Serviceability-Based Slope Factor of Safety using the Shear Strength Reduction (SSR) Method

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ABSTRACT: This paper will apply the Shear Strength Reduction (SSR) method to the determination of slope factor of safety based not only on collapse, but also on exceedance of some serviceability limit. There are several instances in which serviceability issues, such as excessive movements or rotations are critical. The paper will discuss how the Finite Element SSR method can be used to calculate factors of safety against serviceability limits.

1 INTRODUCTION

This paper intends to demonstrate that the Finite Element-based Shear Strength Reduction method (henceforth referred to as the FE-SSR), which has been used to determine safety margins against slope collapse, can be extended to the determination of serviceability limits as well.

Geotechnical excavations and structures, such as retaining walls, bridge abutments, rail embankments, and caverns, are often required to not only avoid ultimate collapse, but to also satisfy serviceability limits. Serviceability limits are performance criteria such as displacements, deflections, rotations, etc. that must be met to guarantee that excavations and structures carry out their intended tasks. For example, for mine open pits and in landslide studies, significant emphasis is placed on slope displacements (Lorig & Varona 2003).

The paper will discuss how the capability for serviceability analysis of the FE-SSR arises from the fact that the Finite Element Method (FEM) satisfies equilibrium and compatibility conditions. Through an example, the paper will demonstrate practical use of FE-SSR to determine the margin of safety against a serviceability failure. The exercise will also highlight one of the primary advantages of the FE-SSR method over conventional limit equilibrium slope stability methods.

2 FE-SSR AND FULL SATISFACTION OF SOLUTION CONSTRAINTS

The Shear Strength Reduction (SSR) method (Dawson et al 1999, Griffith & Lane 1999, Hammah et al 2006, 2005 a, b, 2004, Matsui & San 1992) is a technique that enables the factor of safety of slopes to be calculated with numerical methods such as the FEM and Finite Difference Method FDM). In conventional SSR analysis, a factor of safety is obtained by systematically reducing (or increasing) the shear strengths of slope materials until the slope is brought to the very limit of failure.

It has been demonstrated that the FE-SSR method works for a wide range of problems, including stability problems in blocky rock masses (Hammah et al 2007).

2.1 *FEM Satisfaction of Constraints of Stress-Strain Analysis*

In order to solve for non-arbitrary distributions of stresses and strains induced in continua as a result of loading and/or excavation, the following conditions have to be satisfied (Brady & Brown 1993):

1. Boundary conditions
2. Differential equations governing static stress equilibrium
3. Constitutive equations that relate stresses to strains for the different materials, and
4. Equations regarding strain compatibility.

The FEM, the most widely used numerical analysis method, ensures satisfaction of all the above-outlined constraints. Among the many reasons that account for method's popularity are the abilities to:

1. Handle multiple materials in a single model (material heterogeneity)
2. Readily accommodate non-linear constitutive behaviours, and
3. Model complex boundary conditions.

As a result of the FEM satisfying all the constraints that have to be met in the solution of stress-strain problems, the FE-SSR enjoys several advantages over conventional limit equilibrium methods of slope stability analysis. These advantages include (Griffiths & Lane, 1999):

1. Elimination of *a priori* assumptions on the nature (shape and location of failure surfaces) of failure mechanisms
2. Elimination of assumptions on the inclinations and locations of interslice forces
3. Capability to model progressive failure

4. Calculation of deformations at slope stress levels, and
5. Robustness – ability to perform successfully under a wide range of conditions.

The example problem, described in a later section of this paper, highlights one of the principal strengths of SSR analysis: the ability to account for stress-strain behaviour and different material stiffnesses. These factors are important particularly in problems in which the contrasts in material stiffnesses and behaviour are large.

2.2 *Determination of Safety Margins against Ultimate Collapse*

The solution of an FEM model is stable when all equilibrium conditions are satisfied, and unstable or non-convergent otherwise. As a result, in the FE-SSR method the convergence of the FEM solution is used as the criterion for determining the onset of ultimate slope failure.

Non-convergence within suitably specified number of iterations and tolerance is an appropriate indicator of slope failure. Lack of convergence indicates that stress and displacement distributions that satisfy equilibrium conditions cannot be determined for a given problem. This state of collapse is precisely what occurs during real slope failures, and forms the underlying assumption of limit equilibrium analysis. In FE-SSR analysis, such failure is often characterized by a sudden increase in displacements.

The SSR technique is best described with the Mohr-Coulomb strength criterion, due to the simplicity and linearity of the criterion. For this criterion, the shear strength envelope (defined by the cohesion, c', and friction angle ϕ') can be reduced by a factor, F, and new a cohesion, c*, and friction angle ϕ* determined for the factored shear envelope. Mathematically, this is accomplished by rewriting the equation

$$\frac{t}{F} = \frac{c'}{F} + s_n \frac{\tan f'}{F}.$$

as $\frac{t}{F} = c^* + s_n \tan f^*$,

where $c^* = \frac{c'}{F}$ and $f^* = \arctan\left(\frac{\tan f'}{F}\right)$ are the new factored Mohr-Coulomb shear strength parameters.

