Probabilistic analysis of an MSE wall considering spatial variability of soil properties

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ABSTRACT

The results of probabilistic analyses of a mechanically stabilized earth (MSE) wall with geosynthetic reinforcement are presented. The analyses consider spatial variability of reinforced and foundation soil properties using the 2D non-circular Random Limit Equilibrium Method (RLEM). In this study, it is assumed that the reinforced soil is a purely frictional soil, while the foundation is a cohesive-frictional (c-ϕ) soil. A negative cross-correlation between cohesion and friction angle and a positive cross-correlation between cohesion and unit weight, and between friction angle and unit weight are also assumed. It is shown in this study that, considering only random variability of soil properties results in an overly-conservative probability of failure for design. However, considering spatial variability of soil properties plus cross-correlation between soil input parameters provides an estimate of probability of failure which is in better agreement with the margin of safety implied using a deterministic factor of safety.

INTRODUCTION

The purpose of this study was to investigate the influence of spatial variability of soil properties and cross-correlation between soil input parameters on probability of failure of an example geosynthetic reinforced mechanically stabilized earth (MSE) wall model.

Probabilistic stability analysis results considering spatial variability of soil properties and using limit equilibrium methods (LEM) have been reported in studies by Li and Lumb (1987), El-Ramly et al. (2001), Low (2003), Babu and Mukesh (2004), Cho (2007 and 2010), Hong and Roh (2008), Wang et al. (2011), Ji et al. (2012), Tabbaroki et al. (2013), Li et al. (2014), Javankhoshdel and Bathurst (2014) and Javankhoshdel et al. (2017).

Possible correlations between random values of shear strength parameters that can influence the probability of failure estimates for slopes have been noted by Nguyen and Chowdhury (1985). These correlations are quantified by the cross-correlation coefficient (ρ). Javankhoshdel and Bathurst (2016a) investigated the influence of cross-correlation between soil strength parameters in cohesive and cohesive-frictional (c-ϕ) soil slopes. They showed that probability of failure was less for cohesive soil slopes and positive cross correlation between cohesion and unit weight. Probability of failure also decreased with increasing negative cross correlation between cohesion and friction angle, and increasing positive correlation between cohesion and unit weight, and friction angle and unit weight when all other conditions remain unchanged. Javankhoshdel and Bathurst (2016b) also reported the same trends for the results of probabilistic analyses of simple reinforced slopes with purely frictional soil (cohesion, c = 0) and for c-ϕ soil.

There are few studies that investigate the influence of spatial variability of soil properties for the case of layered slopes. Huang et al. (2010) and Cho (2007) investigated the influence of spatial variability on probability of failure using the Random Finite Element Method (RFEM) and the non-circular Random Limit Equilibrium Method (RLEM), respectively.

The current study uses the MSE wall model presented by Leshchinsky and Han (2004) to compare the results of a probabilistic analysis with and without 2D spatial variability and cross-correlation between soil properties using the RLEM implemented in program *Slide2* 2018 (Rocscience, 2018).

RANDOM LIMIT EQUILIBRIUM METHOD (RLEM)

In the RLEM, a random field is first generated using the local average subdivision (LAS) method developed by Fenton and Vanmarcke (1990) and then mapped onto a grid of elements (mesh). Each mesh element in the random field has different values of soil properties, and cells close to one another have values that are closer in magnitude, based on the value of the spatial correlation length. In each realization, a search is carried out to find the mesh elements intersected by the slip surface. The random soil property values are assigned to the slices whose base mid-point falls within that element. A limit equilibrium approach is then used to calculate factor of safety (F_S) for each trial (simulation). The probability of failure is calculated as the ratio of the number of simulations resulting in $F_s < 1$ to the total number of simulations.

The non-circular RLEM used in this study is a combination of Cuckoo Search (Fister et al. 2014) and the LEM (Morgenstern-Price method). The Cuckoo Search is a fast and efficient global optimization method, which is used to locate critical non-circular slip surfaces. It is not constrained by a fixed initial trial surface geometry.

