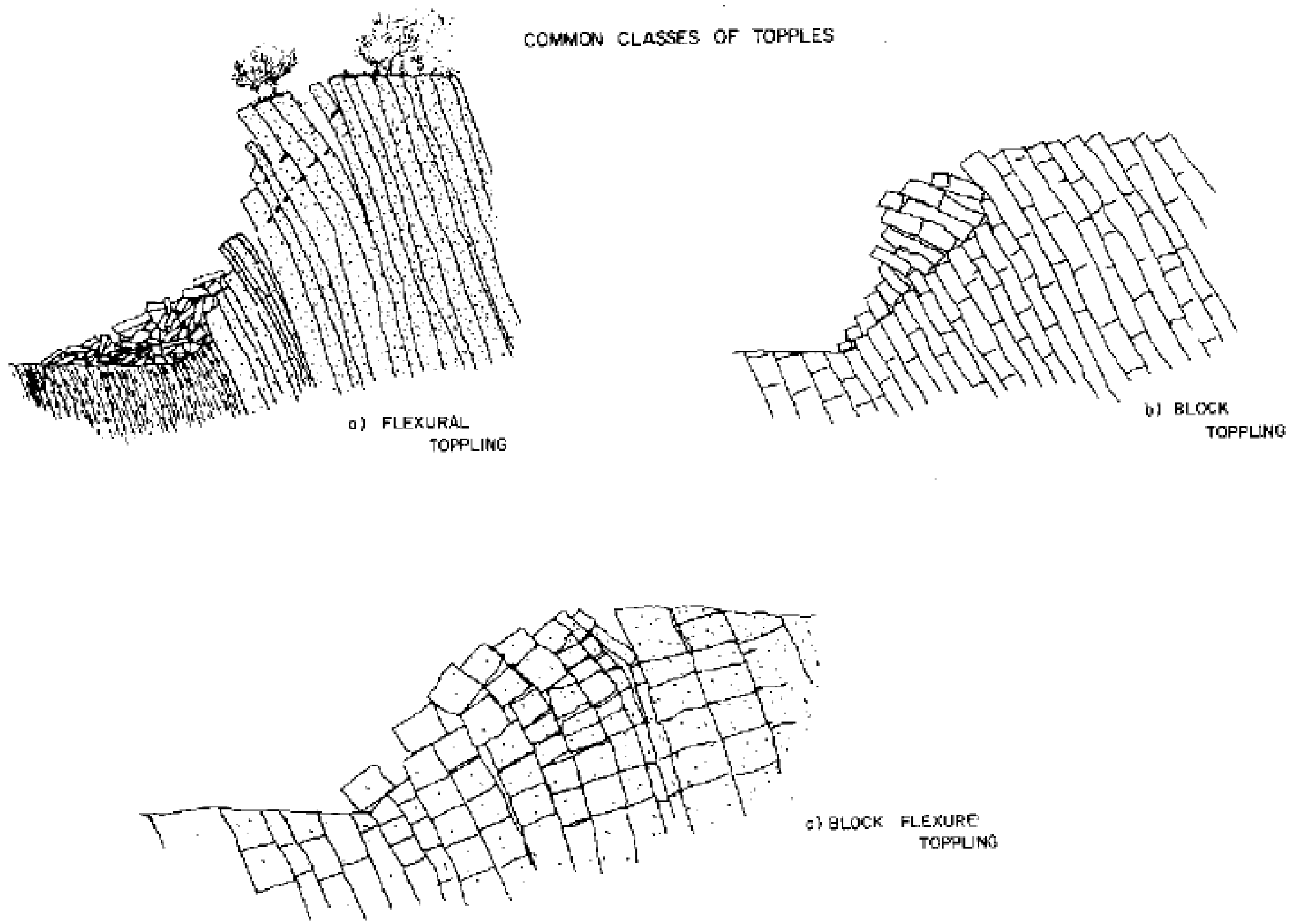
soils with vertical or backward-inclined tension cracks. Since folding is well known as a deformational mechanism in layered rocks, and overturning is recognized as a fundamental failure mode for dams and retaining walls, it is surprising that folding and overturning were not until recently recognized widely in rock slopes. Such failures prove to be widespread in many different kinds of rock masses. Slope toppling does not usually produce high velocities, like some rock slides, but, if uncontrolled, retrogressive failure can encompass a large volume of rock, with deep tension cracks, and considerable rock breakage. Failure begins in the toe region when the slope is deepened or undermined by new excavation or erosion. The material breaks as it comes down and accumulates as a talus, mantling and, apparently, obscuring the mode of failure. Failure can be violent.

Classes of Topples

There are several kinds of failure mechanisms involving overturning of columns. In rocks with one preferred discontinuity system, oriented to present a rock slope with semicontinuous cantilever beams, flexural-toppling can occur, as shown in Figure 1a. Continuous columns break in flexure as they bend forward. Thinner layers transfer load into thicker ones. Sliding, undermining, or erosion of the toe lets the failure begin and it retrogresses backwards, with wide, deep tension cracks. The lower portion of the slope is covered with disoriented and disordered blocks. The progress of cracking and bending ends only when the line of tension cracks intercepts the crest of the slope. The outward movement of each cantilever produces interlayer sliding (flexural slip) and a portion of the upper surface of each bed is exposed in a series of back facing scarps (obsequent scarps). As one ascends the slope, he is confronted with exposed lower surfaces of overhanging beds. Drilling will not discover a seat of sliding, for there is none; it is hard to say where the base of the disturbance lies, for change is gradual. Water levels will vary greatly from one drill hole to another since there may be little or no hydraulic communication across the cantilevers. Flexural-toppling occurs most notably in slates, phyllites, and schists.

Block toppling, depicted in Figure 1b, occurs where the individual columns are divided by widely spaced joints. The toe of the slope, with short columns, receives load from overturning, longer columns above. This thrusts the toe columns forward, permitting further toppling. The base of the disturbed mass is better defined than in the case of flexural toppling; it consists of a stairway generally rising from one layer to the next. The steps of this stairway are formed by cross-joints, which occupy the positions of primary flexural cracks in flexural-topples. Consequently, new rock breakage in flexure occurs much less markedly than in flexural-topples. Interblock caves occur throughout the disturbed zone; water will not be found high within



1. Common classes of topples: a) flexural toppling; b) block toppling; c) block flexure toppling.

the topple due to the openness of the whole joint system. Thick-bedded sedimentary rocks such as limestones and sandstones, as well as columnar jointed volcanics exhibit block-toppling failures.

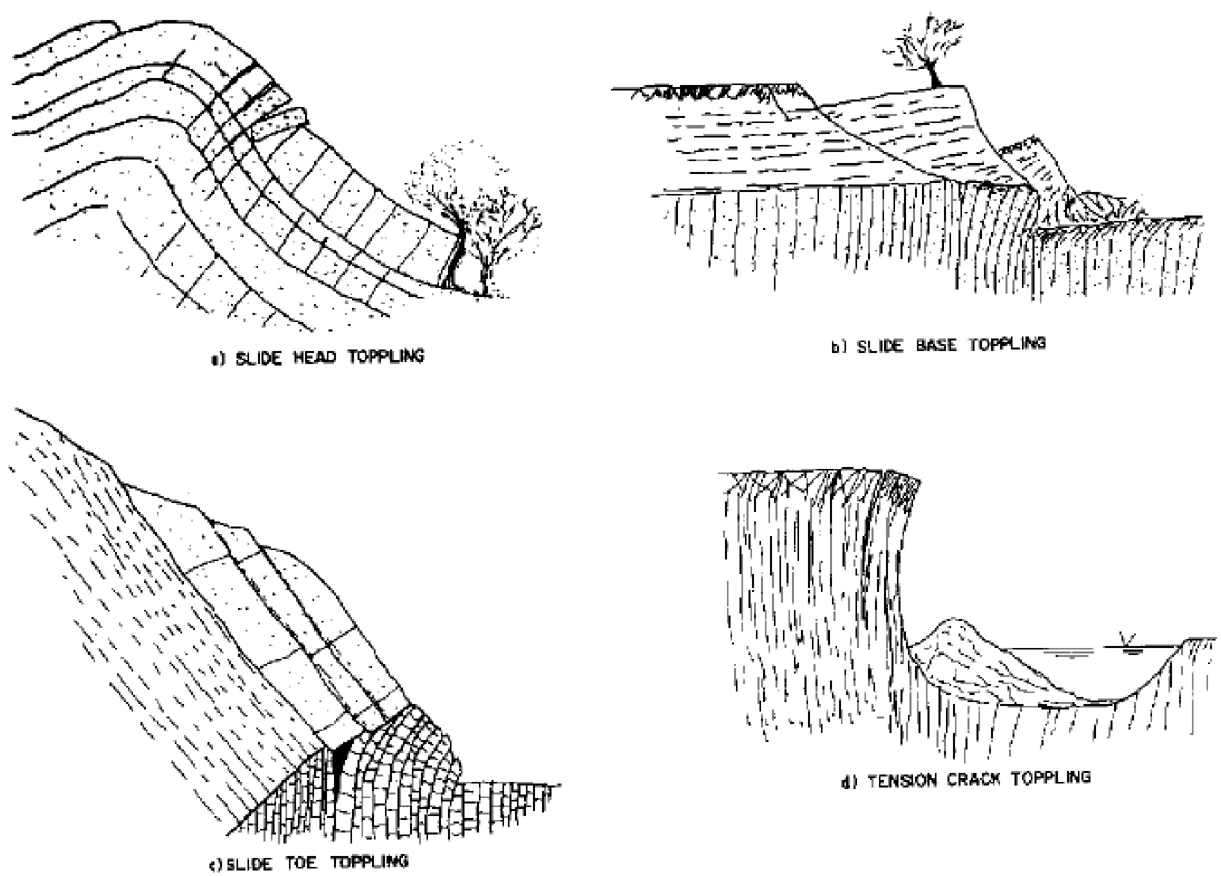
Block-flexure-toppling, depicted in Figure 1c, is characterized by pseudo-continuous flexure of long columns through accumulated motions along numerous cross joints. Sliding is distributed along several joint surfaces in the toe, while sliding and overturning occur in close association through the rest of the mass. Sliding occurs because accumulated overturning steepens the cross joints. There are fewer edge-to-face contacts than in block-toppling but still enough to create a loosened, highly open character within the disturbed zone. Interbedded sandstone and shale, interbedded chert and shale, and thin-bedded limestone, exhibit block-flexure-toppling.

