Stability Analysis of Rock Slopes using the Finite Element Method

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ABSTRACT: This paper examines the application of Finite Element (FE) analysis to the determination of the factor of safety of rock slopes for which strength is modelled by the Generalized Hoek-Brown failure criterion. The paper first outlines the standard shear strength reduction (SSR) technique used for FE slope stability analysis of Mohr-Coulomb slopes. It then examines the difficulties of straightforward application of the method to Generalized Hoek-Brown slopes, and suggests a solution approach that uses equivalent Mohr-Coulomb envelopes in place of the Hoek-Brown failure criterion.

1 INTRODUCTION

In the vast majority of slope stability analysis cases, geotechnical engineers primarily conduct design based on calculated factor of safety values. The factor of safety is a very useful index for determining how close or far away a slope is from failure, and its accuracy is consistent with the accuracy of geotechnical input data.

Due to its power and flexibility, the Finite Element Method (FEM) is increasingly being applied to slope stability analysis. Non-linear finite element models using elastic-perfectly plastic material strength formulations have been used to determine factors of safety. The FEM offers a number of advantages over traditional method-of-slices analysis (Griffiths & Lane, 1999) including:

1. Elimination of a priori assumptions on the shape and location of failure surfaces
2. Elimination of assumptions regarding the inclinations and locations of interslice forces
3. Capability to model progressive failure
4. Calculation of deformations at slope stress levels, and
5. Robustness – ability to perform successfully under a wide range of conditions.

The factor of safety, F, in FE slope stability analysis (Duncan, 1996) is defined as

$$F = \frac{Shear\ strength\ of\ material\ (soil\ or\ rock)}{Shear\ strength\ required\ for\ equilibrium} = \frac{t}{t^*} \tag{1}$$

the same as that for traditional limit-equilibrium analysis.

2 THE SHEAR STRENGTH REDUCTION (SSR) APPROACH FOR FE CALCULATIONS OF SLOPE FACTOR OF SAFETY

The Shear Strength Reduction (SSR) technique of Finite Element (FE) slope stability analysis is a simple approach that involves a systematic search for a stress reduction factor (SRF) or factor of safety value that brings a slope to the very limits of failure.

The SSR technique described in literature assumes Mohr-Coulomb strength for slope materials. The Mohr-Coulomb strength envelope is the most widely applied failure criterion in geotechnical engineering. A unique feature of this linear failure model is the fact that it can be simply and explicitly expressed in both principal (s_1 - s_3) stress space and shear-normal (t - s_n) stress space. The simplicity, explicit representation in both principal and shear-normal stress space, adequate description of strength behaviour for a wide range of materials, and easy-to-obtain parameters of the Mohr-Coulomb criterion account for its popularity.

For Mohr-Coulomb material the factored or reduced shear strength can be determined from the equation

$$\frac{t}{F} = \frac{c'}{F} + \frac{\tan f'}{F}. \tag{2}$$

This equation can be re-written as

$$\frac{t}{F} = c^* + \tan f^*, \tag{3}$$

where $c^* = \frac{c'}{F}$ and $f^* = \arctan\left(\frac{\tan f'}{F}\right)$ are factored Mohr-Coulomb shear strength parameters.

The steps for systematically searching for the critical factor of safety value F that brings a previously stable slope ($F \geq 1$) to the verge of failure are as follow:

Step 1: Develop an FE model of a slope, using the appropriate material deformation and strength properties. Compute the model and record the maximum total deformation.

Step 2: Increase the value of F (or SRF) and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total deformation.

Step 3: Repeat Step 2, using systematic increments of F, until the FE model does not converge to a solution, i.e. continue to reduce material strength until the slope fails. The critical F value just beyond which failure occurs will be the slope factor of safety.

For a slope with a factor of safety less than 1, the procedure is the same except fractional F values will be systematically decremented (translating into increments in the factored strength parameters) until the slope becomes stable.

The principal advantage of the SSR technique is its use of factored strength parameters as input into models, which enable the technique to be used with any existing FE analysis software. All the approach requires of a slope analyst is computation of factored Mohr-Coulomb strength parameters.

