# THE INFLUENCE OF CORRELATION AND DISTRIBUTION TRUNCATION ON SLOPE STABILITY ANALYSIS RESULTS

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ABSTRACT
The paper studies the impact of correlation on the probability of failure of a homogeneous slope with Mohr-Coulomb soil strength, for which cohesion is correlated to friction angle. The paper looks at the influence of different degrees of negative and positive correlations of the strength properties, and of horizontal and vertical seismic accelerations. It also examines the impact of truncation on probability of failure, an aspect that slope modellers must consider carefully, but may be unaware of.

## 1. INTRODUCTION

Presently, most probabilistic slope stability analyses tools do not allow correlation to be modelled extensively. Only few variables (primarily cohesion and friction angle) are allowed to have correlations in software for such analyses. Using the example of a simple homogeneous slope, this paper examines the impact of correlation on computed probabilities of failure. The paper also examines the validity of a rule of thumb in engineering reliability and probabilistic analysis that suggests that if the correlation coefficient of two random variables is less than ±0.3, the variables can be considered statistically independent [1].

For the same slope, the paper also examines the impact of distribution truncation on probabilistic results. Because some parameters in an analysis have valid values only within certain ranges, e.g. friction angle lies between 0 and 90 degrees, when they are represented with unbounded distributions such as the normal distribution, truncation limits are imposed. The paper looks at how truncation affects computed probabilities of failure.

It is not the goal of the paper to comprehensively answer the questions outlined above, or arrive at conclusive guidelines on the modelling of correlations. That would require major study. Rather, it seeks to identify trends that possibly arise from correlation and truncation.

## 2. CORRELATION AND TRUNCATION

### 2.1 Correlation

Correlation is a parameter that measures the degree to which two random variables tend to vary together. Suppose that several pairs of cohesion and friction angle values were measured for a soil. Suppose also that for the data pairs it is observed that as measured cohesion values get higher, measured friction angles also increase. As cohesion values fall, the measured friction angles also tend to fall. In this case the two parameters tend to co-vary, and have a positive correlation.

The paper studies the role of correlation on a slope with Mohr-Coulomb soil strength, for which cohesion and friction angle are correlated. It also looks at correlation between horizontal and vertical seismic coefficients. These two sets of parameters were selected in the study of correlation, because they are the only ones currently implemented in most commercially available slope stability software. Correlation is varied over the range of (-1 to +1).

### 2.2 Truncation

In engineering practice, situations arise where a distribution, which in theory is unbounded at one or both of its ends, is a good fit for observed data that varies over a finite range. In geotechnical engineering, fitting of the normal distribution (the most commonly used statistical distribution) to friction angle data is an example. Random variates from the normal distribution range from $-\infty$ to $+\infty$. On the other hand, valid friction angle values lie between 0 and 90 degrees. Therefore, in applying the normal distribution to friction angle data one would have to truncate the distribution to have values from 0 to 90 degrees.

Truncation can alter the statistics of generated random variables, however. If truncation limits are too narrow, the standard deviation of generated variables can be much smaller than that of the input distribution from which the variables were generated. If the truncation is not symmetric about the mean, or if a distribution is non-symmetric, then truncation can cause generated data to have a mean significantly different from that of the distribution. Therefore, in order to use truncation effectively, its mechanics must be well understood. The paper will look at how truncation affects computed probabilities of failure for the simple slope.

## 3. METHODOLOGY

To obtain a general understanding of the individual and combined influences of correlation and truncation on probabilistic slope stability analysis results, the paper considers the homogeneous slope shown on Figure 1. For the sake of simplicity, the slope is analyzed only with Bishop's method of limit-equilibrium analysis. (*Slide* [2], the slope stability software developed by Rocscience Inc., was used to perform the analyses in the paper.)

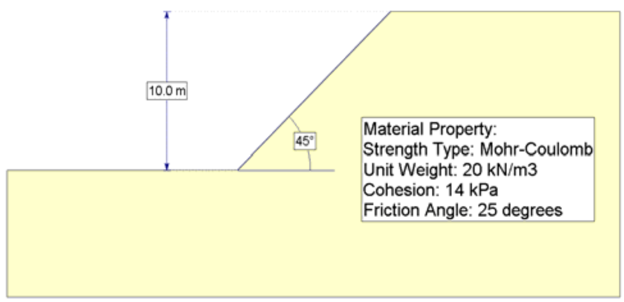


Figure 1. Geometry of slope studied in the paper.