Secondary Toppling

Overturning as a behavior mode may be excited by another, independent phenomenon where overturning would otherwise be unlikely to occur. Figure 2 shows several examples of such secondary toppling modes. As a consequence of a block slide, on a bedding plane, for example, a slide-head-topple may occur as joint blocks overturn into the new void at the slide head.(Figure 2a). A slump confined to soils above a steeply dipping layered rock can cause slide-base-toppling, as shown in Figure 2b. This occurs by virtue of the shear force of the slump acting along the top of the rock. Slide-base-toppling may arise only after slide movement because of continuous drag, or it may precede and trigger slide movement. Similarly, drag of broken rock being drawn off by cave mining can initiate secondary toppling, as in the case discussed by Heslop (1974). These modes are similar to the flexural deformation of steeply dipping layered rocks on steep hillsides underneath downward creeping regolith; "gravity creep" of this type can occur even where the layers dip downslope. Such creep may be termed creep-toppling.

Figure 2c portrays toppling in the toe of a rock slide as a result of load transmitted from the slide. This slide-toe-toppling is like a two block mechanism in which the "active" region is a slide and the passive region is a potentially toppling mass. The slide-toe-topple resembles an overturning retaining wall. In Figure 2c, the two block system exists by virtue of a fault causing sudden dip reversal. The fault gouge is squeezed into the tension cracks, and out onto the slope. A tight syncline can also set the stage for a slide-toe-topple.

The formation of new tension cracks above steep slopes may liberate potentially toppling blocks. Such tension crack toppling can arise in chalk, in volcanic ash, in highly weathered rocks, in damp sands, and in stiff clays. Figure 2d depicts toppling tension cracks in a stream bank several meters high. A similar mode exists



2. Secondary toppling modes: a) slide head toppling; b) slide base toppling; c) slide toe toppling; d) tension crack toppling.

where the toe of a sliding soil or rock mass overhangs a cliff, calves off and falls.

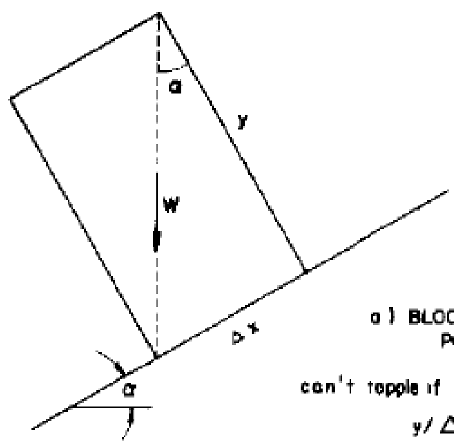
BASIC RELATIONSHIPS

Consider a single block resting initially on a plane inclined α degrees above horizontal (Figure 3a). The block has width Δx, height y, and weight W, and it develops shear and normal reactions along its base equal to W sin α and W cos α. Such a block will not slide if tan $\alpha < \mu$, where μ is the coefficient of friction. If it begins to topple, it will do so about the lower corner so the reactions at the limit of equilibrium of overturning will act through the lower corner. Thus limiting equilibrium, under weight alone, is reached when the weight acts exactly through the block's lower corner; i.e., the block can not topple if $y/\Delta x <$ cot α. The criteria for sliding and toppling of a single block are superimposed in Figure 3, after Hoek and Bray (1974) and Ashby (1971).

In a potentially toppling rock slope, columns interact with one another and there are more degrees of freedom than in the simple example of Figure 3. However, a condition for toppling of contacting columns can be obtained from the kinematic necessity for flexural slip prior to large mass movement. As shown in Figure 4, the state of stress in the rock slope is uniaxial, with σ_1 parallel to the slope face. When the layers slip past each other, σ_1 must be inclined at ϕ with the normal to layers, where ϕ is the angle of friction of layer surfaces. If θ is the slope angle, and α is the angle of the normal to discontinuities (both with respect to horizontal), then, as shown in Figure 4b, the condition for interlayer slip is

$$\theta \geq \phi + \alpha \qquad (1)$$

Equation (1) provides a useful kinematic test for toppling mechanisms, which can be applied together with tests for sliding modes to plan cuts in regularly layered rock masses. The equation shows that for toppling failure to be possible, $\alpha \leq \theta - \phi$, and this means that the pole to a regular discontinuity set must be outside a great circle ϕ degrees below the rock cut (Figure 4c). Since the strike of the discontinuities must be approximately parallel to the strike of the cut slope, this test should be applied only close to the bearing of the dip of the slope; accordingly in Figure 4c the region considered to pass the test of equation (1) has been bounded by vertical small circles 10° from the dip of the cut slope. Toppling is kinematically possible for the given cut slope only for sets of discontinuities whose poles plot within the ruled area. As in the case of sliding modes, the value of a kinematic test is in predicting potentially troublesome cases. Whether or



a) BLOCK IN LIMITING POSITION FOR TOPPLING

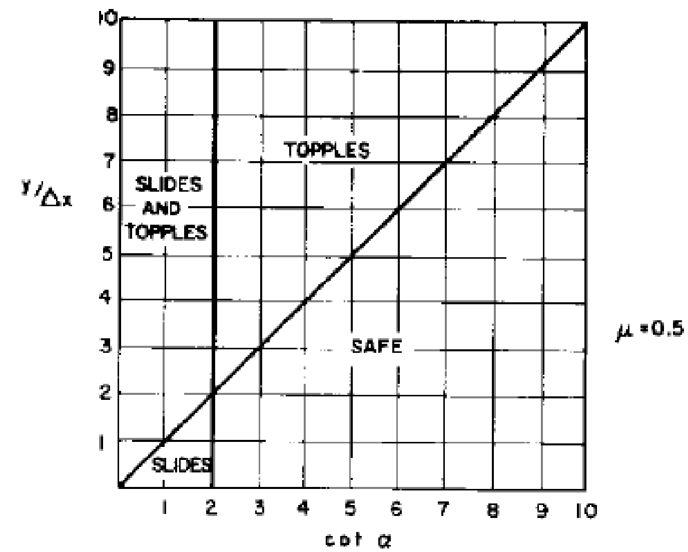
can't topple if :

$y/\Delta x < \cot\alpha$

can't slide if :