3 FE SLOPE STABILITY FOR GENERALIZED HOEK-BROWN MATERIALS

The non-linear Generalized Hoek-Brown criterion is the most suitable strength model for predicting the failure of rock masses, especially in low normal stress ranges. Two of the most significant benefits of the criterion are the links between its parameters and simple geological field observations, and good agreement between its predictions and observed rock strength behaviour. These endow the criterion with great practical value.

The Generalized Hoek-Brown criterion (Hoek, Carranza-Torres, Corkum, 2002) explicitly expresses strength in terms of major and minor principal stresses through the equation

$$s_1' = s_3' + s_{ci}\left(m_b \frac{s_3'}{s_{ci}} + s\right)^a, \tag{4}$$

where s_{ci} is the uniaxial compressive strength of the intact rock material, while

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right),\ s = \exp\left(\frac{GSI - 100}{9 - 3D}\right), \text{ and } a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right).$$

The Geological Strength Index, *GSI*, relates the failure criterion to geological observations in the field. m_i is a material constant for intact rock, while m_b is a reduced value of this constant. s and a are rock mass constants. D is a factor that characterizes the degree to which blasting and stress relaxation have disturbed a rock mass.

Using equations derived by Balmer (1952), it is possible to calculate shear and normal stresses corresponding to points along a Generalized Hoek-Brown. The relationships are as follow [3]:

$$t = \left(s_1' - s_3'\right)\frac{k}{k^2 + 1} \tag{5}$$

$$s_n' = \frac{s_1' + s_3'}{2} - \frac{s_1' - s_3'}{2}\frac{k^2 - 1}{k^2 + 1} \tag{6}$$

where $k^2 = \frac{ds_1'}{ds_3'} = 1 + am_b\left(m_b\frac{s_3'}{s_{ci}} + s\right)^{a-1}$ (the slope of the Generalized Hoek-Brown envelope).

Since the Generalized Hoek-Brown criterion is so useful to rock engineering, a principal objective of the authors' research project was to investigate the possibilities of applying the SSR FE technique to this strength model. To use the SSR technique with this criterion in a manner similar to that done with Mohr-Coulomb strength so that engineers can use existing FE analysis software, it would be necessary to determine factored Hoek-Brown strength parameters.

Using the shear stress equation (5), the factored strength parameters must be obtained from the solution of the following:

$$\frac{t'}{F} = \left(s_1' - s_3'\right)\frac{\sqrt{1 + am_b\left(m_b\frac{s_3'}{s_{ci}} + s\right)^{a-1}}}{2 + am_b\left(m_b\frac{s_3'}{s_{ci}} + s\right)^{a-1}} \times \frac{1}{F} = \left(s_1' - s_3'\right)\frac{\sqrt{1 + am_b^*\left(m_b^*\frac{s_3'}{s_{ci}^*} + s^*\right)^{a-1}}}{2 + am_b^*\left(m_b^*\frac{s_3'}{s_{ci}^*} + s^*\right)^{a-1}} \tag{7}$$

Finding closed-form equations for the factored Hoek-Brown parameters s_{ci}^*, m_b^*, s^* and a^* might be impossible, or at best very challenging. It is possible to determine approximate values of the factored parameters using a numerical scheme, but this will considerably slow down computations. This compounded by the fact that elasto-plastic FE computations using Generalized Hoek-Brown strength are much slower than those for Mohr-Coulomb strength, presently excludes or limits the application of the SSR technique for Generalized Hoek-Brown materials to routine analysis.

4 A SIMPLIFIED APPROACH USING EQUIVALENT MOHR-COULOMB STRENGTH ENVELOPES

To overcome the challenges of performing FE slope stability analysis for rock slopes, the paper proposes an approach that is simple, practical, and accurate. For a specified Generalized Hoek-Brown failure envelope, the approach involves calculation of a Mohr-Coulomb envelope equivalent to the Hoek-Brown model over the slope working range of stresses.

