Probabilistic Analysis of a Tailings Dam using 2D Composite Circular and Non-Circular Deterministic Analysis, SRV Approach, and RLEM

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ABSTRACT
The Random Limit Equilibrium Method (RLEM) is a relatively new method of probabilistic slope stability analysis which uses a combination of 2D random field theory and circular or non-circular limit equilibrium methods. This study uses data available from the Mount Polley Tailings Dam breach in order to 1) compute soil property correlation lengths, 2) calculate deterministic factor of safety (FS), 3) calculate probability of failure (PF) using a Single Random Variable (SRV) probabilistic analysis, and 4) calculate PF using the RLEM, using a search of potential composite circular and non-circular failure geometries. The results from both methods are compared including the influence of the choice of coefficient of variation (COV) of soil input parameters. It is shown that PF increases with increasing COV. Non-circular analyses result in higher PF values when compared to composite circular search methods. It is shown that considering spatial variability of soil properties in the vertical direction reduces the calculated probability of failure from composite circular or non-circular RLEM analyses when compared to the SRV approach.

RÉSUMÉ
La Méthode d'équilibre limite aléatoire (RLEM) est une méthode relativement nouvelle d'analyse probabiliste de la stabilité des pentes qui utilise une combinaison de la théorie des champs aléatoires 2D et des méthodes d'équilibre limite circulaire ou non circulaire. Cette étude utilise les données disponibles de la défaillance de la digue à rejets du *Mount Polley* afin de 1) calculer les longueurs de corrélation des propriétés du sol, 2) calculer le facteur de sécurité déterministe (FS), 3) calculer la probabilité de défaillance (PF) en utilisant une analyse probabiliste à variable unique (SRV), et 4) calculer PF avec la RLEM, en utilisant une recherche de géométries de surfaces de rupture potentielles composites circulaires et non circulaires. Les résultats des deux méthodes sont comparés, y compris l'influence du choix du coefficient de variation (COV) des paramètres du sol. Il est montré que PF augmente avec l'augmentation de COV. Les analyses non circulaires entraînent des valeurs de PF plus élevées par rapport aux méthodes de recherche circulaire composite. Il est démontré que tenir compte de la variabilité spatiale des propriétés du sol dans la direction verticale, réduit la probabilité calculée de défaillance des analyses RLEM circulaires ou non circulaires comparées à l'approche SRV.

1 INTRODUCTION

The purpose of this study is to compare results from slope stability analyses using Limit Equilibrium Methods (LEM). The comparisons are with respect to: 1) factor of safety (FS) from deterministic analysis, 2) probability of failure (PF) from single random variable (SRV) probabilistic analysis, and 3) PF from the Random Limit Equilibrium Method (RLEM), using both composite circular and non-circular LEM. In this study, a version of the program *Slide* v.8, which is currently in development (Rocscience Inc. 2017) was used to carry out the analyses.

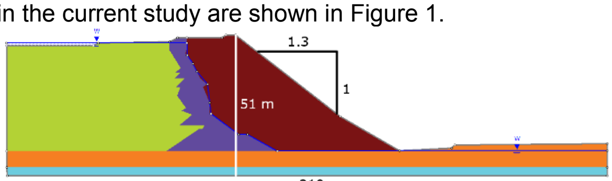
1.1 The Tailings Dam Model

The Mount Polley tailings dam was selected to provide baseline geometry and soil properties for this study (Province of British Columbia, 2015). It is important to note that the purpose of this paper is not to re-analyze the failure of the dam, but to use the dam as a baseline case to compare margins of safety using three different limit equilibrium methods, and study the influence of coefficient of variation (COV) assigned to choice of soil properties for probabilistic analyses. Some soil properties were adjusted from the Mount Polley case study so that detectable values of probability of failure could be computed for the purpose of comparison of deterministic and probabilistic analysis outcomes. The properties used are found in Figure 1.

There are few studies that investigate the influence of spatial variability of soil properties for the case of layered slopes. Huang et al. (2010) and Cho (2007) investigated the influence of spatial variability on probability of failure using the random finite element method (RFEM) and the RLEM, respectively. However, the example slopes presented in those studies are slopes with simple geometries.

The influence of spatial variability on a slope with complex geometry has not been previously investigated, nor have the results been compared to the results of SRV analysis for such a slope.

The model used in this study is taken from Lam (2016), using data provided by Province of British Columbia (2015). The model and baseline material parameters used in the current study are shown in Figure 1.