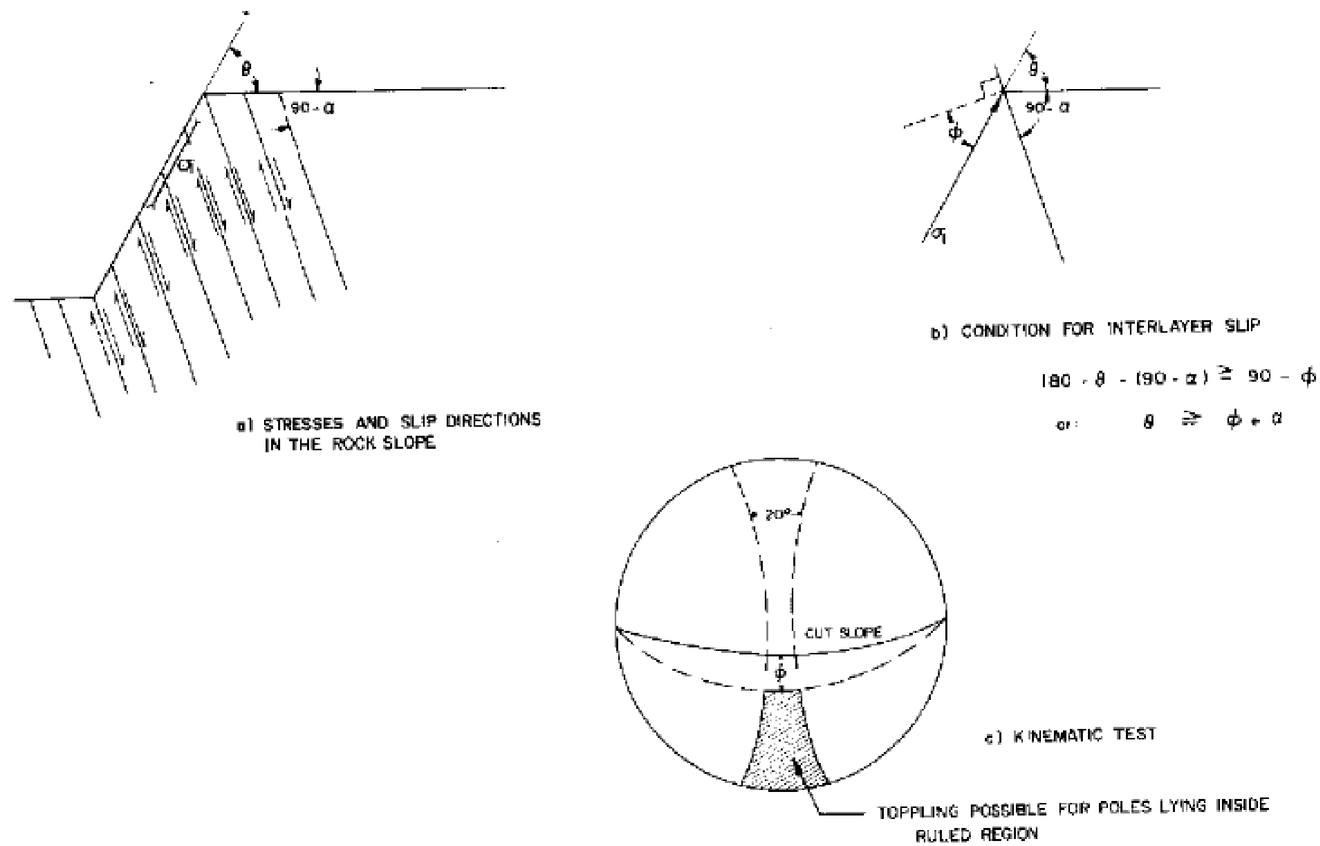
$\phi > \alpha$

or $\frac{1}{\mu} < \cot\alpha$



b) SUPERPOSED CRITERIA FOR SLIDING AND TOPPLING (AFTER HOEK AND BRAY, 1974)

3. Overturning and sliding criteria for a single block on an inclined plane: a) block in limiting orientation for toppling; b) superposed criteria for sliding and toppling [after Ashby (1971) and Hoek and Bray (1974)].



4. Kinematic condition for flexural slip, which precedes toppling: a) stresses and slip directions in the rock slope; b) condition for interlayer slip; c) kinematic test on the lower hemisphere, conformal stereographic projection.

not toppling will occur on a kinematically admissable set depends on the height of the cut, the thickness of the layers, the flexural strength of the rock, the regularity and spacing of cross joints, the degree of development of lateral release joints or tributary valleys perpendicular to the strike of the layers, and other specific system properties. Some of these factors can be considered analytically.

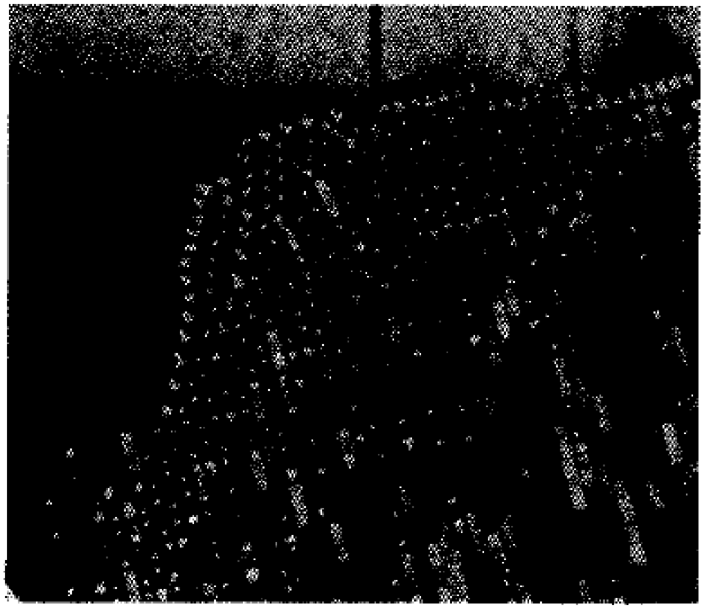
ANALYSIS OF TOPPLING MODES

Methods

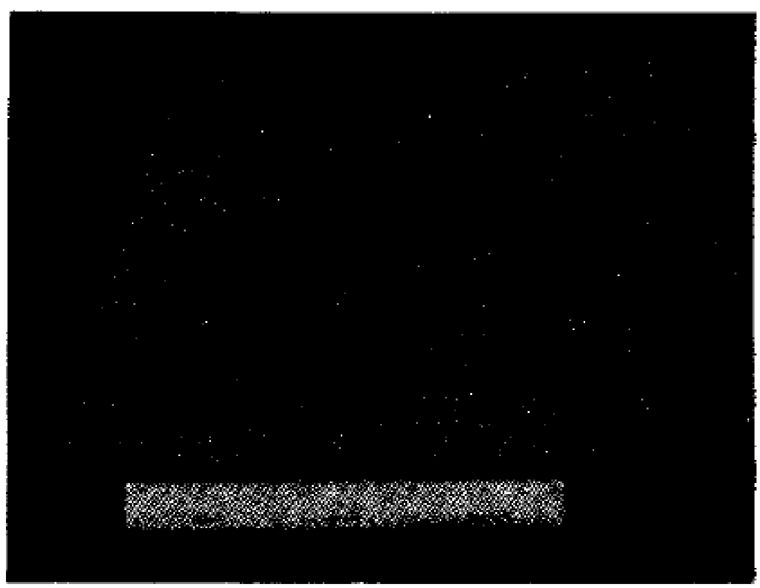
Toppling failures are readily produced in physical models and in fact it is hard to avoid at least secondary toppling when more than one set of joints is present. Barton (1971), Ashby (1971), Hofmann (1972) and others studied toppling slopes in static, physical models. These methods can be tedious but specific design problems can be treated realistically and strengthening measures, if required, can be tried out at reduced scale. Toppling is very easily simulated in base friction models which allow study of some of the parameters of interest if the friction coefficient μ can be duplicated, Ashby (1971), Hammett (1974), Soto (1974), Whyte (1973), and Goodman (1973, 1975). All of the classes of toppling failure described earlier can be studied by physical models. Moreover, as toppling is basically a two dimensional phenomenon, two dimensional models are sufficient. Figure 5 shows models of toppled slopes in tilted and base friction models.

Numerical models of two types can be used as well. Time explicit finite difference methods can follow the large deformations of block toppling and block flexure toppling modes from an initial equilibrium to failure. Cundall (1976) demonstrated this convincingly with computer graphics. The finite element method has perhaps greater capability to handle the flexural toppling mode, wherein the rock columns suffer deformation and failure and accordingly can not be considered to be rigid. A new program written by Marc Hittinger at Berkeley, combining good bending elements with joint elements, is being debugged for flexural toppling problems. Ashby (1971), Byrne (1974), Burman (1971) and Hammett (1974) also studied toppling of rock slopes using numerical methods.

Block toppling can be analyzed by limit equilibrium methods. If the base is flat, the analysis proves complicated, with blocks sliding or overturning both uphill and downhill and tension cracks opening both from the top and from the bottom. In some cases, multiple tension cracks form next to each other in block fans. The variations of toppling modes on a plane base can be appreciated by experimenting with books on an incompletely filled shelf (Figure 6). In the special case of block-toppling on a positively stepped base, the variations are fewer and analysis is

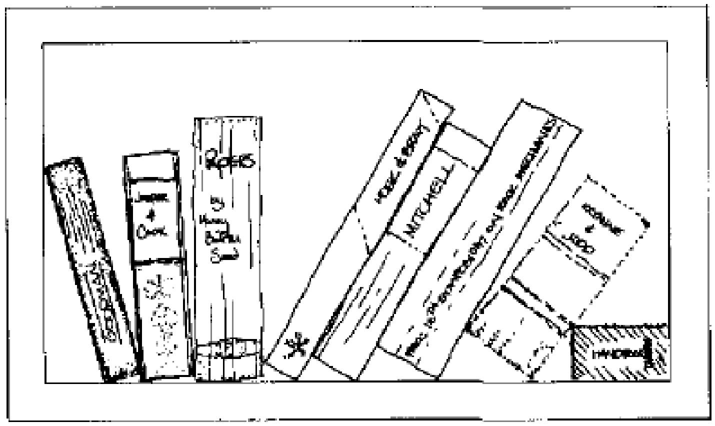


(a)



(b)

5. Models showing toppling failure: a) A tilted block-model; from Hofmann (1972). b) A base friction model; photo by L. Kuykendall.



6. Toppling of books on a shelf.

fairly straight forward.