The Cuckoo Search in this study was used together with the Surface Altering optimization technique (Rocscience, 2018). When used in conjunction with a non-circular search, this optimization method can be very effective at locating (searching out) slip surfaces with lower factors of safety.

In this study, the Morgenstern-Price limit equilibrium method was used with the half sine interslice force function to calculate factor of safety. The bottom of each slice was divided into ten equal increments, and the cell in which the center of each increment landed, was used to assign a value to that increment. The ten values were then averaged, resulting in a weighted average of the cells in contact with the bottom of each slice. This made it possible to use only 50 slices for the probabilistic analysis. A sensitivity analysis confirmed that 10,000 Latin Hypercube simulations were sufficient for all probabilistic analyses.

RETAINING WALL MODEL

The geosynthetic reinforced MSE wall model from Leshchinsky and Han (2004) is shown in Figure 1. In this example, it was assumed that the reinforced soil is purely frictional with $\phi = 34^{o}$, γ = 18 kN/m^3, and c = 0, while the facing blocks and foundation soil are c-ϕ materials with the same ϕ and γ values, and c = 10 kPa. The tensile strength of the reinforcement is assumed to be 15 kN/m.

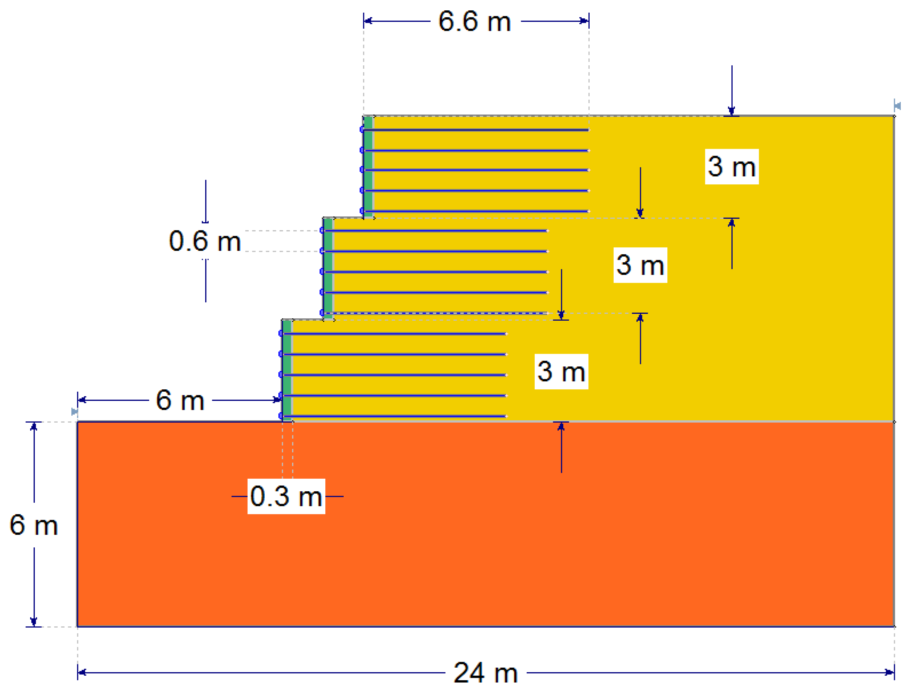

Figure 1. Geosynthetic reinforced MSE wall model.

Table 1 summarizes the mean and coefficient of variation (COV = standard deviation / mean) values for friction angle, cohesion, and unit weight. Lognormal distributions are assumed for all variables. The models considering cross-correlation are assumed to have a coefficient of −0.3 between c and ϕ, and 0.3 between c and γ, and ϕ and γ. These values (and their signs) are within ranges found in the literature (Javankhoshdel and Bathurst 2016a).