The steps for systematically searching for the critical factor of safety value F that brings a previously stable slope ($F \geq 1$) to the verge of failure are as follow:

Step 1: Develop an FE model of a slope with the initial material deformation and strength properties. Compute the model and record the maximum total displacement.

Step 2: Increase the value of F (or *SRF*) and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total displacement.

Step 3: Repeat Step 2, using systematic increments of F, until the FE model does not converge to a solution, i.e. continue to reduce material strength until the slope fails.

The critical strength reduction value which brings the slope to failure (non-convergence) is taken to be the factor of safety.

For a slope with a factor of safety less than 1, the procedure is the same except F values less than 1 will be systematically decreased (translating into increments in the factored strength parameters) until the slope becomes stable. The critical value at which stability occurs would be the factor of safety.

Although the recorded maximum total displacements recorded in the algorithm are not directly used to determine factor of safety against total collapse, a plot of these values against strength reduction factor provides important information. Often, the plot reveals that deformations drastically increase when failure occurs.

2.3 *Determination of Safety Margins against Serviceability Limits*

Because the FEM fully accounts for constitutive behaviour, the FE-SSR method can be used to determine factors of safety against serviceability limits (these limits can be in the form of displacements, strains, stresses or differential settlements). Similar to the determination of safety margins against ultimate collapse, this can be done by plotting values of the required quantity against strength reduction factors. The factor at which the quantity of interest is attained is the factor of safety against the serviceability limit. The example described next illustrates the procedure.

3 SIMPLE EXAMPLE OF FE-SSR APPLICATION TO SERVICEABILITY

This example examines the stability of a slope into which a cavern is to be excavated. The cavern will have a concrete liner with a Young's modulus, E, of 30,000 MPa and Poisson's ratio, μ, equal to 0.2. The liner is assumed to exhibit linear elastic behaviour. The geometry (slope, material layers, and shape of cavern) of the problem are provided in Figures 1 and 2. The properties of the slope materials are as described in Table 1 below.
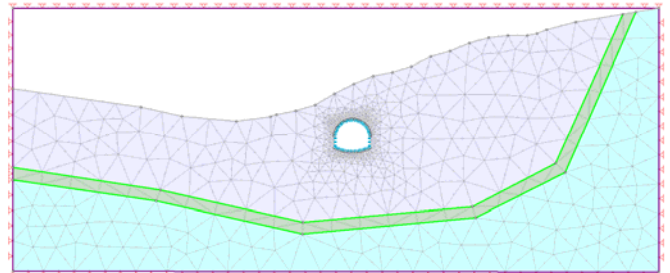

Figure 1: Slope with cavern with geometry of material layers.

In this problem, it is important to limit differential settlements (vertical displacements) between points A and B on the floor of the cabin to no more than 5 cm. The following two questions need to be answered:

1. What is the safety margin against ultimate collapse? and
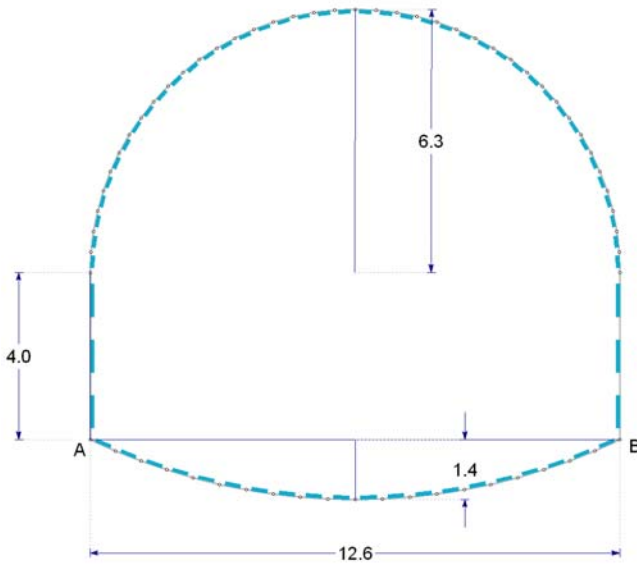2. What is the factor of safety against exceedance of the 5 cm settlement limit?

Figure 2: Geometry of the cavern. Note points A and B for which a differential settlement of 5 cm is the serviceability limit.

Table 1: Properties of Slope Materials

Material	Properties*
Upper Layer	$E = 650$ MPa, $\mu = 0.2$ *Generalized Hoek-Brown strength*: $\sigma_c = 25$ MPa, $m_b = 0.19$ $s = 0.002$, $a = 0.515$ and dilation angle $\psi = 0°$
Weak Middle Seam	$E = 1000$ MPa, $\mu = 0.4$ *Mohr-Coulomb strength*: $\sigma_t = 0.5$ MPa, $\phi = 20°$ $c = 0.5$ MPa, and dilation angle $\psi = 0°$
Lower Layer	$E = 1000$ MPa, $\mu = 0.4$ *Mohr-Coulomb strength*: $\sigma_t = 10$ MPa, $\phi = 35°$ $c = 10$ MPa, and dilation angle $\psi = 0°$

*All the materials are assumed to have elastic perfectly plastic stress-strain behaviour.

