# Three-dimensional Analysis of Underground Wedges under the Influence of Stresses

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**ABSTRACT:**

Analyzing the stability of the three-dimensional wedges created around underground excavations in rock masses is a common problem in rock engineering. Most existing algorithms for wedge failure analysis assume wedges to be in low stress environments. Under such conditions the influence of stresses on the wedge factor of safety can be ignored. In blocky rock masses experiencing high *in situ* stresses, however, such analysis can lead to conservative results. This paper discusses the incorporation of stresses calculated from boundary element analysis into the analysis of wedge stability. The new formulation assumes complete plane strain, allowing the analysis of tunnels arbitrarily oriented with respect to principal stress directions. The approached described in the paper is implemented in the program Unwedge.

## 1. INTRODUCTION

The stability of wedges formed around underground excavations in blocky rock masses is a common problem in rock engineering. The wedges are formed by intersecting discontinuities and the free face created through excavation of an underground opening.

Under the influence of gravity and other forces, roof and wall wedges may fail either by falling, sliding or rotating out of their sockets. The factors that control wedge stability include geometry (the size, shape and spatial location of a wedge), the strength characteristics of the discontinuity planes that create the wedge, and stresses within the rock mass.

Most existing algorithms for underground wedge stability analysis assume that stresses are sufficiently low and can therefore be ignored. This is fine for wedges in low *in situ* stress environments, such as those encountered in shallow excavations [1]. When *in situ* stresses are high, however, exclusion of the effect of stresses leads to error.

The assumption of zero stress results in very conservative results for certain cases. The stability of a wedge in the roof of an excavation is an example. Such a wedge fails by falling under the influence of self-weight. Traditional stability analysis approaches (which do not take stress into account) predict that this wedge has a factor of safety of zero, and can remain in place only when supported.

Many practical situations defy this prediction. Roof wedges located in deep underground excavations have been observed to be stable. Kaiser et al [2] suggest that conventional underground wedge stability analyses are commonly too conservative because they ignore stress.

The stability of wedges at significant depths can be explained by the clamping effect of the surrounding stress field. The components of confining stress normal to the discontinuity planes, which form a wedge, mobilize shear resistance sufficient to keep the wedge in place.

Unfortunately, including stresses in wedge stability analysis is not a trivial exercise. To simplify the problem, techniques that have been suggested use assumptions such as:

1. Two-dimensional wedge and stress geometry, and/or
2. In situ stresses acting on wedges.

These assumptions do not allow comprehensive understanding of stress effects on wedge stability. The geometry of wedges is clearly three-dimensional. The same is true for the state of stress. As well, since excavation in a stressed rock mass causes a re-distribution of stresses around created openings, the stresses acting on the faces of a wedge are often quite different from in situ conditions.

The methodology described in this paper involves calculation of stresses around excavations with the Boundary Element Method (BEM), and incorporation of the stresses into the stability equations for wedges. An example will be provided that illustrates the benefits of including stress in wedge factor of safety calculations.

## 2. BASIC STRESS ASSUMPTIONS OF THE PROPOSED METHOD

The first step of the method proposed in this paper involves calculation of the stresses acting on wedges, taking into account the disturbed stress state around an excavation. The following three assumptions are made to simplify calculations:

1. The rock mass material around an excavation behaves as an elastic continuum
2. The stress state is the same at every tunnel cross-section, and
3. Stresses cannot reduce the factor of safety of a wedge below the unstressed value.

The first assumption ignores the presence of discontinuities during the calculation of stresses. This assumption is typically adequate for modelling stresses (this is especially true for hard rock masses). The second assumption is valid when the ratio of the length of an excavation to its width exceeds six, and sections being analyzed are more than three widths away from the end of the excavation. It excludes the incorporation of stresses into the analysis of end wedges. This is because the spatial distribution of stresses at the ends of excavations is three-dimensional, varying rapidly over short distances in the direction of the excavation axis.

For the stress analysis of geotechnical excavations today, the second assumption allows use of two-dimensional models. These are often simpler to develop, easier to use and faster to compute than their three-dimensional counterparts. As a result, two-dimensional stress approaches are currently more prevalent for practical engineering design and analysis.