Table 1. Probabilistic soil properties of MSE wall

Soil	ϕ (peak friction angle) degrees		c (cohesion) kPa		γ (unit weight) kN/m^3	
	Mean	COV	Mean	COV	Mean	COV
Reinforced soil	34	0.2	0	-	18	0.1
Foundation	34	0.2	33	0.3	18	0.1

Four model cases were investigated in this study: 1) simple probabilistic analysis (no spatial variability) with no cross-correlation between soil parameters; 2) simple probabilistic analysis (no spatial variability) with cross-correlation between soil parameters; 3) more advanced probabilistic analysis (with spatial variability of soil properties) with no cross-correlation between soil parameters; and 4) more advanced probabilistic analysis (with spatial variability of soil properties) with cross-correlation between soil parameters.

Luo and Bathurst (2018) suggested an anisotropic spatial correlation length for the case of reinforced slopes with the vertical correlation length equal to the compaction lift thickness (e.g., 0.2 m) and a horizontal correlation length of infinity. However, for the cases with spatial variability analysis in this study, an isotropic correlation length of 2 m is assumed for simplicity.

2D SPATIAL VARIABILITY ANALYSIS RESULTS

Deterministic Analysis: LEM and Shear Strength Reduction Method

The deterministic factor of safety was found to be $F_S = 1.28$ using the Morgenstern-Price method; the corresponding slip surface is shown in Figure 2. The surface is a composite failure type because it extends through and beyond the reinforced soil zone. The geometry of the critical mechanism can be seen to be non-circular with a slightly concave up alignment over the middle portion. The local deviations from conventional simple bi-linear wedge failure geometry are a characteristic outcome of the search algorithm used here to find the "weakest path" through the problem domain.

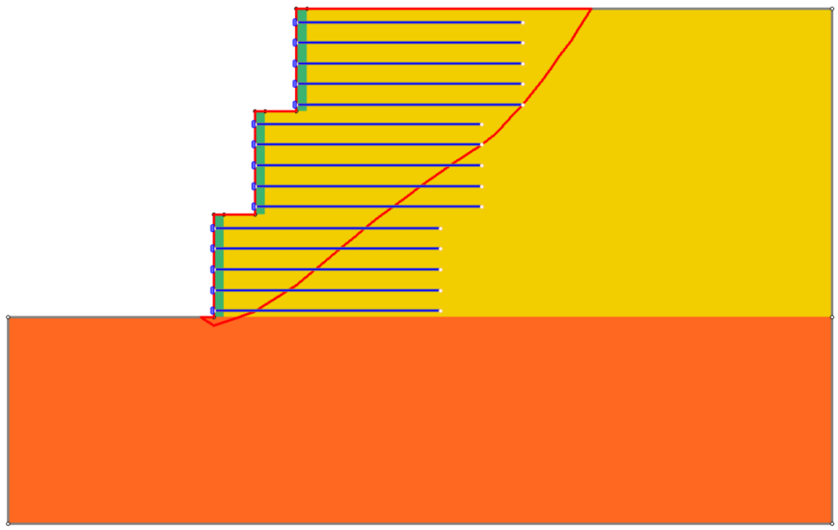

Figure 2. Deterministic critical composite failure surface with F_S = 1.28.

The stability of the model was also examined using the finite element method (FEM) with shear strength reduction as a check on the limit equilibrium method results. In this approach the strength of the soil is reduced in increments until the model fails. The critical shear strength reduction factor was 1.35 and the failure mechanism is shown in Figure 3. The critical failure surface in Figure 2 and the zones of high shear strain in Figure 3 are in good agreement.

Probabilistic analysis: RFEM *without* spatial variability

Figures 4a and 4b show the results of simple probabilistic analyses of the MSE wall without and with cross-correlation between soil properties, respectively. Individual weakest path failure mechanisms can be seen using different randomly selected soil property values for each trial analysis. Figure 4a shows that the mean value of factor of safety F_s = 1.28 which matches the deterministic factor of safety. However, the probability of failure P_f = 16.67% which would be interpreted as unacceptably high for this structure.