LIMIT EQUILIBRIUM ANALYSIS OF TOPPLING ON A STEPPED BASE

Consider the regular system of blocks shown in Figure 7 in which a slope at angle θ is excavated in a rock mass with layers dipping at $90-\alpha$. The base is stepped upward with overall inclination β. The constants a_1, a_2, and b shown in the figure are given by

$$\begin{aligned} a_1 &= \Delta x \tan(\theta-\alpha) \\ a_2 &= \Delta x \tan \alpha \qquad (2) \\ b &= \Delta x \tan(\beta-\alpha) \end{aligned}$$

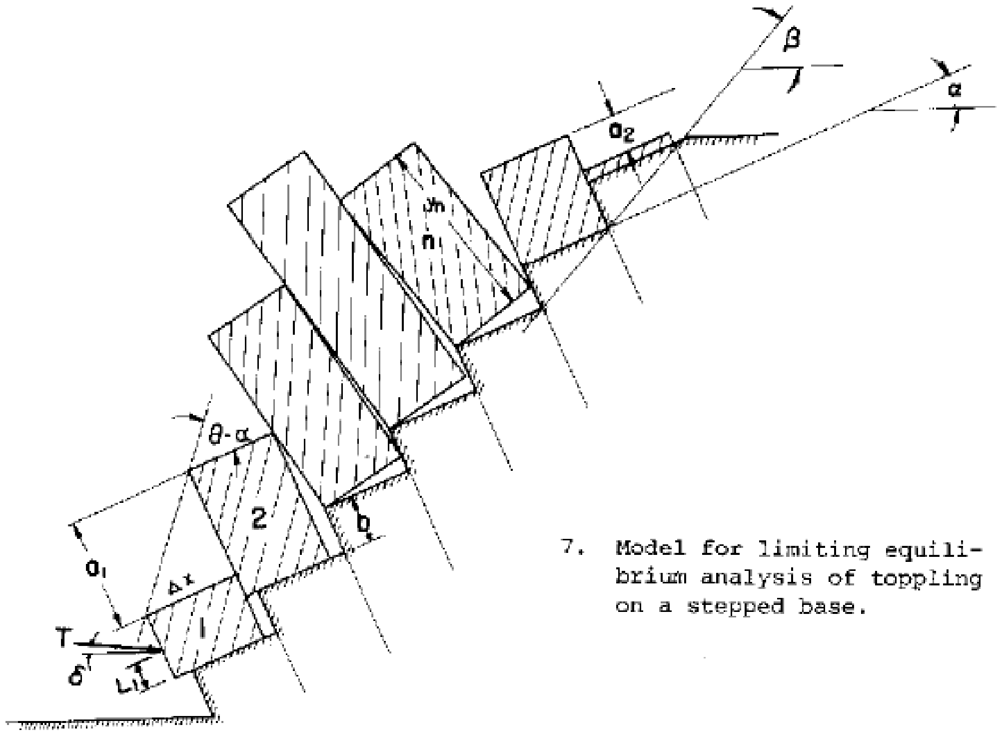
Blocks are numbered from the toe upward. In this idealized model, the height of block n in a position below the crest of the slope is

$$y_n = n(a_1 - b) \qquad (3a)$$

while above the crest

$$y_n = y_{n-1} - a_2 - b \qquad (3b)$$

(It is useful to consider a system with regular geometry but, in practise, less regular systems can be analyzed



7. Model for limiting equilibrium analysis of toppling on a stepped base.

without added difficulty once their geometric parameters are defined.)

In the top of the slope, where blocks are relatively short, $y_n / \Delta x < \cot \alpha$ and blocks are stable unless $\alpha > \phi$. However, in the latter case, all blocks slide and further analysis is not necessary. Below the stable zone, the blocks tend to topple, being restrained from doing so by the normal force transmitted upward from each lower block and the shear forces on the sides of the columns. Proceeding down the slope, as blocks become shorter the toe region is reached wherein again $y_n / \Delta x < \cot \alpha$ and a block will not topple under its own weight. However, toe blocks may still topple under the normal load transmitted from the toppling zone above. Thus both sliding and toppling must be examined within the toe region. The minimum force on the downslope side of block 1 required to prevent both toppling and sliding will be negative if the slope is stable, positive if the slope is unstable, or zero if the slope is exactly at limiting equilibrium. To find the friction coefficient required for limiting equilibrium one can iterate on the choice of μ to find a value making the required toe force come to zero. In practise, iteration usually produces an oscillating series. If the friction coefficient is known, and the slope is found to be unstable, a support force can be calculated to provide equilibrium.

Figure 8 shows the conditions for sliding and toppling in block n. The forces on block n include: W_p, the weight of the block; P_n and Q_n, representing the normal and shear forces acting on the upper side of block n at height M_n; P_{n-1} and Q_{n-1} representing the normal and shear forces acting on the lower side of block n at height L_n; and R_n and S_n representing the normal and shear forces on the base of block n at distance K_n above the lower corner. If block n is tending to topple, the points of application of all forces are known, as shown in Figure 8a.

Below the crest: $M_n = y_n$

$L_n = y_n - a_1$

For the crest block: $M_n = y_n - a_2$ (4)

$L_n = y_n - a_1$

and above the crest: $M_n = y_n - a_2$

$L_n = y_n$

(For an irregular block array, values of y_n, L_n, and M_n can be determined graphically.) Since the sides of toppling blocks are slipping past one another, the shear forces on the block sides are determined by the normal forces and friction coefficient. There are three unknowns: P_{n-1}, R_n, and S_n and the problem is determinate. To prevent toppling, a force $P_{n-1,t}$ is required:

$$P_{n-1,t} = \frac{P_n(M_n - \mu\Delta x) + (W_n/2)(y_n \sin\alpha - \Delta x \cos\alpha)}{L_n} \quad (5)$$

If toppling proves critical for block n,

$$R_n = W_n \cos\alpha - \mu(P_{n-1,t} - P_n) \quad (6)$$

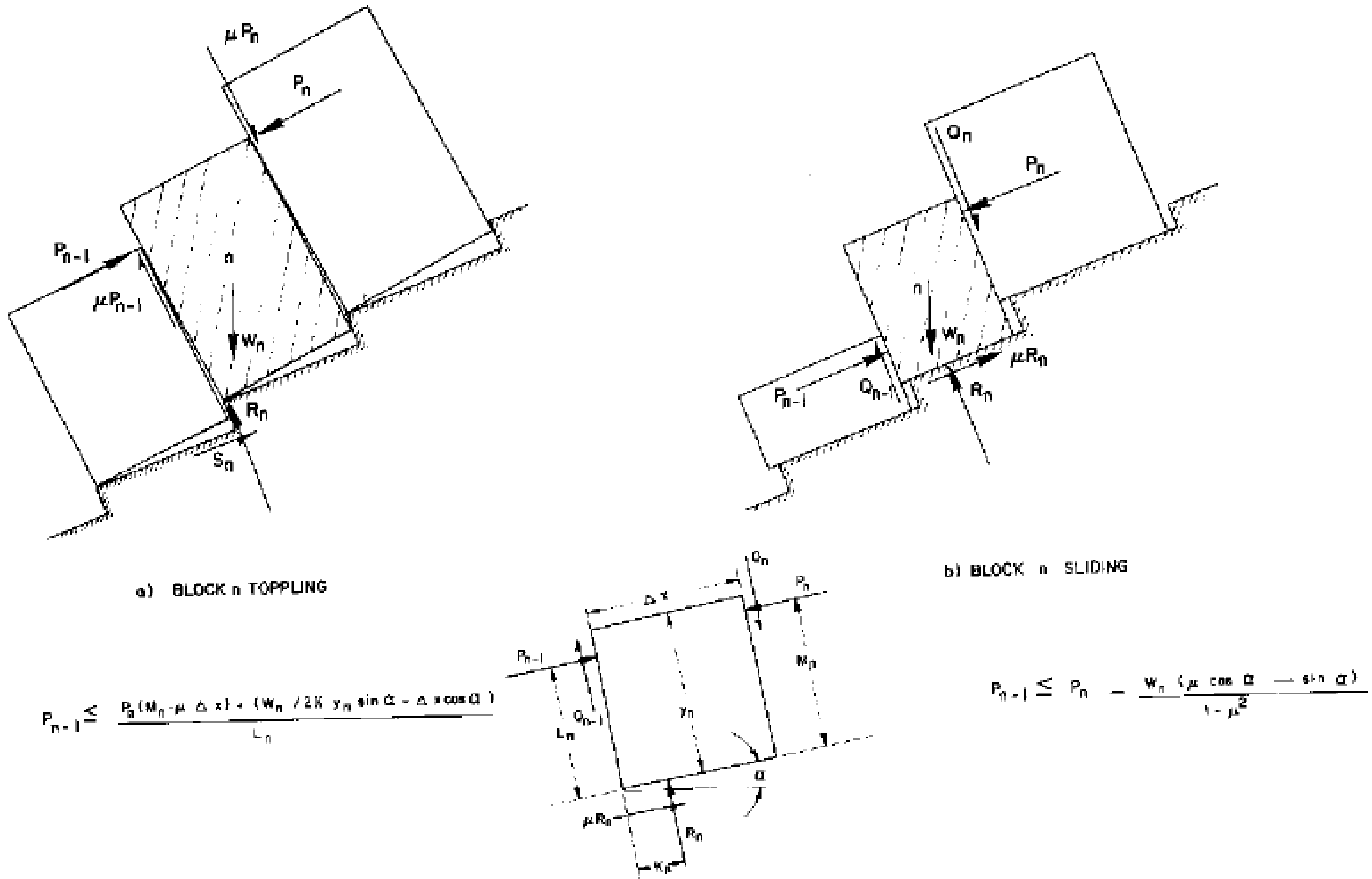
and

$$S_n = W_n \sin\alpha - (P_{n-1,t} - P_n) \quad (7)$$

with the conditions that: $R_n > 0$ (8)

and $|S_n| / R_n \leq \mu$ (9)

It is assumed that the coefficient of friction, μ, along the base of blocks is the same as between blocks. The analysis may be easily modified to allow for independent values of μ.



$$P_{n-1} \le \frac{P_n(M_n - \mu\,\Delta x) + (W_n/2)(y_n \sin\alpha - \Delta x \cos\alpha)}{L_n}$$

$$P_{n-1} \le P_n - \frac{W_n(\mu\cos\alpha - \sin\alpha)}{1-\mu^2}$$

8. Conditions for toppling and for sliding of the nth block.

If $S_n < 0$, the leading corner of block n tends to slide uphill. It will slide into the riser of the stepped base if $S_n < R_n \mu$, mobilizing a force to take the excess base shear.