The logic at the basis of the third assumption is that once a wedge moves, stress relief occurs across its joint planes. This stress relief causes the wedge factor of safety to become the same as the unstressed value.

## 3. COMPLETE PLANE STRAIN ANALYSIS OF UNDERGROUND EXCAVATIONS

Generally the second assumption is known as the (conventional) plane strain condition. Under the conventional definition, strain in the direction perpendicular to the cross-sections of an excavation (the direction along the long axis of the excavation) is assumed to be zero. As well it is assumed that the stress distribution on every cross-sectional plane is identical, and the shear stresses on planes, which contain the long axis (i.e. planes that are perpendicular to the excavation cross-section), have zero values.

The conditions of conventional plane strain are satisfied only when the long axis of an excavation is parallel to a direction of principal stress. In order to optimize design criteria such as the volume of unstable wedges, however, rock engineers often analyze a wide range of excavation orientations. Since the majority of the analyzed orientations do not coincide with any principal stress directions, conventional plane strain is not general enough for calculating stress distributions for wedge stability analysis.

Complete plane strain is a more comprehensive framework for calculating stresses around arbitrarily oriented excavations [3]. Like conventional plane strain, complete plane strain assumes that stress and displacement conditions on every excavation cross-sectional plane are identical. Unlike conventional plane strain however, it allows non-zero values for all six components of the stress tensor at a point.

## 4. STRESS DISTRIBUTIONS AROUND EXCAVATIONS

Numerical methods offer the most generalized means of calculating stresses around excavations. One of the most simple (from the user point of view, but not from the mathematical perspective) numerical approaches is the Boundary Element Method (BEM). The BEM is powerful for analyzing problems that involve elastic, homogeneous materials.

In the BEM only the surfaces in a model are discretized into elements. The method treats the rock mass surrounding an opening as an infinite continuum. Because only surface boundaries are discretized in the BEM, it enjoys several advantages over other numerical methods. The BEM uses much smaller numbers of elements, and thus has shorter computational times. The BEM can also model problems of large geometrical extent since it properly models far-field stress conditions.

Perhaps most important for use with wedge stability analysis, the BEM requires minimal numerical modelling input and user experience. These features facilitate automation of the stress calculations for wedge stability analysis.

## 5. PROCEDURE FOR INCLUDING STRESSES INTO WEDGE STABILITY CALCULATIONS

The method proposed in the paper considers only tetrahedral wedges at the moment. It is implemented in Unwedge, the underground wedge stability analysis program developed by Rocscience Inc. [4].

Figure 1 below shows the geometry of such a wedge in the roof of a tunnel. Discontinuity planes form the three wedge faces ABC, ADB and BCD. In the equations that will follow shortly, these wedge faces are represented with the index $p$ ($p$=1, 2 or 3).

Each discontinuity wedge face is then discretized into $m$ triangular elements as shown on Figure 1b. The elements are indexed with $q$ ($q$=1, 2,…, $m$). On Figure 1, the unit normal vector to the $pq^{\text{th}}$ element, acting into the wedge, is denoted with the symbol $n_i^{pq}$. Each triangular element has an area $a^{pq}$.

From the BEM solution of the stresses around the excavation under analysis, the stress tensor $\sigma_{ij}^{pq}$

$$\sigma_{ij}^{pq} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \quad (1)$$

at the centroid of each element can be calculated.

A stress vector, $\xi_i^{pq}$, acting at the centroid can then be determined from the stress tensor and normal vector to the wedge face using the equation

$$\xi_i^{pq} = \sigma_{ij}^{pq} n_j^{pq}, \quad (2)$$

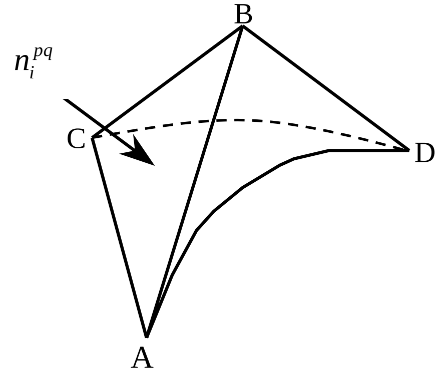
where summation is carried out over repeated subscript indices.