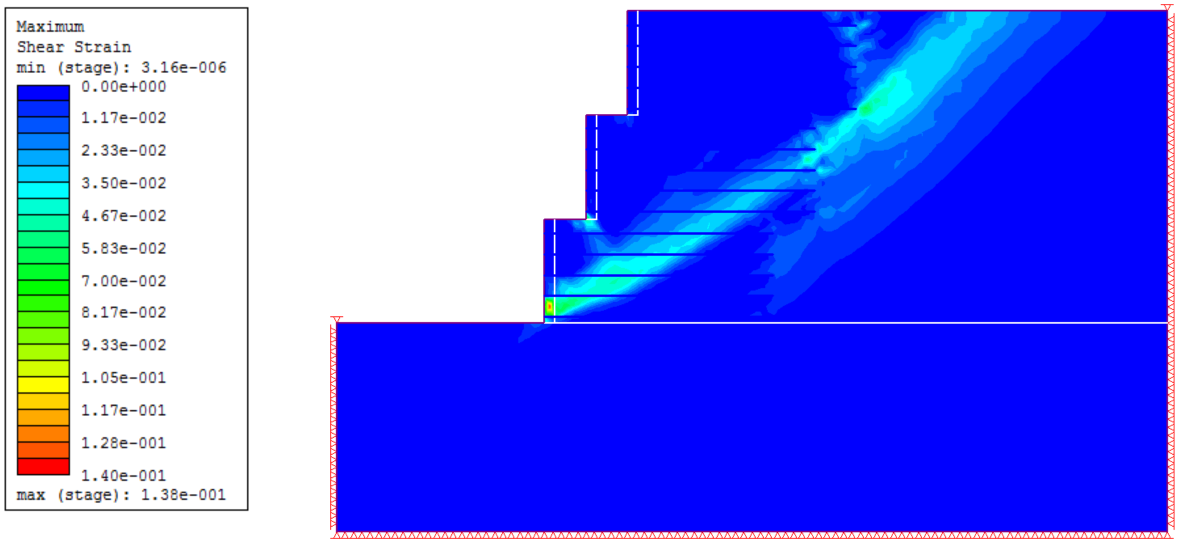

Figure 3. Shear strength reduction method: contours of maximum shear strain. Critical shear strength reduction factor = 1.35. Computed with *RS2* (Rocscience, 2014).

Javankhoshdel and Bathurst (2016a) showed that considering cross-correlation between soil input parameters can reduce the probability of failure. This effect can be seen in Figure 4b using the soil and cross-correlation properties in Table 1; here, mean F_s = 1.28 and P_f = 15.2%, and thus while the reduction in probability of failure is detectable, it is not significant.