Two pairs of correlated material properties – correlated cohesion and friction angle, and correlated horizontal and vertical earthquake accelerations – were studied. Whenever cohesion and friction angles were considered as random variables, horizontal and vertical seismic accelerations were not considered. Whenever the accelerations were modelled as probabilistic variables, the strength parameters were held constant (assigned their respective mean values).

The material of the slope is assumed to have Mohr-Coulomb strength, with a mean cohesion value of 14 kPa and a mean friction angle of 25 degrees. The parameters are assumed to have a coefficient of variation (C.O.V.) of 10%. C.O.V. is the ratio of the standard deviation of a random variable, $\sigma$, to its mean, $\mu$, i.e.

$$\text{C.O.V.} = \sigma / \mu \,. \qquad (1)$$

It is a convenient, dimensionless measure of dispersion. Smaller values indicate smaller amounts of dispersion in random variables, while larger values indicate greater uncertainty. Values ranging from 0.1 to 0.3 are common to engineering random variables [1]. For soils and rock masses, C.O.V.s of up to about 0.75 have been observed for cohesion, while friction angle C.O.V.s have measured around 0.2 to 0.4. From a table of C.O.V.s for a variety of geotechnical properties compiled in [3], it can be seen that most geotechnical parameters and tests have C.O.V.s within the range of 0 to 0.68.

Two different statistical distributions – the normal and lognormal – are used to model cohesion and friction angle. In each model analyzed, the two parameters are assigned the same distribution shape (e.g. when cohesion is assumed normally distributed, friction angle is also assumed to be a normal random variable). Horizontal and vertical accelerations are modelled with the gamma distribution.

In the examples different truncation limits are considered. Some of the truncations are symmetric (in terms of the number of standard deviations away from the mean) while others are not. Three symmetric truncations (at two, three and five standard deviations away from the mean) are applied to both pairs of correlated data. Two non-symmetric truncations – two standard deviations to the left and five standard deviations to the right of the mean values, and its reverse, five standard deviations to the left and two standard deviations to the right of the mean – are applied to the correlated cohesion and friction angle pair only.

In general, a range of five standard deviations from the mean can be considered to model the entire range of variation of a random variable. For the normal distribution, a range of five standard deviations from the mean practically encompasses 100% (the actual number is 99.9999%) data points from the distribution. A range of three standard deviations covers 99.7% of data, while two standard deviations holds of 95.4% of distribution points.

To calculate a probability of failure in each slope example, 20,000 Monte Carlo simulations were performed. This number of simulations sufficiently captures most of the probability of failure levels encountered in the examples.

## 4. RESULTS

The results of systematic changes in correlation coefficients and truncation limits for cohesion-friction angle, and horizontal-vertical seismic acceleration probabilistic data pairs are described next. All the results are tabulated and presented in the Appendix to the paper.

### 4.1 Impact of Correlation

A reduction in probability of failure as correlation coefficient changes from +1 to –1 was observed. The mean factor of safety values remained practically unchanged in all cases, but the dispersion in factor of safety values reduced as correlation coefficient changed from +1 to -1.

For cohesion-friction angle correlation the variation of probability of failure with correlation was non-linear (assuming the variables to be either normally or lognormally distributed). The relationship was practically linear for the gamma-distributed correlated horizontal and vertical seismic accelerations.

The results indicated that for this specific slope example, the probability of failure was more sensitive to correlations of friction angle and cohesion than to correlation of horizontal and vertical seismic coefficients. This might actually be a result of the sensitivity of factor of safety to these parameters. For correlated cohesion and friction angle, correlation coefficients of $+0.25$ in one case produced over a 100% change in probability of failure (over the case of zero correlation). This indicates that the decision to ignore seemingly small correlations might be based on how sensitive a slope is to the correlated parameters, or to the sign (positive or negative) of the sensitivity.

## 4.2 Impact of Truncation

For all the distributions examined (Figures 2 to 4), truncation at three standard deviations yielded results sufficiently close to those for five standard deviation truncation limits. Truncation at two standard deviations, however, generally led to significant reductions in predicted probabilities of failure.

Non-symmetric truncation can also have significant impact on computed probabilities of failure. The two cases examined in the paper yielded wide differences in probability of failure, especially at higher positive values of correlation coefficient. This is evident on Figure 5 (Table 4 in the Appendix contains all the numerical data). The results show that the effects of non-symmetric truncation on the mean and standard deviation of computed factors of safety are very pronounced. Since probability of failure however changes substantially, it can be concluded that truncation alters the distribution of computed factor of safety values.