Material Name	Color	Unit Weight (kN/m3)	Strength Type	Cohesion (kPa)	Phi (deg)	Water Surface
Core		20.5	Mohr-Coulomb	0	35	Water Surface
Rock		22	Mohr-Coulomb	100000	0	None
Tailings		18	Mohr-Coulomb	0	30	Water Surface
Upper Till		21	Mohr-Coulomb	0	35	Water Surface
Upper Glaciolacustrine		20	Mohr-Coulomb	25	15	Water Surface

Figure 1. The Mount Polley tailings dam model and the material properties used in this study.

1.2 Spatial Correlation Length

Liu and Chen (2010), Lloret-Cabot et al. (2014), and Pieczynska-Kozlowska (2015), measured spatial correlation length using field CPT data. They showed that, due to the larger number of measurements available in the vertical direction, the calculated value of vertical correlation length can be found relatively accurately. The horizontal correlation length on the other hand is harder to determine, particularly when measurement locations are far apart.

In this study, vertical correlation length was calculated using a method outlined by Vanmarcke (1977) which estimates the correlation length by minimizing the error between the theoretical and empirical correlation models. The effect of horizontal correlation length was not considered in this study, meaning that it was assumed to be infinity.

1.3 Slope Stability Analysis Using LEM

In this study, the Morgenstern-Price limit equilibrium method was used with the half sine interslice force function to calculate factor of safety. All results are computed for composite circular LEM and non-circular LEM. Composite circular surfaces are used instead of circular failure mechanism to ensure that the slip surface goes through the glaciolacustrine layer, as it did in the real failure case.

1.3.1 Deterministic Analysis

In a deterministic slope stability analysis, the material parameters shown in Figure 1 are assumed to be constant values that do not vary within each material unit. The FS values are computed as described in the previous section. The shear strength reduction (SSR) method using program *RS*^{2} (Rocscience Inc. 2014) was also used in the current study as a check on deterministic analysis results using Morgenstern-Price limit equilibrium method.

1.3.2 SRV Approach

The Single Random Variable (SRV) approach considers soil variability by defining a distribution for each soil parameter and carrying out Monte Carlo simulations. Each realization used a single set of sampled soil property values in the LEM (deterministic) analysis method described previously to compute the corresponding factor of safety in each simulation. The PF is calculated as the ratio of the number of simulations resulting in FS<1 to the total number of simulations. In this study 10,000 simulations were used in all probabilistic analyses in order to compute a reliable estimate of PF for the case of seven random variables (Table 2).

1.3.3 RLEM

Probabilistic stability analyses results considering spatial variability of soil properties and using LEM have been reported in studies by Li and Lumb (1987), El-Ramly et al. (2001), Low (2003), Babu and Mukesh (2004), Cho (2007 and 2010), Hong and Roh (2008), Wang et al. (2011), Ji et al. (2012), Tabbaroki et al. (2013), Li et al. (2014), Javankhoshdel and Bathurst (2014) and Javankhoshdel et al. (2017).

Javankhoshdel et al. (2017) used a circular slip limit equilibrium method and random field theory to investigate the influence of spatial variability of soil properties on probability of failure. Tabbaroki et al. (2013) used a non-circular limit equilibrium approach together with random field theory to consider spatial variability in their probabilistic analyses.

In the RLEM, a random field is first generated using the local average subdivision (LAS) method developed by Fenton and Vanmarcke (1990) and then mapped onto a grid of elements (mesh). Each mesh element in the random field has different values of soil properties, and cells close to one another have values that are different in magnitude, based on the value of the spatial correlation length. In each realization, a search is carried out to find the mesh elements intersected by the slip surface. Random soil property values are assigned to all slices whose base mid-point falls within that element. A limit equilibrium approach (Morgenstern-Price method) is then used to calculate factor of safety for each realization. The PF is calculated as the ratio of the number of simulations resulting in FS<1 to the total number of simulations.

Composite Circular RLEM

In the composite circular RLEM, a circular search with enabled composite surfaces is used to calculate the factor of safety in each realization.

Non-Circular RLEM

The non-circular RLEM used in this study is a combination of a refined search and the LEM approach (Morgenstern-Price method). The refined search is based on circular surfaces that are converted to piece-wise linear surfaces. The search for the lowest factor of safety is refined as the search progresses. An iterative approach is used, so that the results of one iteration, are used to narrow the search area on the slope in the next iteration.

The refined search in this study was used together with an additional optimization technique. The optimization is based on a Monte Carlo technique, often referred to as "random walking" (Greco 1996). When used in conjunction with a non-circular search this optimization method can be very effective at locating (searching out) slip surfaces with lower safety factors.