4.1 *Determination of Equivalent Mohr-Coulomb Parameters*

Hoek, Carranza-Torres & Corkum (2002) provide a method for calculating Mohr-Coulomb cohesive strength and friction angle equivalent to a Generalized Hoek-Brown failure envelope. They define the equivalent Mohr-Coulomb criterion as that, which over a specified stress interval, minimizes the area between linear model and the Hoek-Brown curve. Geometrically this involves fitting a Mohr-Coulomb that makes the sum of positive areas (areas above the Mohr-Coulomb line) equal the sum of negative areas (areas below the line). (See Figure 1 below.)

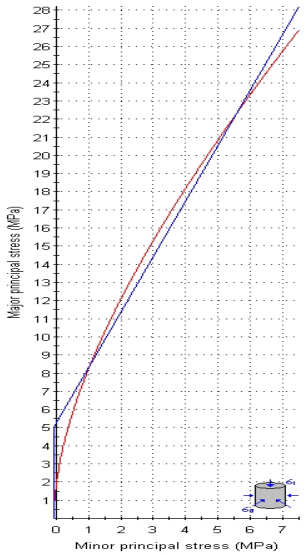

Figure 1. Definition of Mohr-Coulomb strength equivalent to a Generalized Hoek-Brown envelope over a stress interval. (The image was captured from RocData, a program for geotechnical strength data analysis developed by Rocscience Inc.)

The resulting formulas for calculating the equivalent Mohr-Coulomb parameters are

$$\varphi' = \sin^{-1}\left[\frac{6am_b(s + m_b\sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma'_{3n})^{a-1}}\right], \text{ and}$$

$$c' = \frac{\sigma_{ci}\left[(1+2a)s + (1-a)m_b\sigma'_{3n}\right](s + m_b\sigma'_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1 + \left(6am_b(s + m_b\sigma'_{3n})^{a-1}\right)/(1+a)(2+a)}} \tag{8}$$

where $\sigma'_{3n} = \frac{\sigma'_{3\max}}{\sigma_{ci}}$. The fitting procedure occurs over a stress range from σ_t, the tensile strength to the maximum compressive stress $\sigma'_{3\max}$ in the slope.

The maximum compressive strength is calculated from the equation

$$\sigma'_{3\max} = 0.72\sigma'_{cm}\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{0.91},$$

where γ is the rock mass unit weight, H is the slope height, and σ'_{cm} is the global "rock mass strength". It is calculated from the formula

$$\sigma'_{cm} = \sigma_{ci}\frac{(m_b + 4s - a(m_b - 8s))(m_b/4 + s)^{a-1}}{2(1+a)(2+a)}.$$

Once the parameters of the equivalent Mohr-Coulomb failure criterion are obtained, the SSR FE technique described above is applied in straightforward fashion.

5 AN EXAMPLE

We illustrate the merits of the suggested approach on an example of a 10 m homogeneous rock slope with a 35.5° slope angle (to the horizontal). The Generalized Hoek-Brown parameters of the slope rock mass are provided in Table 1.

The slope, which has a height of 10 m, is assumed loaded under gravity with a horizontal to vertical stress ratio of 1. Using *RocData* (2003), a program for geotechnical strength analysis developed by Rocscience that implements the approach for establishing equivalent Mohr-Coulomb

parameters, we obtain a maximum slope stress s'_{3max} value of 0.189 MPa, a cohesion value of 0.02 MPa and a friction angle of 20.89 degrees.

Table 1. Properties of the Slope Rock Mass

Property	**Value**	**Property**	**Value**
Young's modulus, E (MPa)	5000	Disturbance factor, *D*	0
Poisson's ratio, ν	0.3	Parameter m_b	0.067
Weight, γ (MN/m^3)	0.025	Parameter *s*	2.5 x 10^{-5}
Uniaxial compressive strength σ_{ci} (MPa)	30	Parameter *a*	0.619
GSI	5	Dilatancy angle, ψ (deg)	0
Intact rock parameter m_i	2		

Table 2. Maximum Total Displacements

#	**Strength Reduction Factor (*SRF*)**	**Maximum Total Displacement (m) x 10^{-4}**
1	1.0	3.919
2	1.05	4.062
3	1.10	4.219
4	1.15	4.438
5	1.20	12.41

Next we apply the SSR FE technique for Mohr-Coulomb material to determine the factor of safety of the slope. *Phase*[2] (2002), a Rocscience FE program, was used for the analysis. Table 2 below provides the sequence of shear strength reduction values, corresponding factored Mohr-Coulomb parameters, and resulting maximum total displacements of the slope that helped establish the slope's factor of safety. At a strength factor value of 1.2, deformations significantly increase and indicate failure. The last *SRF* value of 1.15 at which the slope is stable is therefore assumed to be the factor of safety.