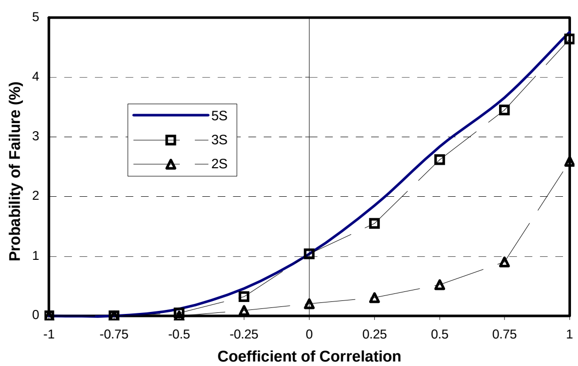


Figure 2. Variation of slope probability of failure with correlation coefficient for cohesion and friction angle. Cohesion and friction angle are both assumed normally distributed.

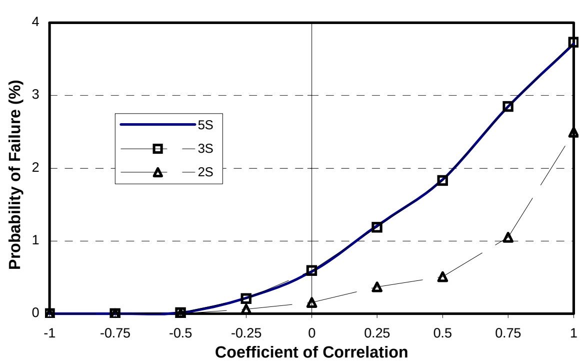


Figure 3. Variation of slope probability of failure with correlation coefficient for cohesion and friction angle. Cohesion and friction angle are both assumed lognormally distributed.

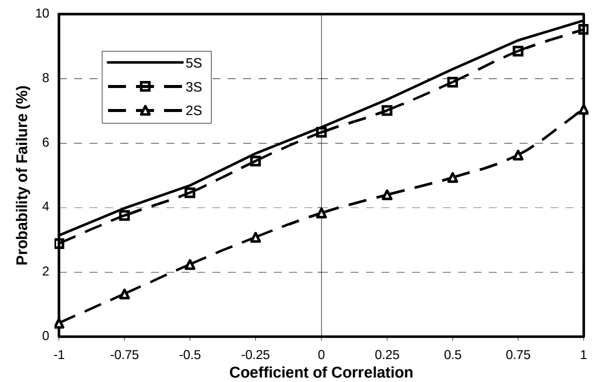


Figure 4. Variation of slope probability of failure with correlation coefficient for horizontal and vertical seismic accelerations. Both accelerations are assumed to have gamma distributions.

As seen on Figure 5, over the possible range of correlation coefficients (from –1 to +1), there were significant differences between the two non-symmetric truncations of cohesion and friction angle distributions.

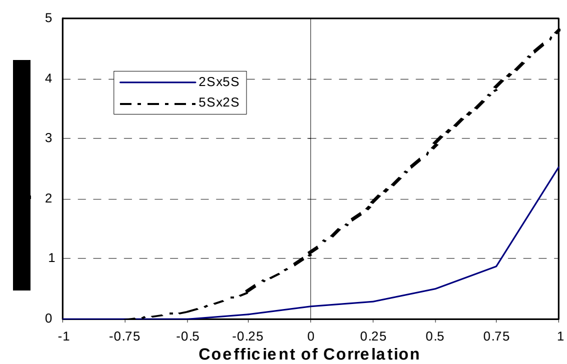


Figure 5. Impact of non-symmetric truncation (of cohesion and friction angle distributions) on computed probability of failure values.

## 5. CONCLUDING REMARKS

Results of the numerical experiments conducted in the paper show that correlation has significant impact on computed probabilities of failure for the slope example analyzed. Positive correlation induces higher probabilities of failure, while negative correlation reduces the probability of failure. As well, the degree of impact correlation has on results depends on the sensitivity of the slope to the probabilistic variables in a slope problem. The more sensitive the slope is to a set of correlated parameters, the more severe is the impact of correlation on failure probabilities.

Results of the numerical experiments conducted in the paper show that truncation can alter the statistical characteristics (mean standard deviation and distribution shape) of computed values. They show that the wider the range of truncation is, the less is its effect on the statistical characteristics of a distribution. This leads us to the following recommendations: if you realize that the characteristics of truncated samples significantly differ from those of the original complete distribution, it is best to use distributions that are inherently restricted to a specified range. The beta, triangular, and uniform distributions are examples of distributions that range from a minimum value a to a maximum value b.