2 MEASURING CORRELATION LENGTH

Vanmarcke (1977) outlined a method that can be used to estimate the correlation length from CPT data by minimizing the error between the theoretical and empirical correlation models. The empirical correlation model, $\hat{\rho}(\tau)$

$$\hat{\rho}(\tau_k) = \frac{1}{\hat{\sigma}^2(n-k)}\sum_{t=1}^{n-k}(x_t-\hat{\mu})(x_{t+k}-\hat{\mu}) \qquad [1]$$

where k is the number of lag distances (τ) between two points, $\hat{\sigma}$ is the estimated standard deviation of the detrended CPT data, n is the number of observations, x_t is the detrended tip resistance value at location t, and $\hat{\mu}$ is the estimated mean of the detrended CPT data.

In this study the Markov correlation model [Equation 2] was used to calculate the theoretical correlation values.

$$\rho(\tau_k) = \exp\{\frac{-2|\tau_k|}{\theta}\} \qquad [2]$$

where θ is the correlation length.

Using the method described above, the vertical correlation length was measured at nine different CPT locations and found to range between 0.3 m and 1.8 m, with most values falling closer to 1 m; hence, a vertical correlation length of 1 m was used in this study. A square mesh size of 0.5 m was used in the random field to accommodate this correlation length. Tip resistance data from CPT RCP14-106 is shown in Figure 2, this case having resulted in a correlation length of 0.86 m.

Due to less data in the horizontal direction as noted earlier, the effect of horizontal correlation length was not considered in this study, by letting the value of horizontal correlation length be 1000 m. It should be noted that using an isotropic correlation length value of 1000 m in the RLEM yields the same results as a SRV analysis for a model of this scale; hence it is a good assumption for the homogeneous soil condition.

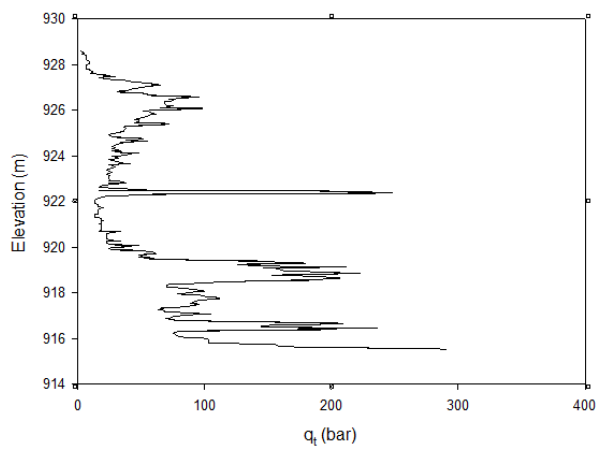


Figure 2. Tip resistance data from CPT RCP14-106. The vertical correlation length calculated for this case was 0.86m.

3 LIMIT EQUILIBRIUM ANALYSIS

In this study, the Morgenstern-Price limit equilibrium method was used in all analyses with the half sine interslice force function. As a result of a sensitivity analysis, 100 slices were used for all composite circular search simulations and 200 slices were used for all non-circular search simulations. 10,000 simulations were used for all probabilistic analyses.

In this study, three different combinations of the COV of soil properties were selected (Table 1). In case A, COVγ is set to the typical maximum value (0.1) and COVs of ϕ and c are assumed to be the same as COVγ. In case B, COVγ is set to 0.1, COVϕ is set to a typical maximum value (COVϕ = 0.2) and COV$_c$ is set equal to COVϕ. In case C, COVγ is set to 0.1, COVϕ is set to 0.2, and COV_c was increased to 0.3 to compare the results with case B. Table 2 shows the random variable parameters used in case A. Lognormal distributions are assumed for all random variables. The core and rock materials were assumed to have the constant material properties defined in Figure 1.

Table 1. COV values used in probabilistic analyses.

Case	COVγ	COVϕ	COV_c
A	0.1	0.1	0.1
B	0.1	0.2	0.2
C	0.1	0.2	0.3

Table 2. Random variable parameters used in case A. Lognormal distributions are used with all variables.

Material	Property	Mean	Std. Dev.
Tailings	Unit Weight	18 kN/m^3	1.8 kN/m^3
Tailings	Friction Angle	30 deg	3.0 deg
Upper Till	Unit Weight	21 kN/m^3	2.1 kN/m^3
Upper Till	Friction Angle	35 deg	3.5 deg
Upper Glaciolacustrine	Unit Weight	20 kN/m^3	2.0 kN/m^3
Upper Glaciolacustrine	Friction Angle	20 deg	2.0 deg
Upper Glaciolacustrine	Cohesion	25 kPa	2.5 kPa

Note: standard deviation of soil properties corresponds to assumed COV = 0.1

3.1 Deterministic Analysis

The results of composite circular and non-circular deterministic analyses are shown in Figure 2. The factor of safety for the composite circular analysis was 1.35 while the factor of safety for the non-circular analysis was 1.27 (i.e. lower).