The stress vector $\xi_i^{pq}$ can be resolved into two components: a normal vector $N_i^{pq}$ and a shear vector $S_i^{pq}$. The magnitude of the normal vector is calculated from the expression

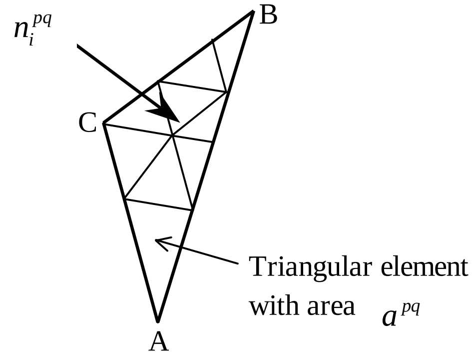
$$N^{pq} = \xi_i^{pq} n_i^{pq}. \quad (3)$$

The shear vector is determined as

$$S_i^{pq} = \xi_i^{pq} - N^{pq} n_i^{pq} \quad (4)$$




a. Tetrahedral wedge in tunnel roof




b. Discretization of discontinuity plane into triangular elements

Figure 1. Typical geometry of a tetrahedral wedge. Three of its faces – ABC, ADB and BCD – are formed by discontinuity planes.

Since the magnitude of the shear stress $\left\| S_i^{pq} \right\|$ acting on an element cannot exceed the shear strength $\tau^{pq}$ (the computation of $\tau^{pq}$ is discussed later), the following condition is checked for and ensured:

$$\left\| S_i^{pq} \right\| \le \tau^{pq} \quad (5)$$

The normal and shear forces acting on an element can be easily computed by multiplying the respective stress vectors with the element area $a^{pq}$.

The wedge stability algorithm [4] used in the paper groups all forces acting on a wedge into two classes: active forces and passive forces. Active forces are those that drive the wedge to failure while passive forces are those that resist failure. Examples of active forces include self-weight, water pressures and seismic forces. Passive forces include bolt pressures and shear resistance force of shotcrete support.

The resultant force, $Q_i$, computed from the stresses acting on a wedge is grouped with the active forces. It is calculated from the equation

$$Q_i = \sum_{p=1}^{3}\sum_{q=1}^{m} a^{pq}\left(N^{pq}n_i^{pq} + S_i^{pq}\right) \qquad (6)$$

For a simple wedge analysis, which does not involve support, seismic loads, or groundwater pressures, the total active force $A_i$ is

$$A_i = W_i + Q_i, \qquad (7)$$

where $W_i$ is the weight vector of the wedge.

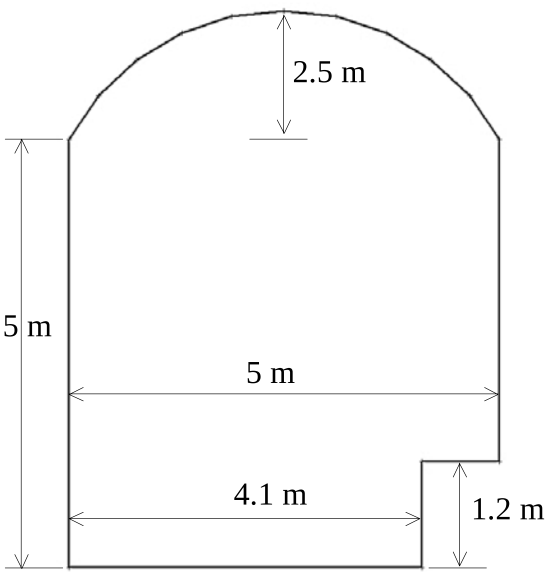



Figure 2. Cross-sectional geometry of tunnel.

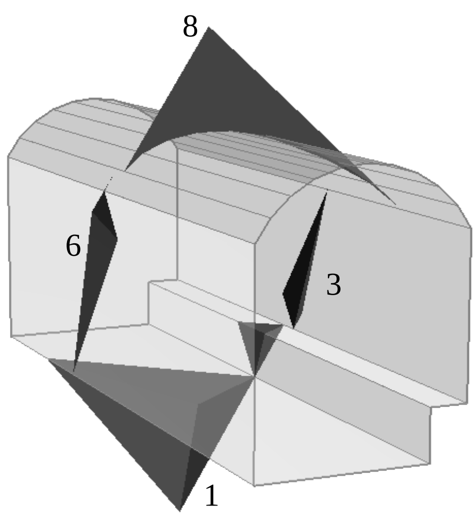



Figure 3. Wedges formed around perimeter of excavation (Example 1).

