The Effect of Rigid Body Impact Mechanics on Tangential Coefficient of Restitution

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ABSTRACT
In current practices of rockfall simulations, Lumped Mass Analysis (LMA) is being used extensively. For a lumped mass model, the normal coefficient of restitution (R_n) and tangential coefficient of frictional resistance (R_t) are required as input parameters. R_n is classical parameter known as material property that is determined by rigidity of the contacting slope. R_t is an experimental parameter that is measured by vegetation and slope material. The range of suggested values for R_t is relatively large compared to the range of suggested values for R_n. For instance, the suggested R_n values for talus and firm soil slope range from 0.1 to 0.2 while the suggested R_t values range from 0.5 to 1.0. It is uncertain in some situations what R_t value should be used for materials with this wide range of suggested values when simulating rockfall in LMA. In this paper, we show that the empirical parameter R_t can be obtained through the Rigid Body Impact Mechanics (RBIM) using only the material parameters R_n and dynamic friction coefficient (μ). In RBIM, we introduce the effect of size and shape of the rock and its interaction with the slope to calculate rockfall trajectory and obtain R_t as a result. The R_t results obtained in RBIM is applied to LMA for comparison. The study shows that LMA can be modelled using RBIM. Also, it expands the analysis to expand the domain of applicability to wide range of situations with various dynamic conditions. The theoretical R_t values obtained using the RBIM are tabulated against the empirical values used in practice for LMA in various conditions.

RÉSUMÉ
Les simulations de chutes de blocs sont actuellement basées sur le modèle de masse localisée (LMA). Les données d'entrée nécessaires pour l'analyse sont le coefficient normal de restitution (R_n) et le coefficient tangentiel de résistance à la friction (R_t). R_n est un paramètre classique qui est déterminé par la rigidité de la pente au contact du bloc et qui est donc généralement dicté par les propriétés du matériau rencontré lors de la chute. R_t est un paramètre expérimental qui est établi selon le type de végétation et de matériaux de la pente le long du parcours. La gamme de valeurs proposées pour R_t est relativement étendue par rapport celle proposée pour R_n. Par exemple, la valeur de R_n proposée pour les talus et les sols denses d'une pente varie de 0,1 à 0,2, alors que la valeur de R_t proposée varie de 0,5 à 1,0. Dans le cadre d'une analyse de la masse localisée (LMA), un questionnement peut survenir lors de la sélection de la valeur appropriée de R_t à utiliser pour les matériaux lors de la simulation des chutes de blocs Cet article présente comment obtenir le paramètre empirique R_t par la méthode «Rigid Body Impact Mechanics» (RBIM) en utilisant uniquement les paramètres de matériaux de R_n et le coefficient de friction (μ). La méthode «Rigid Body Impact Mechanics» (RBIM), permet d'introduire l'effet de la taille et la forme du bloc, ainsi que l'interaction du bloc avec la pente, afin de déterminer des valeurs de R_t. Les valeurs obtenues sont utilisées dans le modèle de masse localisée (LMA), et l'essai est concluant. Les résultats obtenus corroborent non seulement les valeurs empiriques utilisées dans la pratique courante, mais permettent d'élargir le domaine d'applicabilité à différentes conditions dynamiques. Les valeurs théoriques obtenues à l'aide du RBIM sont mises sous forme de tableau et comparées aux valeurs empiriques utilisées dans la pratique avec le modèle de masse localisée (LMA), pour diverses situations.

1 INTRODUCTION

Rockfall is a natural event that occurs when rocks break away from a slope. It has the potential for enormous damage to human life and infrastructure. Lumped Mass Analysis (LMA) is widely used in many rockfall simulations. LMA considers the rock as dimensionless point mass and calculates rockfall trajectory based on user-input material parameters such as normal coefficient of restitution (R_n) and tangential coefficient of restitution (R_t).

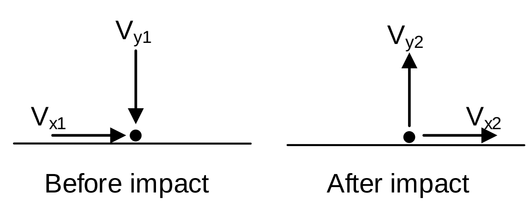

Figure 1. Lumped mass model of rockfall impact.