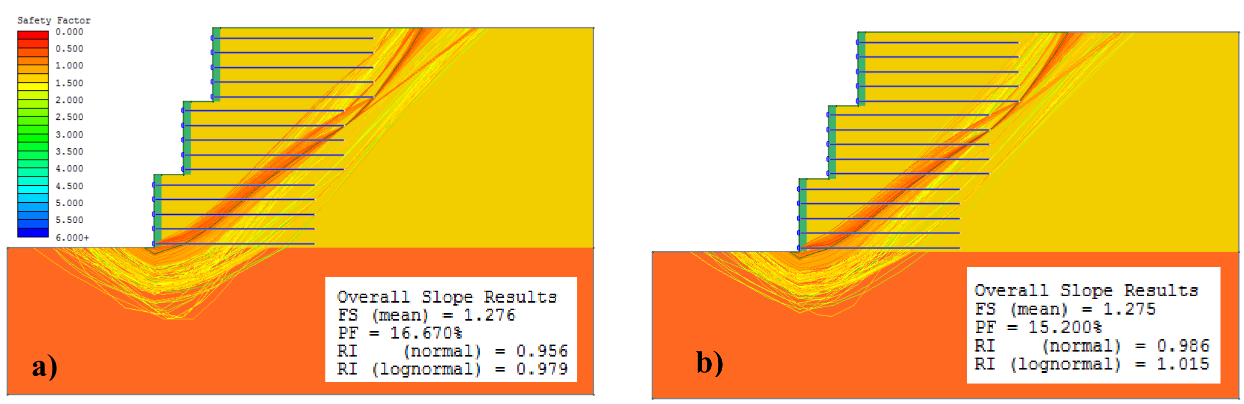

Figure 4. Results of probabilistic analysis *with no* spatial variability and a) with no cross-correlation b) with cross-correlation.

Probabilistic analysis: RFEM *with* spatial variability

Figures 5a and 5b, present the results of more advanced probabilistic analyses considering spatial variability of soil properties without and with cross-correlation between soil properties, respectively.

It can be seen from Figure 5a that there is a small reduction in the mean factor of safety compared to Figure 4a with no spatial variability and no cross-correlation. However, there is a significant reduction in the value of probability of failure from P_f = 16.67% in Figure 4a to P_f = 0.18% in Figure 5a. To explain the influence of spatial variability on these two outcomes, histograms of factor of safety for the cases in Figures 4a and 5a are presented in Figures 6a and 6b, respectively. It can be seen in Figure 6b that considering soil spatial variability causes the standard deviation of F_s to decrease from about 0.29 to about 0.1. Thus, the probability of failure reduces regardless of the slight reduction in the mean F_s; in other words it is the standard deviation of F_s that controls P_f and not the mean of F_s.

Figure 5b shows results with spatial variability and cross-correlation between soil properties. It can be seen in this figure compared to Figure 5a that, similar to Figure 4b and Figure 4a, the probability of failure decreases slightly when cross-correlation between soil parameters is considered.

An interesting observation from Figure 5b is that the probability of failure is about 0.1%. The predominant failure mechanisms are mostly internal failure type. It is reasonable to assume that the margin of safety is largely controlled by the strength and pullout capacity of the geosynthetic layers. In conventional reliability-based design, and load and resistance factor design (LRFD) calibration, a target probability of failure for reinforcement rupture or pullout is 1% (Allen et al. 2005; Bathurst et al. 2018). This value may appear high, but recall that geosynthetic layers are highly strength-redundant; if one layer fails in rupture or pullout, the other layers can compensate. Assuming that this target value of probability of failure applies to analyses of the type shown here, it can be argued that the combination of spatial variability together with cross-correlation between soil parameters leads to an acceptably safe design outcome. Ignoring cross-correlation of soil properties as in the earlier example, leads to what is likely to be judged an unacceptable design outcome in probabilistic terms.