Once all active forces have been determined, they are used in the method presented in Chapter 9 of [5] to determine the unit vector in the sliding direction (failure mechanism), $s_i$, for the wedge.

In order to compute a factor of safety for the wedge, the final quantities required are the forces due to the shear strengths and tensile strengths of the joint faces. The magnitude of the resisting force in the sliding direction due to the shear strength of joint *p*, $J^p$, is computed from the equation

$$J^p = \sum_{q=1}^{m} \tau^{pq} a^{pq} \cos\theta^p \qquad (8)$$

where $\tau^{pq}$ is the shear strength of the $q^{th}$ element of wedge face *p*, $a^{pq}$ is the area of the $q^{th}$ element of the wedge face *p*, and $\theta^p$ is the angle between the direction of sliding and the $p^{th}$ joint.

The shear strength $\tau^{pq}$ is calculated using the normal stresses acting on joint elements and the strength characteristics of the joint. For example, if a joint is assumed to have Mohr-Coulomb strength, then the shear strength is determined from the formula

$$\tau^{pq} = c^p + N^{pq}\tan\phi^p, \qquad (9)$$

where $c^p$ and $\phi^p$ are the cohesion and friction angle of the joint, respectively.

The magnitude of the tensile strength force, $T^p$, in the sliding direction mobilized on a wedge face is determined from the equation

$$T^p = \sum_{q=1}^{m} \sigma_t^p a^{pq} \sin\theta^p, \qquad (10)$$

where $\sigma_t^p$ is the tensile strength of the $p^{th}$ joint.

The following equation is then used to compute the factor of safety, *F*, of the wedge:

$$F = \frac{\sum_{p=1}^{3}\left(J^p + T^p\right)}{A_i s_i} \qquad (11)$$

## 6. EXAMPLES

To illustrate the influence of stress on the stability of underground wedges, we will analyze two cases of a tunnel with its long axis oriented at a trend of 45° and a plunge of 0°. The rock mass has a unit weight of 0.0265 MN/m$^3$, a Young's modulus of 5000 MPa, and a Poisson's ratio of 0.25. The cross-sectional geometry of the tunnel is shown on Figure 2 (all figures shown below are captured from Unwedge).

### *6.1. Example 1*

In Example 1, three joint sets with the following orientations exist in the rock mass:

| **Joint** | **Orientation** | |
|---|---|---|
| | **Dip** | **Dip Direction** |
| #1 | 60° | 30° |
| #2 | 30° | 150° |
| #3 | 40° | 270° |

The joints all have the same Mohr-Coulomb strength parameters: a cohesion value of 0.1 MPa and a friction angle of 30$^o$.

The *in situ* principal stresses at the site have the magnitudes and directions shown in Table 1.

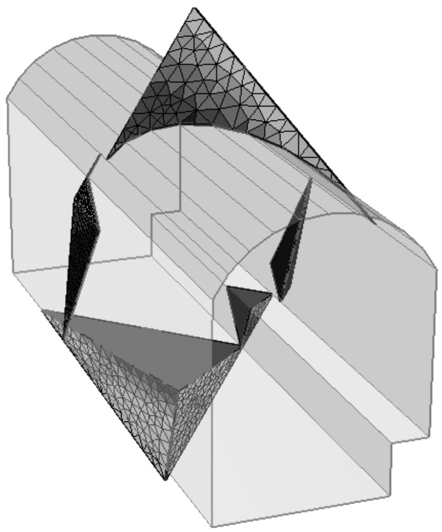


Figure 4. Contours of normal stress on wedge faces.