Figure 1 shows the velocity components of falling rocks before and after the impact. These velocities are related to the coefficients of restitution with simple relationships

$$V_{y2} = R_n V_{y1} \quad [1]$$

$$V_{x2} = R_t V_{x1} \quad [2]$$

The Colorado Rockfall Simulation Program (CRSP) is one of the commonly used rockfall simulation software packages that use LMA. It provides slope material data from experimental records that was conducted near Rifle, Colorado. The CRSP model defines rebound and friction characteristic of the slope as R_n and R_t respectively, where the value of R_n is determined by the elasticity of the slope material and the value of R_t is determined by vegetation cover and surface roughness (Jones et al. 2000).

In the CRSP model, there is relatively large range of suggested coefficient values for R_t compared to R_n. Although CRSP states that the wide range of suggested values for R_t are suitable for covering diverse slope types, extensive calibration is required to determine which values should be used for a user created model. This creates more work for the user and increases possibility for error in which R_t value should be used. Since LMA is limited to dimensionless rocks, any eccentricity of the rock is not taken into consideration. To accommodate this limitation the CRSP model provides additional empirical factors based on observed data, such as friction function and scaling factor.

A two dimensional (2D) Rigid Body Impact Mechanics (RBIM) approach to rockfall simulation is being developed as an alternative to LMA. RBIM takes hypothetical two dimensional rock shapes into account and calculates the impulse that occurs during the rock to slope contact period. Using this impulse, the translational and rotational velocities are calculated at the contact point. This allows RBIM to take eccentric impact configuration into consideration. The two material input parameters are normal coefficient of restitution (R_n) and dynamic friction (μ). Dynamic friction is the tangent of the friction angle, which can be obtained from the experimental data. We can then calculate the apparent R_n (R_n*) and apparent R_t (R_t*), which are R_n and R_t associated with centroid of the rock based on the impact result.

The study aims to investigate the differences between models using a LMA and a RBIM analysis to show that RBIM can capture the same behavior as LMA. Since RBIM does not require R_t as an input parameter it frees the user from extensive calibration required to select the correct R_t value. Furthermore, LMA is limited to spherical shaped rocks with empirical factors for taking rock shape into consideration. We show that this causes underestimation of rockfall runout distance using RBIM. Therefore, using RBIM for rockfall simulation allows for a wider variety of impact configurations to be taken into consideration.

1.1 RBIM Outline

To demonstrate that apparent R_t (R_t*) is the resulting velocity of rockfall trajectory using impact mechanics, we first introduce its general concept applied in rockfall simulation. RBIM are being used in rockfall simulation so that we can realistically model the trajectory of arbitrarily shaped rocks. The RBIM approach first find the impact point where rock-slope contact occurs, calculates the impact impulse, and goes through a coordinate transformation to calculate the outgoing trajectory of the rock at the centroid of the body. Figure 2 illustrates an elliptical shaped rock with the coordinates of outgoing velocities at the contact point as R_n and R_t and velocities at the centroid of the body as apparent R_n and apparent R_t.

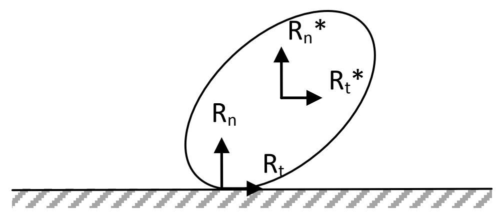

Figure 2. Description of apparent and regular R_n and R_t.

The general principles presented in Impact Mechanics by Stronge (2000) have been applied. RBIM considers impulse reaction of the rock during the instantaneous contact period with the slope to determine the critical events of the rock. These critical events include slip, stick, or reversal behavior during compression and restitution phase of the impulse duration. RBIM uses the normal coefficient of restitution during these two separate phases of the contact period to calculate the terminal impulse. With this terminal impulse, the outgoing velocities at the contact point can be calculated.

In order to verify that RBIM and LMA produce equivalent results, we have reduced the size of the rock in the RBIM model to small scale so that a rock will behave similarly to a point mass. Also, the CRSP model explains that sphere is often used for rock shape in LMA because it yields maximum volume for given radius. Thus, sphere has been used in RBIM model. For a sphere, R_n is the same as R_n* because the ratio of change in velocity at the contact point is the same at the centroid in normal direction to the slope. Then, RBIM model is created with these modification and the results are compared with LMA model. This comparison allows us to validate RBIM is equivalent with projectile motion of LMA.