For this problem, even the first question cannot be effectively answered with limit equilibrium methods; those methods would not be able to consider failure surfaces that traverse the cavern. As well they would be unable to account for the supporting influence of the cavern liner.

The FE-SSR method, on the other hand, has no difficulties providing an answer to the problem. The SSR algorithm implemented in the 2D Finite Element program *Phase*[2] (Rocscience 2003) was used to perform analysis.

The failure mechanism determined by the FE-SSR method is visible on the contours of maximum shear strain shown on Figure 3. A close-up of the mechanism around the cavern is displayed on Figure 4. The figures also show the manner in which the cavern deforms. The failure mechanism does not simply consist of a single continuous shear band, but of three bands that define two wedges. This ultimate collapse mechanism has factor of safety equal to 3.48.
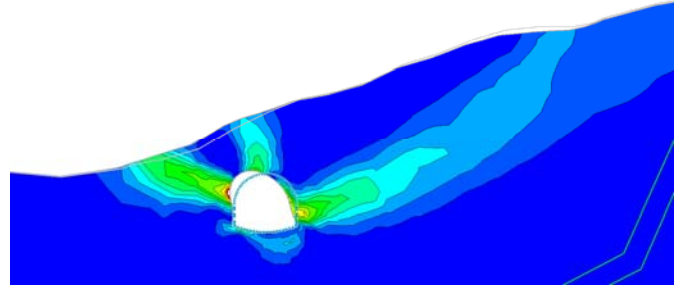

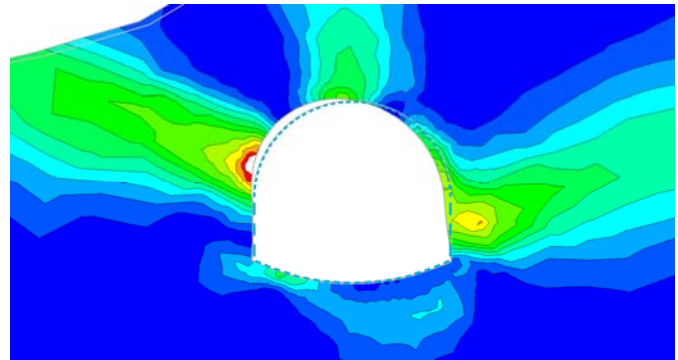Figure 3: Contours of maximum shear strain showing the failure mechanism of the slope

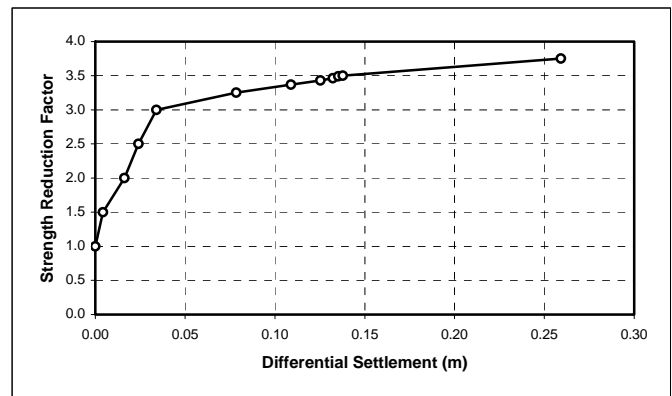Figure 4: Closer look at maximum shear strains around cavern.

Figure 5: Plot of strength reduction factor against differential settlement of points A and B.

Figure 5 gives the plot of strength reduction factor versus differential settlement used to answer the second question.

From the plot, the factor of safety against exceedance of the 5 cm differential settlement is estimated to be around 3.1. Several other serviceability factors of safety could have been estimated from the same analysis.

4 CONCLUSIONS

In addition to calculating safety margins against ultimate collapse, the FE-SSR method can be used for applications involving the determination of factors of safety regarding serviceability limits. In the paper, the method was applied to estimation of factor of safety against exceedance of differential settlements of the floor corners of a cavern excavated in a slope. All this is possible because the FEM offers solutions that satisfy all the constraints of stress-strain problems.

This example illustrates two important attractions of the FE-SSR method. Firstly, the FE-SSR method can be applied to non-slope problems. In the example, mechanism of ultimate collapse involved not only the slope, but also the cavern. As well, the serviceability limit was related to the cavern, and not directly to the slope. This indicates that the SSR method can be used, for example, to estimate the factor of safety of a reinforced tunnel cross-section.

Secondly, there are few limits to the serviceability states the FE-SSR method can be used to analyze. In the papers sample problem, for example, the method could have been used to estimate a factor of safety for axial forces in the liner exceeding a threshold value.

There may be several other useful applications of the SSR method which have not been explored. It is our hope that such applications will be brought to light. This will help geotechnical engineers to more fully harness the potential of the SSR method.

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