If block n tends to slide, the side forces Q, are not known nor are their points of application. Assuming P_n, Q_n, and M_n were known from the previous calculation step, there are five new unknowns: forces Q_{n-1}, P_{n-1}, and R_n, and distances L_n and K_n (Figure 8). Though the problem is indeterminate, $P_{n-1,s}$ required to prevent sliding of block n can be determined if an assumption is made about the magnitude of Q_{n-1}. If we assume that $Q_{n-1} = \mu P_{n-1}$ then the normal force required to prevent sliding of block n is

$$P_{n-1,s} = P_n - \frac{W_n(\mu \cos \alpha - \sin \alpha)}{1 - \mu^2} \tag{10}$$

R_n is given by (6) with $P_{n-1,s}$ in place of $P_{n-1,t}$ and $S_{n-1} = \mu R_{n-1}$. It should be noted that the above assumption has no effect on the problem as regards the values of forces within the toppling zone, the condition for limiting equilibrium, computation of support force, etc., and identical results would be obtained adopting any other reasonable assumption regarding forces in the sliding zone.

If $P_{n-1,t} > P_{n-1,s}$ block n tends to topple and $P_{n-1} = P_{n-1,t}$. For analysis of the next block, set $P_n = P_{n-1}$ and proceed. If both $P_{n-1,t}$ and $P_{n-1,s}$ are negative, the slope is stable. To determine μ required for equilibrium, reduce μ and start again at the highest block. If P_{n-1} is positive for block 1, the slope is unstable. Start again with a larger value of μ, or compute a support force to achieve stability. In the examples below, a cable has been installed through block 1 at a distance L_1 above its base; the cable is inclined δ degrees below horizontal, and anchored a safe distance below the base. The tension in this cable required to prevent toppling of block 1 is

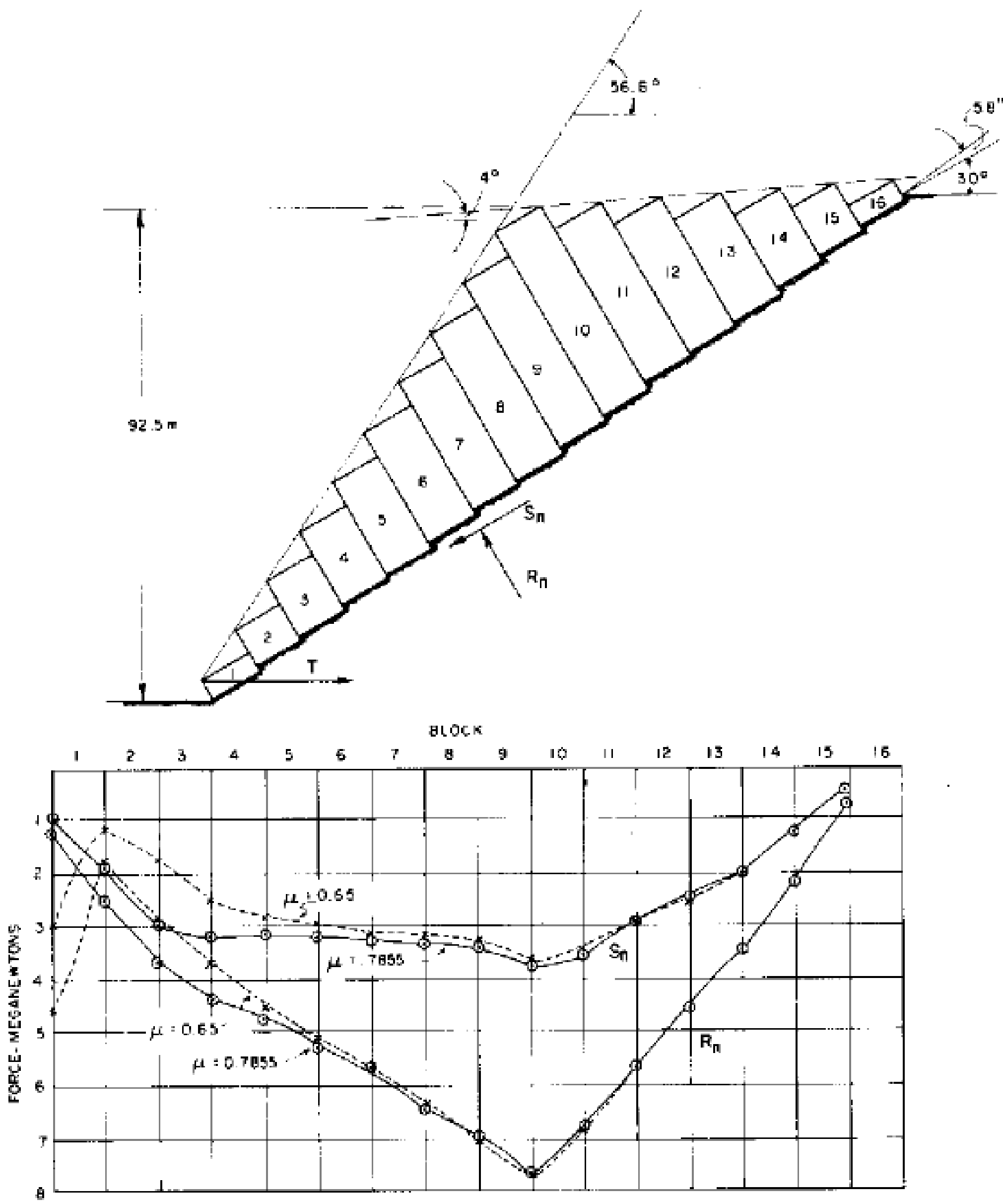
$$T_t = \frac{(w_1/2)(\sin \alpha \cdot y_1 - \cos \alpha \cdot \Delta x) + P_1(y_1 - \mu \Delta x)}{L_1 \cos(\alpha + \delta)} \tag{11}$$

while the tension in the cable to prevent sliding of block 1 is

$$T_s = \frac{P_1(1-\mu^2) - w_n(\mu \cos \alpha - \sin \alpha)}{\mu \sin(\alpha + \delta) + \cos(\alpha + \delta)} \tag{12}$$

These equations show that T_t and T_s are theoretically minimum when $\delta = -\alpha$, and $\delta = \phi - \alpha$ respectively. The actual value of δ will depend on practical considerations. The tension to be installed is the greater of T_t and T_s. Then the normal and shear force at the base of block 1 are respectively:

$$R_1 = \mu P_1 + T \sin (\alpha + \delta) + W_1 \cos \alpha \qquad (13)$$



9. Example 1 - Limiting equilibrium of a toppling slope.

TABLE 1 - Example 1a - $\mu = 0.7855$

n	y_n	$y_n/\Delta x$	M_n	L_n	$P_{n,t}$	$P_{n,s}$	P_n	R_n	S_n	S_n/R_n	Mode
16	4.0	0.4			0	0	0	866	500	0.577	
15	10.0	1.0			0	0	0	2165	1250	0.577	STABLE
14	16.0	1.6			0	0	0	3464	2000	0.577	
13	22.0	2.2	17	22	0	0	0	4533.4	2457.5	0.542	
12	28.0	2.8	23	28	292.5	-2588.7	292.5	5643.3	2966.8	0.526	
11	34.0	3.4	29	34	825.7	-3003.2	825.7	6787.6	3520.0	0.519	
10	40.0	4.0	35	35	1556.0	-3175.0	1556.0	7662.1	3729.3	0.487	
9	36.0	3.6	36	31	2826.7	-3150.8	2826.7	6933.8	3404.6	0.491	
8	32.0	3.2	32	27	3922.1	-1409.4	3922.1	6399.8	3327.3	0.520	TOPPLING
7	28.0	2.8	28	23	4594.8	156.8	4594.8	5872.0	3257.8	0.555	
6	24.0	2.4	24	19	4837.0	1300.1	4837.0	5352.9	3199.5	0.598	
5	20.0	2.0	20	15	4637.5	2013.0	4637.5	4848.1	3159.4	0.652	
4	16.0	1.6	16	11	3978.1	2234.1	3978.1	4369.4	3152.5	0.722	
3	12.0	1.2	12	7	2825.6	2095.4	2825.6	3707.3	2912.1	0.7855	
2	8.0	0.8	8	3	1103.1	1413.5	1413.5	2471.4	1941.3	0.7855	SLIDING
1	4.0	0.4	4	-	-1485.1	472.2	472.2	1237.1	971.8	0.7855	