Although in theory, it may be possible to generate data that truly conform to a truly truncated distribution, research on such simulation is ongoing, and is far from mature. As progress is made in this area, geotechnical engineering will be a major beneficiary due to its needs for data truncation. The authors believe that the trends identified through this analysis will generally hold true for more complex slopes.

## 6. REFERENCES


1. Haldar, A. and S. Mahadevan, *Probability, reliability and statistical methods in engineering design*, John Wiley & Sons, New York. 2000.

2. Slide v5.0, Program for limit-equilibrium slope stability analysis, Rocscience Inc. 2002.

3. Duncan, J.M., Factors of safety and reliability in geotechnical engineering, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 126, No. 4,pp. 307-316. 2000.


## APPENDIX
## Tables of Factor of Safety and Probability of Failure Results

Table 1. Cohesion and friction angle both assumed to be normally distributed, and to have C.O.V. of 10%.

| **Correlation Coefficient** | **Mean Factor of Safety** | **Factor of Safety Standard Deviation** | **Probability of Failure (%)** |
|---|---|---|---|
| Truncation of 5 standard deviations on both sides of mean | | | |
| 1 | 1.215 | 0.1309 | 4.76 |
| 0.75 | 1.213 | 0.1227 | 3.66 |
| 0.5 | 1.215 | 0.114 | 2.84 |
| 0.25 | 1.215 | 0.1046 | 1.85 |
| 0 | 1.214 | 0.09341 | 1.04 |
| -0.25 | 1.214 | 0.08269 | 0.45 |
| -0.5 | 1.214 | 0.06912 | 0.12 |
| -0.75 | 1.214 | 0.052 | 0 |
| -1 | 1.213 | 0.02466 | 0 |
| Truncation of 3 standard deviations on both sides of mean | | | |
| 1 | 1.215 | 0.1294 | 4.64 |
| 0.75 | 1.215 | 0.1205 | 3.45 |
| 0.5 | 1.215 | 0.112 | 2.62 |
| 0.25 | 1.213 | 0.1027 | 1.55 |
| 0 | 1.214 | 0.09406 | 1.049 |
| -0.25 | 1.214 | 0.08177 | 0.32 |
| -0.5 | 1.214 | 0.06858 | 0.05 |
| -0.75 | 1.214 | 0.05169 | 0 |
| -1 | 1.213 | 0.02422 | 0 |
| Truncation of 2 standard deviations on both sides of mean | | | |
| 1 | 1.214 | 0.1146 | 2.59 |
| 0.75 | 1.214 | 0.1023 | 0.905 |
| 0.5 | 1.214 | 0.09525 | 0.525 |
| 0.25 | 1.214 | 0.08883 | 0.305 |
| 0 | 1.214 | 0.08351 | 0.205 |
| -0.25 | 1.213 | 0.07387 | 0.09 |
| -0.5 | 1.213 | 0.06356 | 0 |
| -0.75 | 1.213 | 0.04854 | 0 |
| -1 | 1.213 | 0.02151 | 0 |

Table 2. Cohesion and friction angle both assumed to be lognormally distributed, and to have C.O.V. of 10%.

| Correlation Coefficient | Mean Factor of Safety | Factor of Safety Standard Deviation | Probability of Failure (%) |
|---|---|---|---|
| Truncation of 5 standard deviations on both sides of mean | | | |
| 0.99 | 1.215 | 0.131 | 3.71 |
| 0.75 | 1.215 | 0.1228 | 2.845 |
| 0.5 | 1.215 | 0.1141 | 1.845 |
| 0.25 | 1.213 | 0.1049 | 1.21 |
| 0 | 1.214 | 0.095 | 0.58 |
| -0.25 | 1.214 | 0.08321 | 0.215 |
| -0.5 | 1.214 | 0.06981 | 0.015 |
| -0.75 | 1.214 | 0.0528 | 0 |
| -0.99 | 1.213 | 0.02721 | 0 |
| Truncation of 3 standard deviations on both sides of mean | | | |
| 0.99 | 1.213 | 0.128 | 3.73 |
| 0.75 | 1.212 | 0.1186 | 2.845 |
| 0.5 | 1.212 | 0.1104 | 1.835 |
| 0.25 | 1.212 | 0.1016 | 1.19 |
| 0 | 1.213 | 0.09219 | 0.595 |
| -0.25 | 1.213 | 0.0814 | 0.205 |
| -0.5 | 1.213 | 0.06835 | 0.015 |
| -0.75 | 1.213 | 0.0515 | 0 |
| -0.99 | 1.213 | 0.02582 | 0 |
| Truncation of 2 standard deviations on both sides of mean | | | |
| 0.99 | 1.208 | 0.1139 | 2.495 |
| 0.75 | 1.207 | 0.1021 | 1.05 |
| 0.5 | 1.207 | 0.09482 | 0.51 |
| 0.25 | 1.207 | 0.08849 | 0.37 |
| 0 | 1.208 | 0.08157 | 0.155 |
| -0.25 | 1.208 | 0.07347 | 0.06 |
| -0.5 | 1.209 | 0.06287 | 0 |
| -0.75 | 1.21 | 0.0479 | 0 |
| -0.99 | 1.211 | 0.02182 | 0 |