Table 1. Principal *in situ* stresses in rock mass.

| Principal Stress | Magnitude (MPa) | Orientation | |
|---|---|---|---|
| | | **Trend** | **Plunge** |
| $\sigma_1$ | 20 | 5$^o$ | 85$^o$ |
| $\sigma_2$ | 15 | 185$^o$ | 5$^o$ |
| $\sigma_3$ | 7 | 95$^o$ | 0$^o$ |

Figure 3 shows the wedges formed around the perimeter of the tunnel. Wedge #1 is in the floor, #3 is on the right sidewall, #6 is in the left sidewall, while #7 and #8 are in the roof. Wedge #7 is very small and will therefore not be discussed in the rest of this section. Wedge #1 will not fail since it is in the floor so it will also not be discussed further.

When stress is omitted from the analysis we obtain the following factor of safety values for the wedges:

| Wedge | Factor of Safety |
|---|---|
| #3 | 23.973 |
| #6 | 26.458 |
| #8 | 0.0 |

Notice that the roof wedge (#8) has a factor of safety of zero.

We next analyze the wedges with the disturbed stress state around the excavation taken into consideration. Figure 4 shows the contours of normal stress on the faces of the wedges.

The factors of safety obtained are as follow:

| Wedge | Factor of Safety |
|---|---|
| #3 | 23.973 |
| #6 | 26.458 |
| #8 | 4.505 |

Although the *in situ* stresses for the problem are quite low, they substantially change the computed factors of safety for the roof wedge. The results indicate no changes in the factor of safety of the wedges 3 and 6. This is because, as noted earlier, the proposed formulation does not allow the stressed factor of safety to be lower than the unstressed value.

### *6.2. Example 2*

Example 2 involves three joint sets with the following orientations:

| Joint | Orientation | |
|---|---|---|
| | **Dip** | **Dip Direction** |
| #1 | 30$^o$ | 30$^o$ |
| #2 | 43$^o$ | 150$^o$ |
| #3 | 50$^o$ | 270$^o$ |

The joints all have the same Mohr-Coulomb strength parameters: zero cohesion and a friction angle of 30$^o$.

The *in situ* principal stresses at the site are the same as those for Example 1 (Table 1).

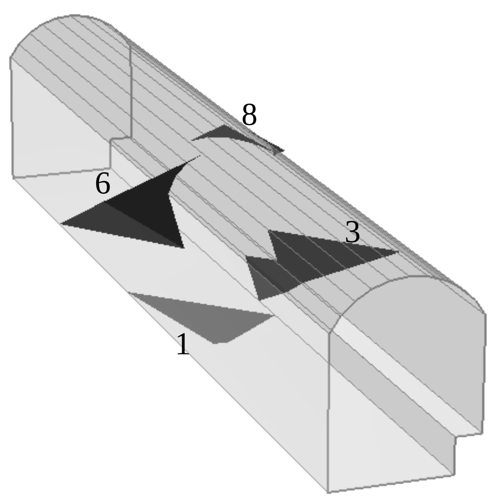



Figure 5. Wedges formed around perimeter of excavation (Example 2).

The wedges formed around the excavation are shown on Figure 5. Wedge #1 is in the floor, #3 is on the right sidewall, #6 is on the left sidewall, while #7 and #8 are in the roof. Again Wedge #7 will be omitted from discussions due to its very small size.

Wedge #1 does not fail since it is in the floor. Wedges #3, #6 and #8 have the following factors of safety values when stress is not considered in the analysis:

| Wedge | Factor of Safety |
|---|---|
| #3 | 1.277 |
| #6 | 0.619 |
| #8 | 0.0 |

When stress is included in the analysis, the factor of safety values increase to:

| Wedge | Factor of Safety |
|---|---|
| #3 | 3.155 |
| #6 | 3.461 |
| #8 | 0.751 |

This time, all wedges benefit from the effects of stress clamping.

## 7. CONCLUSION

As seen from the examples provided above, stresses can considerably alter the factor of safety values computed for underground wedges. Whereas traditional analysis gives very conservative results for some wedges, the method described in this paper gives more realistic results.

The greatest value of the suggested approach does not lie in the exact factor of safety values calculated, but in the fact that it can be used to study the varying influence of stress on stability. In addition, since the procedure for incorporating stress into wedge stability uses all six components of the stress tensor, it can be combined with a full three-dimensional stress analysis approach. Such a combination would allow the analysis of wedges occurring anywhere along the long axis of an excavation, including those located at the ends.

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