TABLE 2 - Example 1b - μ=0.650

n	y_n	$y_n/\Delta x$	M_n	L_n	$P_{n,t}$	$P_{n,s}$	P_n	R_n	S_n	S_n/R_n	Mode
16	4.0	0.4			0	0	0	866	500	0.577	
15	10.0	1.0			0	0	0	2165	1250	0.577	STABLE
14	16.0	1.6			0	0	0	3464	2000	0.577	
13	22.0	2.2	17	22	0	0	0	4573.0	2457.5	0.537	
12	28.0	2.8	23	28	292.5	-599.2	292.5	5706.4	2952.7	0.517	
11	34.0	3.4	29	34	839.8	-470.2	839.8	6868.3	3491.6	0.508	
10	40.0	4.0	35	35	1598.2	-86.2	1598.2	7800.2	3676.8	0.471	
9	36.0	3.6	36	31	2921.4	508.8	2921.4	7004.9	3285.6	0.469	
8	32.0	3.2	32	27	4135.8	1940.9	4135.8	6370.8	3142.4	0.493	TOPPLING
7	28.0	2.8	28	23	4993.4	3264.2	4993.4	5745.7	3013.1	0.524	
6	24.0	2.4	24	19	5480.3	4230.8	5480.3	5134.6	2905.3	0.566	
5	20.0	2.0	20	15	5575.0	4826.6	5575.0	4547.4	2834.2	0.623	
4	16.0	1.6	16	11	5240.8	5030.3	5240.8	3747.4	2435.8	0.650	
3	12.0	1.2	12	7	4406.1	4805.0	4805.0	2810.5	1826.8	0.650	SLIDING
2	8.0	0.8	8	3	3205.3	4478.2	4478.2	1873.7	1217.9	0.650	
1	4.0	0.4	4		685.7	4260.3	4260.3	4641.7	3017.1	0.650	

and

$$S_1 = P_1 - T\cos(\alpha + \delta) + w_1 \sin\alpha \qquad (14)$$

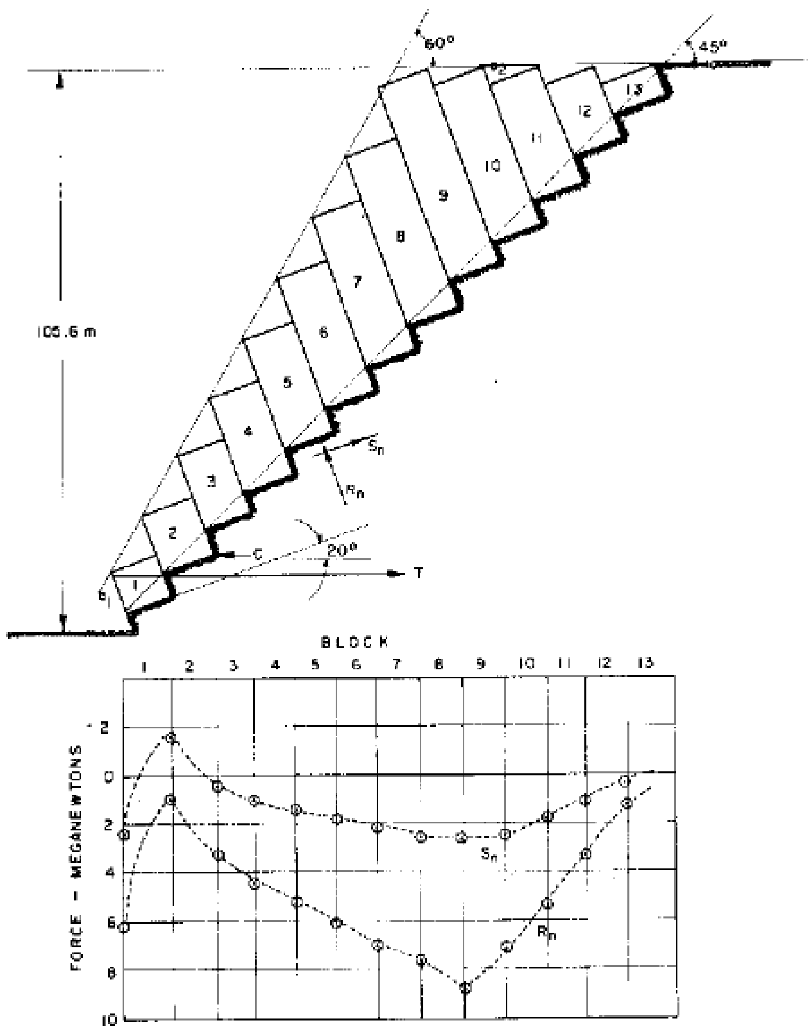
with the conditions (8) and (9).

An idealized example is shown in Figure 9. A rock slope 92.5 m high is cut on a 56.6° slope in a layered rock mass dipping 60° into the hill. A regular system of 16 blocks is shown on a base stepped 1 meter in every 5 ($\beta - \alpha = 5.8°$). The constants are: $a_1 = 5.0$ m, $a_2 = 5.0$ m, $b = 1.0$ m, $\Delta x = 10.0$ m, and $\gamma = 25$ kilo Newtons/m³. Block 10 is at the crest, which rises 4° above horizontal. Since $\cot\alpha$ is 1.78, blocks 16, 15, and 14 comprise a stable zone for all cases in which $\phi > 30°$, ($\mu > 0.577$). In example 1a, μ is set as 0.7855. P_{13} is then equal to 0 and P_{12} calculated as the greater of $P_{12,t}$ and $P_{12,s}$ given by (5) and (10) respectively. As shown in Table 1, $P_{n-1,t}$ turns out to be the larger until a value of $n = 3$, whereupon $P_{n-1,s}$ remains larger. Thus blocks 4 through 13 constitute the potentially toppling zone and blocks 1 to 3 constitute a sliding zone. The force required to prevent sliding in block 1 tends to zero and the slope is very close to the limit of equilibrium. The installed tension required to stabilize block 1 is 0.5 k Newtons, as compared with the maximum value of P (in block 5) equal to 4836 k Newtons (all the force calculations assume a meter of slope crest length).

In contrast, when the friction coefficient, μ, is 0.650, example 1b, there are 4 blocks in the toe region, as shown in Table 2. The required tension in a supporting cable, installed horizontally through block 1, is 2013 k Newtons (per meter length of slope crest). This is not a large number, demonstrating that support of the "keystone" is remarkably effective in increasing the degree of stability. Conversely, removal or weakening of the keystone for a slope near failure can have dire consequences, ultimately extending very far from the point of disturbance. The support force required to stabilize the slope corresponding to removal of the first n toe blocks can, of course, be calculated with equations (11) and (12), substituting P_{n+1} for P_1.

Now that the distribution of P forces has been defined in the toppling region, the forces R_n and S_n on the bases of columns can be calculated using (6) and (7); and assuming $Q_{n-1} = \mu P_{n-1}$, R_n and S_n can also be calculated by these equations for the sliding region. Tables 1 and 2, and Figure 9 show distribution of forces for both examples throughout the slope. Conditions (8) and (9) are satisfied everywhere. This is not the case in the next example.

In example 2 (Figure 10 and Table 3), a 105.6 m high cut on a 60° slope is comprised of 13 blocks with $\alpha = 20°$ and $\beta = 45°$. The distribution of block heights, which is not exactly as given by (3a) and (3b) is listed in Table 3. With the friction angle of 0.393, blocks 11, 12, and 13 are stable and only block 1 tends to slide (even through the first 5 blocks have $y_n / \Delta x < \cot\alpha$). The required cable



10. Example 2 - Limiting equilibrium analysis of a toppling slope.

TABLE 3 - Example 2 - μ=0.393

n	y_n	$y_n/\Delta x$	M_n	L_n	$P_{n,t}$	$P_{n,s}$	P_n	R_n	S_n	S_n/R_n	Mode
13	6.0	0.6	1.3	6.0	0	0	0	1404.8	511.3	0.3640	
12	14.3	1.4	10.6	14.3	0	0	0	3354.7	1221.0	0.3640	STABLE
11	22.6	2.3	18.9	22.6	0	0	0	5304.6	1930.7	0.3640	
10	30.9	3.1	27.2	30.9	0	0	0	7197.2	2494.8	0.3466	
9	39.2	3.9	35.5	29.8	145.6	-249.1	145.6	8942.7	2684.3	0.300	
8	34.5	3.5	34.5	26.1	811.3	-170.4	811.3	7894.4	2414.5	0.3058	T O
7	30.8	2.1	30.8	22.4	1346.7	533.0	1346.7	7046.9	2168.9	0.3078	P
6	27.0	2.7	27.0	18.7	1808.8	1098.5	1808.8	6192.9	1906.4	0.3078	P
5	23.3	2.3	23.3	14.9	2214.4	1590.7	2214.4	5325.2	1609.4	0.3022	L I
4	19.6	2.0	19.6	11.2	2598.1	2026.4	2598.1	4424.9	1229.3	0.2778	N
3	15.9	1.6	15.9	7.5	3043.0	2440.2	3043.0	3423.5	591.8	0.1729	G
2	12.1	1.2	12.1	3.7	3806.4	2915.2	3806.4	1020.7	-1700.8	-1.666	
1	8.4	0.8	8.4	0	6224.6	3708.6	6224.6	6075.0	2387.5	0.393	SLIDING

tension to secure block 1, and consequently to secure the whole slope, is 4846 k N. In block 2, however, the ratio of S_n to R_n is -1.6, meaning that this block tends to slide uphill as it overturns; block 2 sould bump into the next riser along the base and mobilize the additional required base force.

THE FACTOR OF SAFETY FOR LIMITING EQUILIBRIUM ANALYSIS

The factor of safety for toppling can be defined by dividing the friction coefficient believed to apply to the rock layers ($\mu_{available}$) by the friction coefficient required for equilibrium with the given support force T ($\mu_{required}$).

$$F.S. = \frac{\mu_{available}}{\mu_{required}} \tag{15}$$

If, for example, the best estimate of the friction coefficient is 0.800 for the rock layer surfaces sliding on each other, the factor of safety in example 1A, with the 0.5 K Newton support force in block 1, is 0.800/0.7855 = 1.02. With the support force of 2013 K Newtons in example 1B, the factor of safety is increased to 0.800/0.650 = 1.23. Probability of safety can also be introduced, with the same relationship to factor of safety as discussed by Goodman (1975).