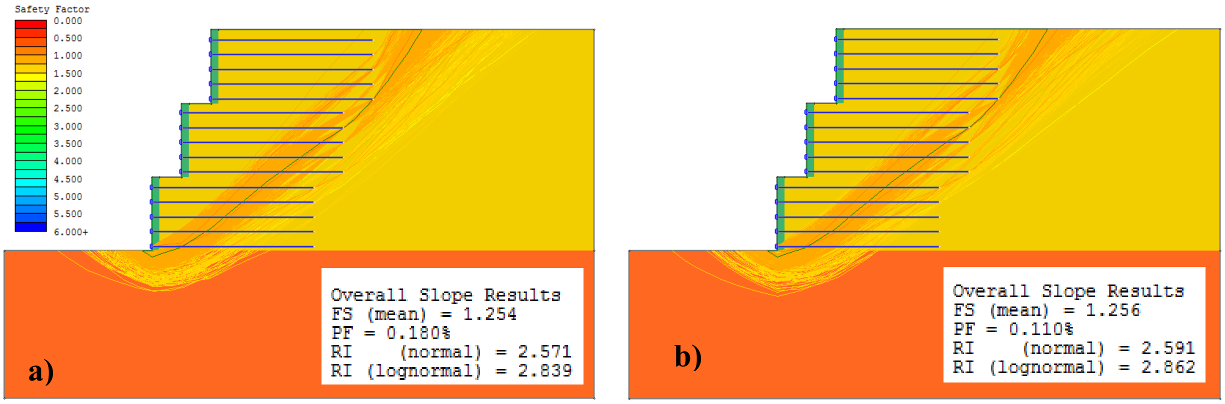

Figure 5. Results of probabilistic analysis *with* spatial variability and a) with no cross-correlation b) with cross-correlation.

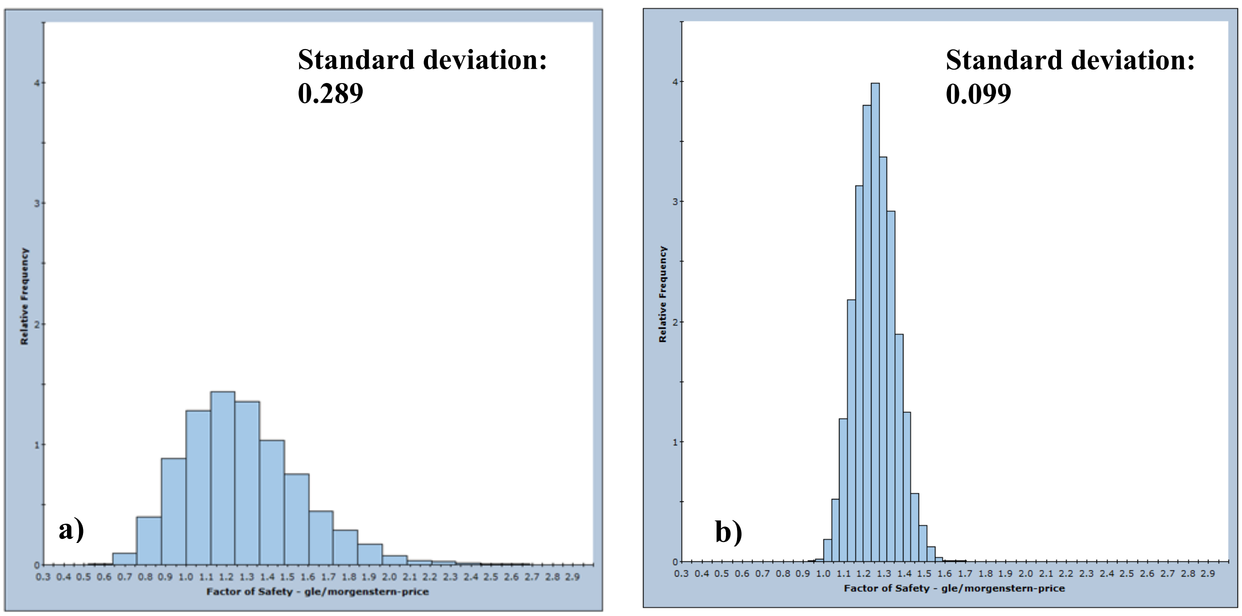

Figure 6. Histogram of F_S for a) no spatial variability and no cross-correlation (see Figure 4a) b) with spatial variability and with cross-correlation (see Figure 5a).

As mentioned earlier, Surface Altering optimization was used in this study together with the Cuckoo search method to find the critical factor of safety for each trial calculation. Figure 7 shows one example of a random field with its corresponding critical slip surface and factor of safety. The simulation shown was selected to have F_s close to the mean F_s. The failure surface can be seen to pass through weakest path (the blue regions) which demonstrates the effectiveness of optimization techniques combined with the RLEM method in this study to identify critical mechanisms and thus compute maximum probabilities of failure.