Table 3. Horizontal and vertical seismic accelerations assumed to be gamma distributed with C.O.V. of 10%.

| Correlation Coefficient | Mean Factor of Safety | Factor of Safety Standard Deviation | Probability of Failure (%) |
|---|---|---|---|
| Truncation of 5 standard deviations from both sides of mean | | | |
| 0.98 | 1.02 | 0.01547 | 9.805 |
| 0.75 | 1.02 | 0.0149 | 9.185 |
| 0.5 | 1.02 | 0.01438 | 8.3 |
| 0.25 | 1.02 | 0.01375 | 7.35 |
| 0 | 1.02 | 0.01316 | 6.495 |
| -0.25 | 1.02 | 0.01243 | 5.68 |
| -0.5 | 1.019 | 0.01173 | 4.69 |
| -0.75 | 1.019 | 0.01104 | 3.985 |
| -0.98 | 1.019 | 0.01028 | 3.145 |
| Truncation of 3 standard deviations from both sides of mean | | | |
| 0.98 | 1.02 | 0.0152 | 9.52 |
| 0.75 | 1.02 | 0.01469 | 8.855 |
| 0.5 | 1.02 | 0.01413 | 7.89 |
| 0.25 | 1.02 | 0.0135 | 7.005 |
| 0 | 1.02 | 0.01299 | 6.33 |
| -0.25 | 1.02 | 0.01226 | 5.445 |
| -0.5 | 1.02 | 0.01157 | 4.46 |
| -0.75 | 1.019 | 0.01089 | 3.76 |
| -0.98 | 1.019 | 0.01006 | 2.885 |
| Truncation of 2 standard deviations from both sides of mean | | | |
| 0.98 | 1.02 | 0.01356 | 7.045 |
| 0.75 | 1.02 | 0.01288 | 5.63 |
| 0.5 | 1.02 | 0.0125 | 4.94 |
| 0.25 | 1.02 | 0.01208 | 4.4 |
| 0 | 1.02 | 0.01173 | 3.84 |
| -0.25 | 1.02 | 0.01113 | 3.085 |
| -0.5 | 1.02 | 0.01054 | 2.235 |
| -0.75 | 1.02 | 0.0098 | 1.325 |
| -0.98 | 1.019 | 0.00902 | 0.415 |

Table 4. Results for non-symmetric truncation. Cohesion and friction angle assumed to be normally distributed with C.O.V. of 10%.

| Correlation Coefficient | Mean Factor of Safety | Factor of Safety Standard Deviation | Probability of Failure (%) |
|---|---|---|---|
| 5 standard deviations on left side of mean and 2 standard deviations on right side. | | | |
| 1 | 1.2215 | 0.1233 | 2.52 |
| 0.75 | 1.224 | 0.1136 | 0.89 |
| 0.5 | 1.224 | 0.10566 | 0.51 |
| 0.25 | 1.223 | 0.09755 | 0.285 |
| 0 | 1.22 | 0.08906 | 0.21 |
| -0.25 | 1.22 | 0.07907 | 0.085 |
| -0.5 | 1.217 | 0.06688 | 0 |
| -0.75 | 1.215 | 0.05045 | 0 |
| -1 | 1.213 | 0.0215 | 0 |
| 2 standard deviations on left side of mean and 5 standard deviations on right side. | | | |
| 1 | 1.207 | 0.122 | 4.805 |
| 0.75 | 1.204 | 0.1118 | 3.84 |
| 0.5 | 1.204 | 0.104 | 2.895 |
| 0.25 | 1.205 | 0.09617 | 1.925 |
| 0 | 1.206 | 0.08791 | 1.09 |
| -0.25 | 1.208 | 0.07768 | 0.445 |
| -0.5 | 1.21 | 0.06586 | 0.12 |
| -0.75 | 1.211 | 0.04975 | 0 |
| -1 | 1.213 | 0.02151 | 0 |