Once a column overturns by a small amount, the friction required to sustain it from turning further increases. Thus, a slope just at limiting equilibrium is meta-stable. However, a rotation equal to $2(\beta - \alpha)$ will convert the edge-to-face contacts along the sides of the columns into continuous face contacts. Accordingly, the friction coefficient required to prevent further rotation will drop sharply, possibly even below that required for an initial equilibrium. The choice of safety factor, therefore, depends on whether or not some deformation can be tolerated. Many of the natural toppling failures visited in the field displayed full face-to-face contacts in the lower portions, suggestive that motion had ended when this condition had been restored.

EXAMPLES OF TOPPLING FAILURE

A few examples of toppling failures are discussed in published literature. Zischinsky (1966) described ancient flexural-topples of schist, phyllites, and amphibolites in the Austrian Alps. Zaruba and Mencl (1969) showed two ancient slide-head-topples in Czeckoslovakia.* In one, a slump in horizontal, Cretaceous, clayey marls initiated block-flexural-toppling above the head scarp, along vertical joints in the overlying sandstone. In the second example, gliding of large sandstone blocks down the dip of the underlying claystone created a graben at the head, into which toppled 50 meter high sandstone columns defined by persistent joints. Zaruba and Mencl also give examples of slide-base-toppling and creep-toppling.

Heslop (1974) described toppling of schists, cherts, and serpentinites in the hanging wall of a caving mine in Swaziland. Underground mining by longwall shrinkage stoping and sub-level caving of a steeply dipping orebody has produced subsidence by flexural-toppling. Flexural slip was observed underground on the shear surface forming the sides of columns. In-situ stresses were measured and the major principal stress was determined to be directed perpendicularly to the columns.

Bukanovsky, Rodriguez, and Cedrun (1974) described a block-toppling failure which occurred on bedding in limestone, marl, and sandstone ("flysch deposits") in a Spanish highway cut. The beds dipped 56° into the hillside which was cut at 45° to a depth of 35 meters. Flexural slip was observed on bedding planes and wide tension cracks developed as far as 30 meters behind the crest of the cut slope. The slope was stabilized by cutting it back to 34°, so as to be normal to the bedding; this increased the cut height to 35 meters.

Along the North Devon seacoast, England, a series of Carboniferous age sandstone and shale layers has experienced a number of toppling failures. One of these was described by de Freitas and Watters (1973) and others are discussed by Baynes (1975). Most of the North Devon cases are block-flexure-topples while in several instances synclinal folds set up slide-toe-topples. Several topples are intermediate between the three classes as sandstones present numerous cross joints while the more compliant shales tolerate considerable bending. Sea cliff erosion, aided by softening of the shales, triggers toe failure, initiating overturning of the strata above. Figure 11 shows several examples of topples on the North Devon coast. In Figure 12, the slope angles and dips have been plotted for ten North Devon toppling failures as well as for five safe

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*Pages 178 and 187; pages 6, 35, and 36.

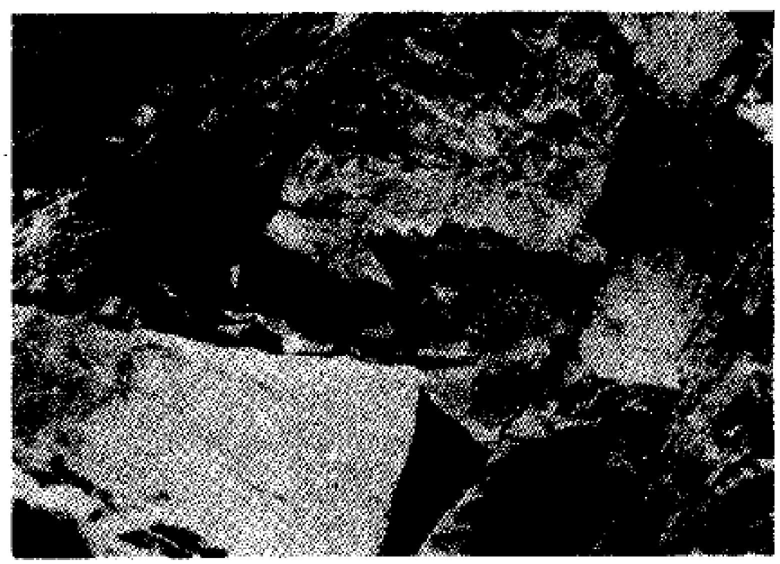
(a)

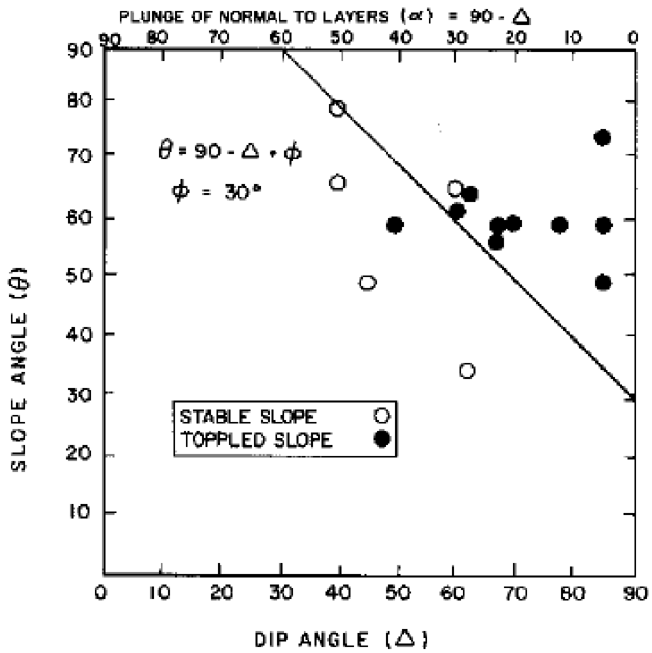
11. Examples of block flexure topples in North Devon:
a) South of Hartland Quay ("St. Catherine's Tor #1" of Baynes, 1975);
b) Just south of Hartland Quay;
c) North of Hartland Quay;
d) Detail in the toe region of a topple west of Clovelly.

(c)

(b)

(d)





12. Plot of slope angle versus dip of the strata for toppled and stable slopes along the North Devon Coast.

slopes in the same formation. Baynes (1975) determined the friction angle of saw-cut sandstone surfaces in these rocks to be 26° while the residual friction angle of unsheared sandstone surfaces was found to be 31°. Equation (1) with φ equal to 30° is represented by a line on Figure 14, producing a surprisingly successful division between safe and unsafe slopes.

A large, post-glacial tension-crack-topple in South East Wales was also described by de Freitas and Watters. This occurred in flat-lying Carboniferous sandstones and shales 40 meters thick overlying a coal seam. A rock block of perhaps 40,000 cubic meters was partially undermined by a slide of softened shales in the toe of the bluff below the coal seam. The block then overturned through 20° and slid downwards. In 1967, a somewhat similar class of topple occurred about one year after excavation of a 20 meter deep building foundation in Dallas, Texas. An eight meter high block of Austin chalk, resting on Eagleford shale, and bounded by a tension crack 5 to 6 meters behind the face, toppled forward about 3°, undermining a part of a downtown street. Water pressure accumulating in the tension crack may have been a factor in that case.

Slate and schist are especially prone to toppling failures because well developed cleavage or schistocity is

(a)

13. A flexural topple of slate in Dnorwic quarry, N. Wales: a) Viewed from above (photo by Evert Hoek); b) Viewed from below.

(b)

often steeply inclined. Several sizable topples in slate occur in Cottonwood Canyon, near Salt Lake City. Topples up to 200,000 cubic meters in volume can be found in the Vishnu Schist near Clear Creek, in the Grand Canyon, Arizona; flexural toppling occurs in the grand Canyon where the schistocity dips steeply into the hillside while slide-toe-toppling occurs where the schistocity dips toward the creek bottom. In each case, lateral release of the toppling mass was permitted by erosion of tributary valleys perpendicular to the strike.

De Freitas and Watters described a natural topple over 1000 meters long in schists and more massive metamorphic rocks (granulites) at Glen Pean, Scotland. The schistocity is inclined 80° into the hillside and the surface averages 45° in its lower half, flattening above. The bases and tops of columns are formed by continuous joints inclined about 38° towards the valley. A block roughly 200 meters long has slid along these joints at the toe of the slope and the columns above rotated forward through 30°; toppling columns developed numerous obsequent scarps, some of which traverse the entire width of the topple. The Glen Pean topple is estimated to encompass some 30 million cubic meters. The authors presumed it resulted from valley deepening by an advancing glacier, followed by removal of toe support as a consequence of glacial retreat.

Large flexural topples occur in slate quarries of North Wales. Figure 13a shows a topple in Dnorwic quarry on steeply plunging slaty cleavage. The wide, deep tension crack of this topple continues beyond its lateral boundaries all the way to the distant ridge; surface or underground excavation in the toe of that ridge (off the photo to the right) could thus trigger an even larger topple. Figure 13b is a view of the topple from below, showing the back face of the tension crack and the overturned slate columns. The destruction of leading columns by flexural cracking is also evident. It can be appreciated that persons unfamiliar with this mode of failure might choose to refer to this cliff as "talus" and to the failure mode as "rock falls." Some features of topples in Penn Rynn quarry, North Wales, are shown in Figure 14. Rotated columns at the head of a large tension crack are shown in Figure 14a. Below the scarp, the columns are broken and resemble a talus. Figure 14b shows a wide tension crack at the head of a developing topple. Individual topples involve rock volumes of up to 50,000 cubic meters. Since the strike of the slaty cleavage parallels the long dimensions of the quarry, there are many instances of toppling. According to quarryman Ted Oliver, the formation of tension cracks gives ample warning of the toppling failure to come and the accelerated rock movement occurs over a minimum period of four hours.* Figure 14c shows obsequent scarps up to

*The Welsh quarrymen refer to these topples as "Toflu dros ei drolud" meaning "toppling over the foot."