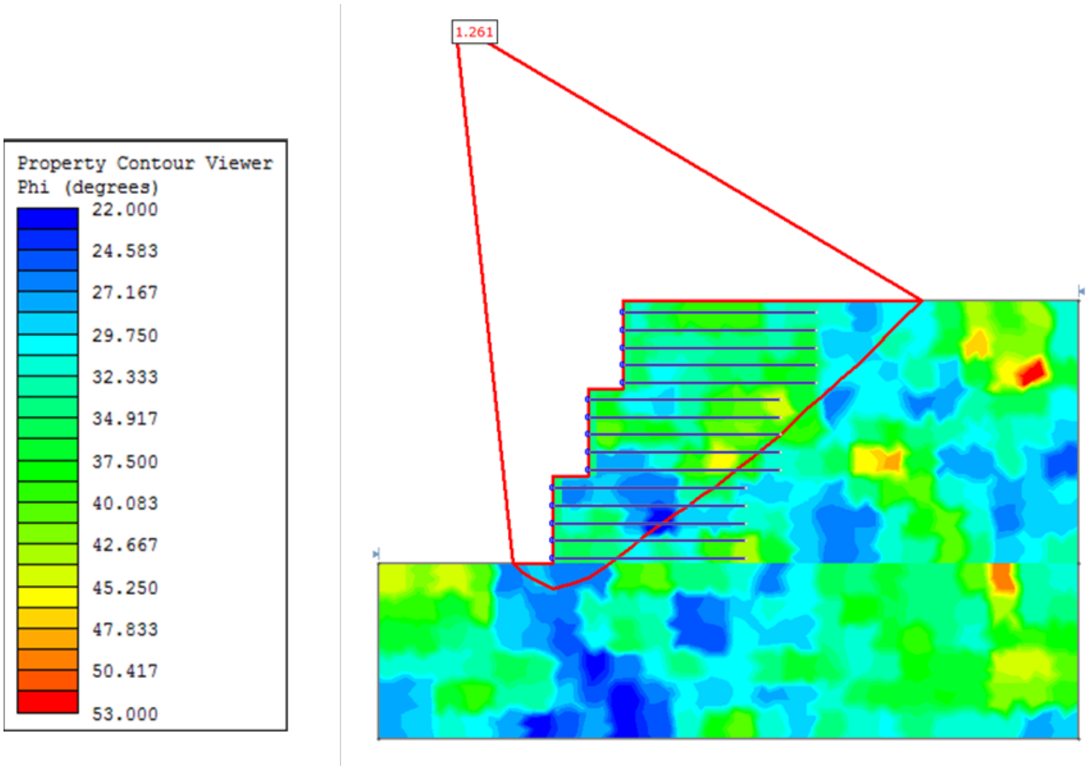

Figure 7. Random field and corresponding failure surface and factor of safety.

CONCLUSIONS

This study focused on probabilistic analysis of a geosynthetic reinforced MSE wall with cross-correlation between soil properties and with and without spatial variability of soil properties.

It was shown that for a factor of safety of about 1.3 and ignoring spatial variability results in a high probability of failure even with cross-correlation between soil properties. However, for the same wall considering spatial variability *plus* cross-correlation between soil parameters reduces the probability of failure. The lower probability of failure for this wall example is judged to be a better estimate of the true probability of failure of the wall because it is in closer agreement with the margin of safety of the wall implied by the magnitude of deterministic factor of safety computed for the same structure.

The study also highlights the advantage of optimization techniques together with a search method for soils with spatial variability analysis to find the weakest failure path through the problem domain. This is important because non-circular limit equilibrium methods applied to soil walls (or slopes) with spatial variability that cannot locally adjust trial failure surfaces using optimization techniques, cannot be guaranteed to find the most critical mechanism; hence analysis outcomes may be non-conservative.

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