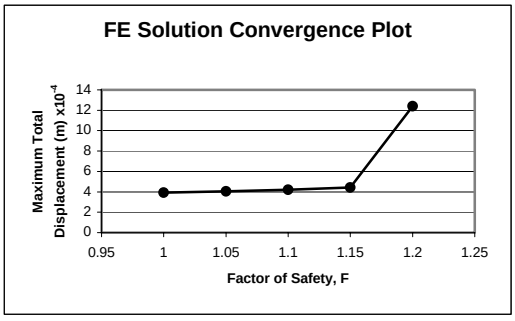

Figure 3. Plot of strength reduction factor against slope maximum total displacement.

Figure 4 below shows the contours of maximum shear strain in the slope. The deformation pattern exhibited on the figures is consistent with that predicted by traditional limit-equilibrium analysis – a circular failure surface passing through the slope toe.

The results of the FE analysis were compared to answers obtained from the Bishop and Spencer limit-equilibrium methods (Table 3) computed in *Slide* (2002), a slope stability program developed by Rocscience. For each limit-equilibrium method two cases were considered: one that modelled material strength with the Generalized Hoek-Brown criterion, and a second that used the equivalent Mohr-Coulomb parameters. The FE factor of safety result agreed very well with those produced by the limit-equilibrium methods.

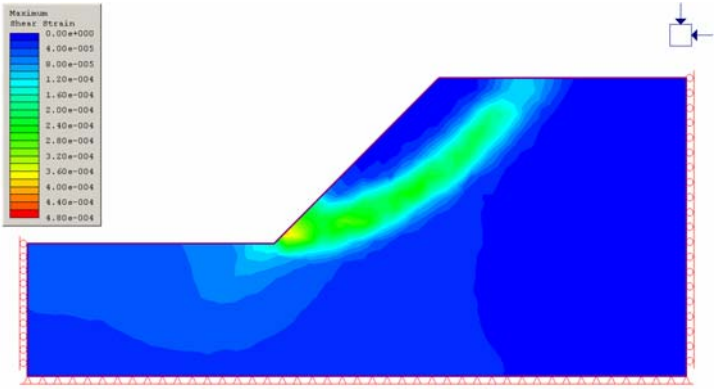

Figure 4. Contours of maximum shear strain in slope.

Table 3. Factor of Safety Results from FE and Limit-Equilibrium Analysis

#	**Method**	**Strength Model**	**Factor of Safety**
1	FE SSR technique	Equivalent Mohr-Coulomb	1.15
2	Bishop's simplified method	Generalized Hoek-Brown	1.153
3	Spencer's method	Generalized Hoek-Brown	1.152
4	Bishop's simplified method	Equivalent Mohr-Coulomb	1.158
5	Spencer's method	Equivalent Mohr-Coulomb	1.154

6 CONCLUSION

The paper presents a simple approach for performing the FE SSR analysis of rock slopes for which strength is modelled with the Generalized Hoek-Brown failure criterion. The paper shows that determining factored Hoek-Brown parameters, as is done for the Mohr-Coulomb, would be at best cumbersome and would slow down computations considerably. It suggests an approximate method that involves first determining a Mohr-Coulomb envelope equivalent to a Hoek-Brown model, and then applying the resulting equivalent cohesion and friction angle values in the standard SSR technique.

It may be possible, however, to directly incorporate the SSR technique for Generalized Hoek-Brown strength into a FE computational engine, since in that case there will be no need to explicitly determine factored Generalized Hoek-Brown parameters.

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