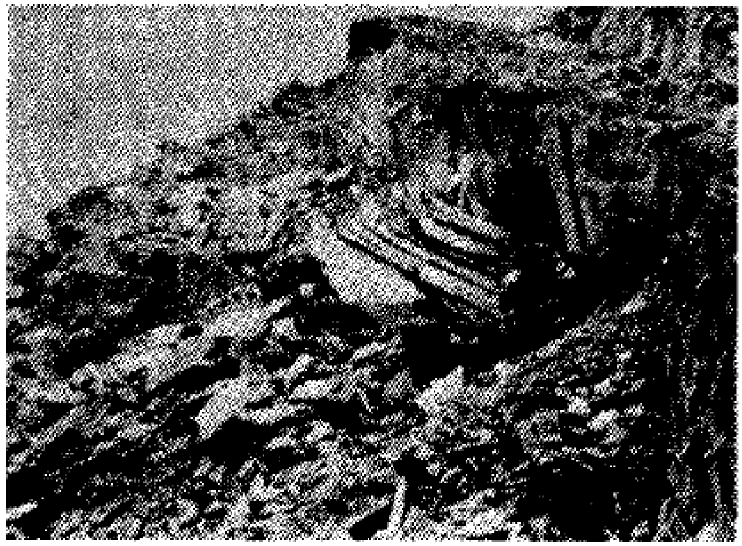
1 meter in height on a toppling bench.

Figure 15 shows part of the wall of a limestone quarry about 120 meters deep, near Philadelphia. The quarry was being used for water supply and a water treatment plant, seen at the top of Figure 15a, had been installed at the quarry's edge. Water was lifted into the plant by means of four wells with submersible turbine pumps intercepting a tunnel at the foot of the slope. These were destroyed in a failure which extends up to 14 meters behind the original face. The thin-bedded Elbrook limestone under the plant, forming this wall of the quarry, contacts the more massive Ledger dolomite about 25 meters below the crest. The beds strike parallel to the wall and dip 53° to 57° into the face. Twelve, and then, three days before the failure, there were episodes of unusually high turbidity and after the failure several new sink holes appeared. Two boreholes drilled within the cracked zone intersected the water table at a depth of 31 meters--still 33 meters above the level of water in the quarry lake. Figure 15b shows the stepped surface across the layers, which resembles that of new flexural cracking. Slight bending of one layer, and cantilever support of the clay overburden by the top-most layer can also be seen. These features all point towards toppling as the failures mode in this case.

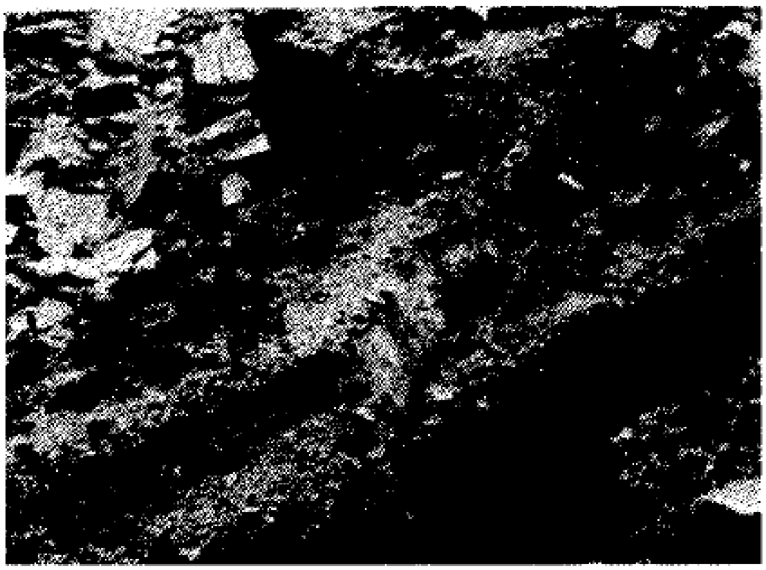
DISCUSSION AND CONCLUSIONS

Toppling of rock layers is an important mode of potential failure which must not be overlooked. It can involve a large volume of rock in serious deformations far distant from a slope face, e.g. up to four times the slope height. Toppling is distinct from conventional sliding modes in that there is no single, basal sliding surface; however, sliding must occur simultaneously with overturning between the columns as well as at the bases of blocks in the toe region. Toppling of rock slopes is accompanied by distinctive geomorphic features which make it possible to identify it as the applicable mechanism in most cases. These include obsequent scarps, deep tension cracks, flexural cracking, and normal fault type flexural slip. Moreover, as opposed to sliding, displacements are greater at the slope crest and show forward rotation. Nevertheless, toppling has been appreciated as a relevant mode of behavior only rather recently and relatively few cases have been published. The authors have begun a worldwide inquiry for additional cases and would welcome information on new examples. The catalogue of experiences now includes toppling failures in California, Arizona, Utah, Texas, Colorado, Pennsylvania, Wisconsin, Manitoba, British Columbia, Ontario, Guatemala, Colombia, Britain, Sweden, Italy, Spain, Czeckoslovakia, Austria, Swaziland, Australia and China. These failures involve shale, sandstone, slate,

(a)



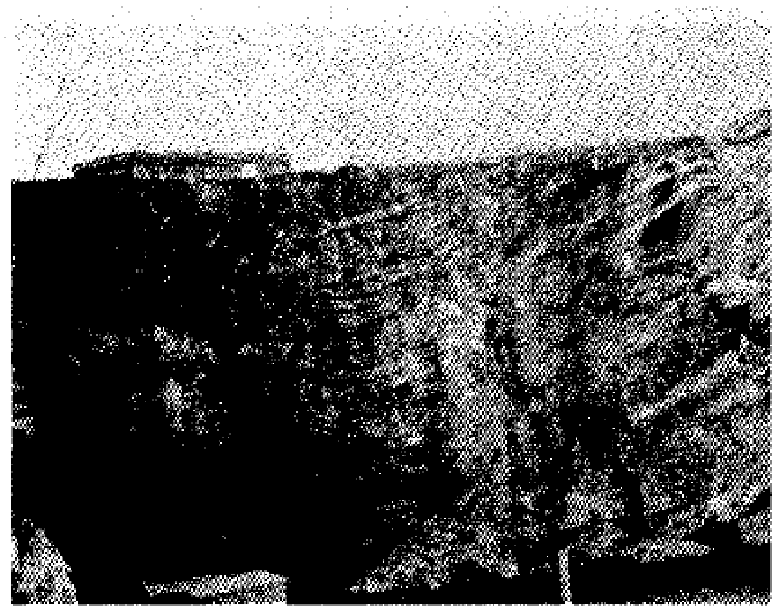
(b)

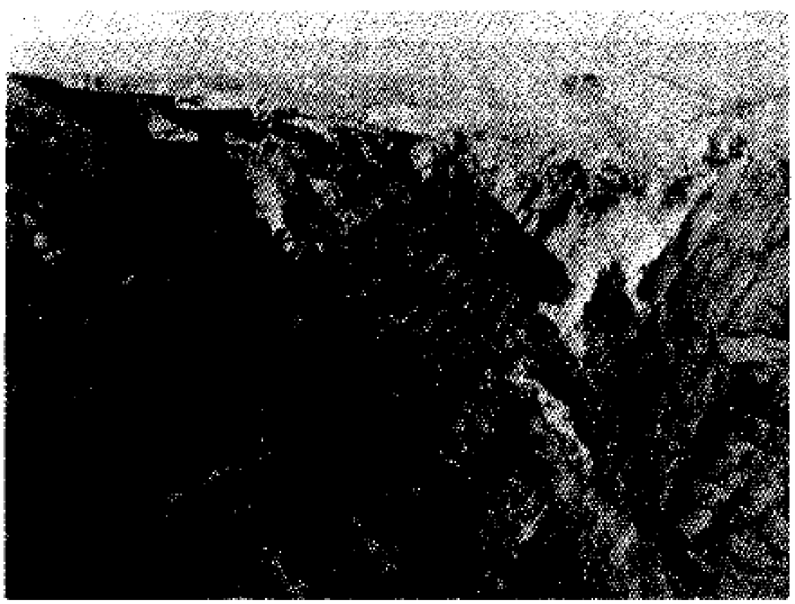


(c)

14. Toppling in Penn Rynn slate quarry, N. Wales: a) Rotated columns at the head and broken columns below; b) A tension crack; c) Obsequent scarps on a toppling bench.

(a)





(b)

15. Slope failure in the wall of a limestone quarry being used for water supply: a) Note contact of thin-bedded and thick-bedded formations; b) Close up of upper part of slope suggests toppling mechanism.

schist, amphibolite, granitics, serpentinite, limestone, and volcanics. Toppling is especially important in steep slopes in layered or foliated rock. Thus it is particularly important in quarries and open pit mines but significant toppling failures also occur in natural slopes. Toppling can be combined with sliding modes in a variety of ways, as indicated by the discussion of "secondary toppling modes" and it relates not only to slopes but to structural foundations and underground openings. Suffice it to say that our understanding and appreciation of this behavioral mode is but in its infancy.

ACKNOWLEDGEMENTS

We wish to thank Mr. M. de Freitas and Dr. Evert Hoek for guiding us to localities of toppling failures in Great Britain. Research support of the National Science Foundation, through grant ENG 74 - 12407, is greatly appreciated.

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