2 APPLICATION

In order to compare the results of the LMA to the RBIM, we have created a simple model for each analysis methods. The LMA model requires two user-input parameters, R_n and R_t. Conversely, RBIM model requires two parameters R_n and dynamic friction, μ. The initial condition of the simplistic rockfall model is shown in Figure 3 below.

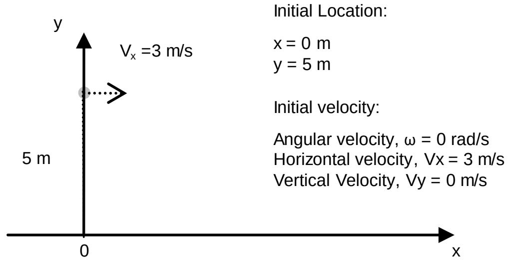

Figure 3. Simple model layout.

Figure 3 shows the geometry layout of the model with the initial conditions. We have set the common parameters, R_n value to be 1.0 and dynamic friction coefficient to be 0.1. This is equivalent to the tangent of Friction angle 5.7 degrees, which is determined to be used for comparison purposes.

2.1 RBIM and LMA

Since we can calculate the R_t* value for each bounce of the rock using RBIM, these R_t* values are used as an user-input parameter, R_t, for the LMA.

The R_t* result in the RBIM model with small spherical rock is 0.714 for first bounce and R_t* is 1.0 for rest of the bounces onwards. The result shows that the first bounce loses its energy in translational velocity and other bounces have no translational energy loss. Although this behavior seems counterintuitive at first, we can use impact mechanics to explain this behavior.

When a rock that has no initial rotational velocity makes contact on a surface with friction, the rock loses some of its translational velocity during the period of contact with the slope and gains rotational energy as a result. Stronge (2000) describes this behavior as slip-stick transition during compression where the rock slips during compression phase, causing it to rotate along the slope. If the initial rotational velocity is small and coefficient of friction is sufficiently large, slip halts during compression phase and the contact points stick to the surface during the remainder of the contact period, which generates the rolling motion at that instantaneous impact as a result. The magnitude of the rotational velocity depends on the condition of the rock after the impact. For this condition, the terminal outgoing tangential velocity after the impact can be expressed in relation to rotational velocity as

$$V_{t2} = \omega_2 * r \qquad [3]$$

Where V_{t2} is the outgoing tangential velocity, r is the radius of the rock and ω is the rotational velocity. Once the rock has gained sufficient rotational velocity to establish this relationship, and the friction coefficient is large enough for stick behavior of subsequent impacts then there is no additional transfer of translation into rotational energy as the ratio in Eq. 3 holds. Based on these results from our RBIM model, the LMA model is created with the same geometry. A R_n value of 1.0, friction angle of 5.7 degrees (dynamic coefficient of 0.1), and a R_t value of 0.714 for the first bounce and 1.0 for subsequent bounces.

3 ANALYSIS RESULTS

The bounce height, total kinetic energy, and rotational kinetic energy results are plotted and compared with these two models as shown in Figure 4 to Figure 6 respectively. The legends in the graph indicate LMA and RBIM results as shown in the graph.

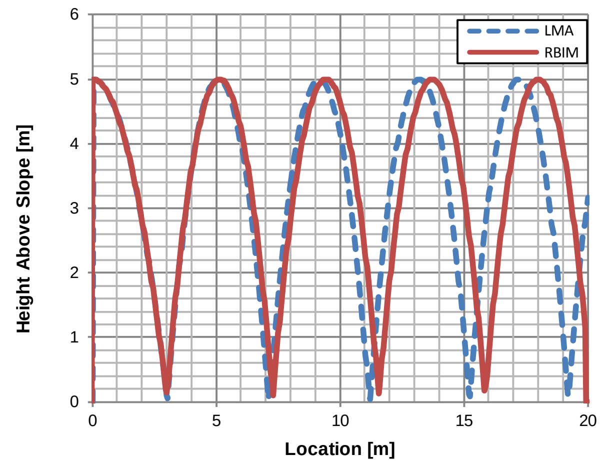

Figure 4. Bounce height envelope versus rock path location.

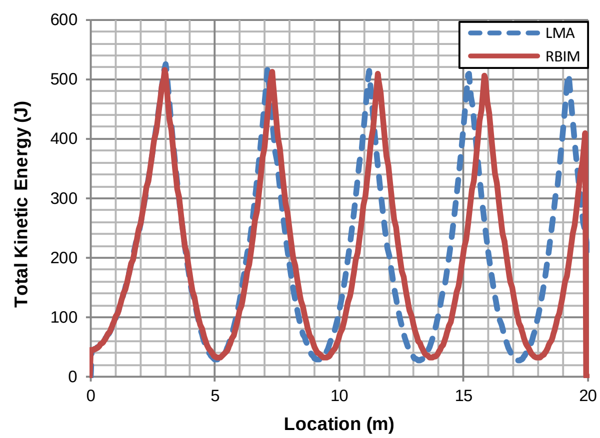

Figure 5. Total kinetic energy versus rock path location.

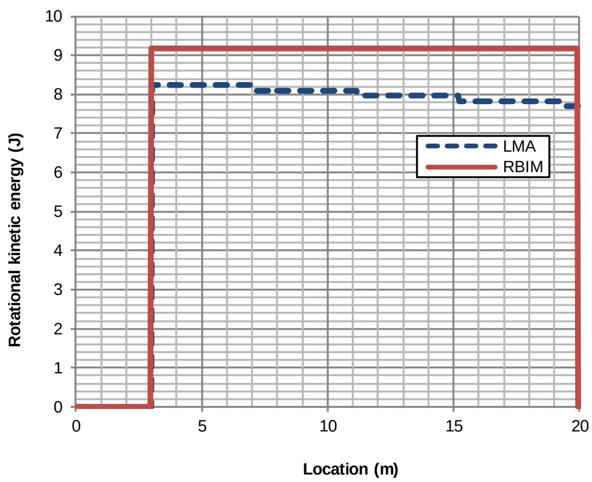

Figure 6. Rotational kinetic energy versus rock path location.

From the graphs above, we can see that although the two models have the same initial conditions, the difference in the results between the two models seem to grow on each bounce. The difference grows with increase in rock path location as shown in the graphs. This is due to the empirical factors that are applied to the LMA on all bounces of the rock to account for the rock shape. Thus, this causes rock to travel less distance and lose more energy as it bounces on the slope as shown in Figure 4 and Figure 5 respectively. The rotational kinetic energy in LMA and RBIM is significantly different for the first bounce as shown in Figure 6. This difference is investigated using the energy balance equations where the conservation of energy was checked for the first bounce of the rock to analyze how the LMA and the RBIM model calculate their outgoing trajectory.

3.1 Energy balance equation for the RBIM

The general energy balance equation in tangential velocity can be written as

$$\frac{1}{2}mv_{x1}^2 + \frac{1}{2}I\omega_1^2 = \frac{1}{2}mv_{x2}^2 + \frac{1}{2}I\omega_2^2 + Friction\ loss \quad [4]$$

Where v_{x1}, ω_1 are the initial tangential and rotational velocity respectively, v_{x2} and ω_2 are the final tangential and rotational velocity respectively.

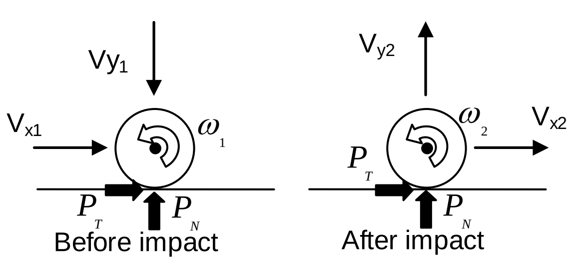

Figure 7. Rigid body mechanics of the new algorithm.

Figure 7 shows the components of velocities before and after the impact. P_t and P_n are the forces at the contact region. The contact point lies at the periphery away from the centroid of the rigid body.

From the RBIM model with spherical rock, the moment of inertia, initial and final rotational velocity can be written as

$$I = \frac{2}{5}mr^2, \omega_1 = 0, \omega_2 = \frac{v_{x2}}{R} \quad [5]$$

The friction energy loss is expressed in ratio of moment of inertia to kinetic energy. The general energy balance equation can be shown as

$$\frac{1}{2}mv_{x1}^2 = \frac{1}{2}mv_{x2}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right) * \frac{v_{x2}^2}{r^2} + \frac{2}{7}\left[\frac{1}{2}mv_{x1}^2\right] \quad [6]$$

$$v_{x1}^2 = v_{x2}^2 + \left(\frac{2}{5}\right) * v_{x2}^2 + \frac{2}{7}[v_{x1}^2]$$

$$\left(\frac{25}{49}\right) v_{x1}^2 = v_{x2}^2$$

$$0.714 * v_{x1} = v_{x2} \quad [7]$$

We get an apparent R_t value of 0.714 with a spherical rock, which is the fractional change in tangential velocity from before to after the impact. We can get the tangential outgoing velocity (V_{x2}) as the following with initial tangential velocity of 3m/s.

$$0.714 * 3\ m/s = v_{x2}$$

$$v_{x2} = 2.142\ m/s$$

Taking this outgoing tangential velocity, we can calculate rotational kinetic energy with energy balance equation to confirm rotational kinetic energy calculated from the RBIM.

$$Rotational\ kinetic\ energy = \frac{1}{2}I\omega_2^2 \quad [8]$$

$$Rotational\ kinetic\ energy = \frac{1}{2}\left(\frac{2}{5}mr^2\right) * \frac{v_{x2}^2}{r^2}$$

$$= \frac{1}{5}(10)(2.142)^2$$

$$= 9.176\ J$$

Where mass of the rock, *m*, of 10 kilograms has been used in the model. The rotational kinetic energy from energy balance equation is very close to the results we've obtained from the RBIM model which is 9.184J, as shown in Figure 6.

3.2 Energy balance equation for LMA

The LMA model uses two empirical equations from the CRSP model. These equations are derived from experimental data observed in the field and are applied to account for shape and size of the rock. The empirical equations are shown below

$$f(F) = \text{friction function} \quad [9]$$

$$f(F) = R_t + \frac{1 - R_t}{\left(\frac{V_{t1} - \omega_1 R}{20}\right)^2 + 1.2}$$

$$SF = \text{scaling factor} \quad [10]$$

$$SF = \frac{R_t}{\left(\frac{V_{n1}}{250 R_n}\right)^2 + 1}$$

Where V_{t1} is the initial tangential velocity and V_{n1} is the initial normal velocity. Solving Eq. 9 and Eq. 10, we get f(F) = 0.9123 and SF = 0.7024. LMA does not take theoretical rotational energy loss during impact into account but applies these empirical factors to calculate the outgoing velocity. With these empirical factors, the energy balance equation is shown as

$$\left[\frac{1}{2} m {v_{x1}}^2 + \frac{1}{2} I {\omega_1}^2\right] * f(F) * SF = \frac{1}{2} m {v_{x2}}^2 + \frac{1}{2} I {\omega_2}^2 \quad [11]$$

$$\left(\frac{1}{2} m {v_{x1}}^2\right) * (0.9123) * (0.7024) = \frac{1}{2} m {v_{x2}}^2 + \frac{1}{2} * \left(\frac{2}{5} m r^2\right) * \frac{{v_{x2}}^2}{r^2}$$

$$\left(\frac{5}{7}\right) ({v_{x1}}^2) * (0.6408) = {v_{x2}}^2$$

$$v_{x2} = 2.03\ m/s$$

The configuration for velocity components are the same as shown in Figure 7. A combined empirical factor of 0.6408 is applied to energy balance equation instead of 0.714 which is the theoretical R_t* calculated from the RBIM approach. If this combined empirical factor was applied in the RBIM method, then the lumped mass model and the RBIM model would have the identical results. Using the calculated outgoing tangential velocity to calculate rotational kinetic energy we can see that

$$\text{Rotational kinetic energy} = \frac{1}{2} I {\omega_2}^2$$

$$= \frac{1}{2}\left(\frac{2}{5} m r^2\right) * \frac{{v_{x2}}^2}{r^2}$$

$$= 8.24\ J$$

Figure 6 shows that LMA has rotational kinetic energy of 8.236J at the first bounce of the rock, which is at 3 meters of rock path location. The rotational kinetic energy result calculated using energy balance equation is similar to what we have obtained in LMA. Based on the energy balance equation, it is interesting to observe that the only difference between the LMA and the RBIM**,** when both methods are simulated as point mass analysis, is the empirical factors that are applied to initial balance equation in LMA. Since LMA applies same empirical factor for all bounces of the rock, this makes each bounce of the rock a shorter distance than the RBIM result. As discussed earlier, we have provided reasons behind how the rock gains rotational velocity using impact theory. There may be cases where tangential velocity is unaffected by the contact with the slope due to the correct ratio of translation to rotational velocity and a sufficiently large friction coefficient. Applying the empirical factors built into LMA to this situation doesn't take the impact mechanics into account and might cause an underestimation in rock trajectory in the results.

3.3 Empirical R_t values and Theoretical R_t results

The Table 1 shows the original suggested tangential coefficient input values from the CRSP manual for LMA.

Table 1. Suggested Tangential Coefficient Input Values (Jones et al. 2000)

Description of Slope	Tangential Coefficient (R_t)
Smooth hard surfaces and paving	0.90 - 1.0
Most bedrock and boulder fields	0.75 - 0.95
Talus and firm soil slopes	0.65 - 0.95
Soft soil slopes*	0.5 - 0.80

*Soft soil slope coefficients were extrapolated from other slope types due to lack of data.

These values are based on observed data from a site near Rifle, Colorado. The CRSP manual suggests users to calibrate to their specific site from much wider range of tangential coefficient (0.50 – 1.0) to take into account the wide variety of slope types that are possible. Table 2 shows the results of apparent R_t (R_t*) values obtained for various shapes of the rock in RBIM.

Table 2. Apparent R_t results for rocks with various shapes in the RBIM

Shape of the rock	Minimum tangential Coefficient (R_t)	Maximum tangential Coefficient (R_t)	Average Tangential Coefficient (R_t)
Sphere	-	-	0.714
Circle	-	-	0.667
Rhombus*	0.629	1.337	0.929
Hexagon*	0.670	1.169	0.987
Oval*	0.624	1.325	0.987

*The R_t results are obtained from one thousand rocks thrown at arbitrary impact angles.

It is important to note that these results are obtained from rock-slope impact condition where the rock experiences slip-stick transition during compression as discussed in Eq. 3. The R_t values for the non-circular shapes are obtained from throwing one thousand rocks with arbitrary impact angles. An illustration of the impact angle for ellipse is shown in Figure 8. In Figure 8, *a* is the radius along major axis; *b* is the radius along the minor axis; θ indicates the impact angle.

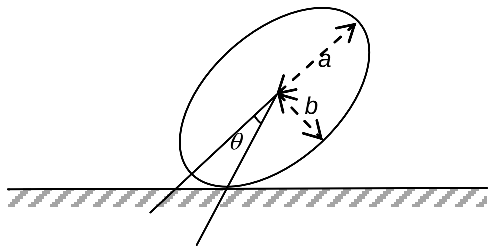

Figure 8. Position of the rock body just before the impact.

The apparent tangential coefficient R_t* varies depending on the impact angle. In Table 2, the RBIM provides users with range of R_t values for various shapes of the rock with arbitrary impact angles. The RBIM has shown that apparent R_t (R_t*) can be calculated from two parameters R_n and µ. Through the energy balance equation we are able to verify that RBIM has equivalent result to LMA if the same empirical factors are applied to the RBIM.

4 DISCUSSION AND RECOMMENDATIONS

(i) We have used simple geometry to compare the impact mechanics results with LMA. The aim of the paper is to show that R_t* is the result of rockfall trajectory based on dynamics of the rock mechanics with rock-slope impact.

(ii) We are able to show that RBIM can be used to model LMA with small rock size. Two different user input variables were required in RBIM, normal coefficient of restitution (R_n) and dynamic friction coefficient (µ).

(iii) The model results show that LMA and RBIM have slightly different results. Through the energy balance equation and impact mechanics, we found that LMA applies empirical factors for all rock impacts and that RBIM does not apply the same empirical factors.

(iv) If RBIM takes consideration of the same empirical factors that are applied in LMA, both of the methods have the matching results.

(v) The RBIM analysis has the capacity to analyze rocks with different moment of inertia, whereas LMA was limited to spherical rocks with empirical factors. These empirical factors sometimes cannot capture some of the theoretical behavior of the rocks with impact mechanics. The empirical R_t values and theoretical apparent R_t results are tabulated in Table 1 and 2.

(vi) This comparison between RBIM and LMA builds credibility on the RBIM for its ability to simulate point mass analysis, as well as potential to expand the domain of rockfall simulation to variety of dynamic situations with non-circular shaped rocks.

(vii) Further study of how to use probabilistic RBIM analysis with a variety of shapes to match the original empirical data that the CRSP LMA is based is an important next step.

(viii) The appropriate empirical factors need to be applied to RBIM in the future to account for the field data. It should also match with the LMA result for the point mass scale analysis.

5 ACKNOWLEDGEMENT

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