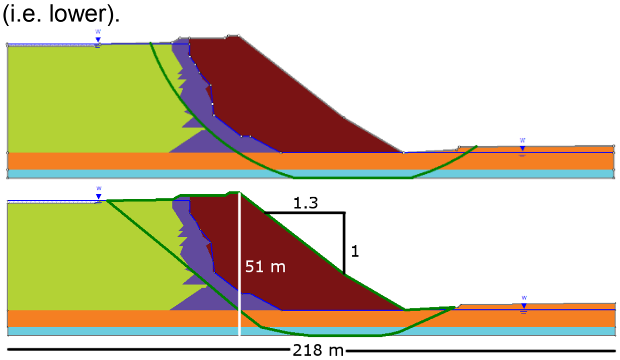


Figure 3. Critical slip surface geometry from deterministic analyses: composite circular (top), non-circular (bottom).

A deterministic SSR analysis was also computed in order to verify the LEM results. Figure 4 shows the maximum shear strain contours of the model, which resulted in a shear strength factor (SRF) of 1.26. The surface in Figure 4 is in agreement with the non-circular slip surface, as are the SRF and FS values.

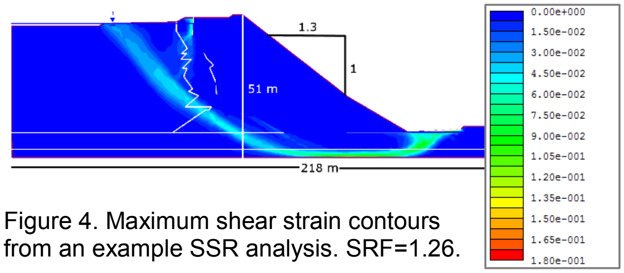


Figure 4. Maximum shear strain contours from an example SSR analysis. SRF=1.26.

3.2 SRV Analysis

The PF values from SRV analyses are summarized in Figure 5a. Probability of failure values are shown to increase with increasing COV, as expected. It should be noted that the change in PF from COV_B to COV_C is not as pronounced as that from COV_A to COV_B because only the glaciolacustrine material in the slope is cohesive-frictional (the others being only frictional) (Table 2).

Figure 5b shows the mean FS results for the SRV analysis. As expected, the non-circular LEM search has lower mean FS values (and higher PF values) when compared to the composite circular LEM approach.

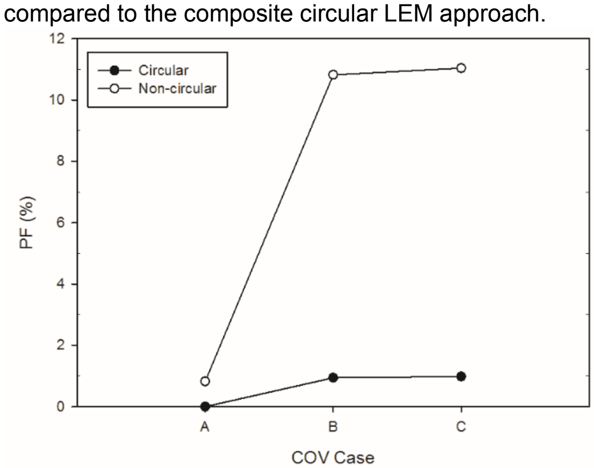


Figure 5a. SRV probability of failure (PF) results plotted against COV case for composite circular and non-circular analyses.

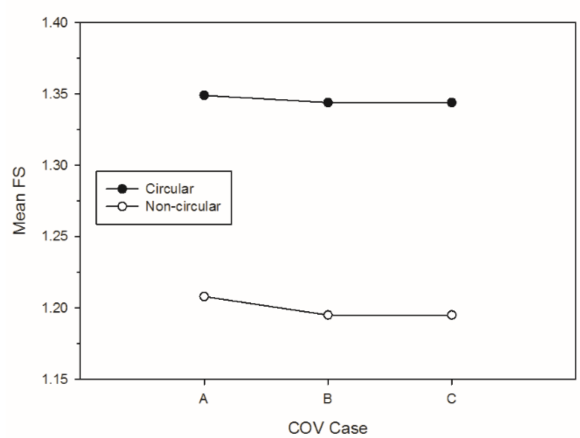


Figure 5b. SRV mean FS results plotted against COV case for composite circular and non-circular analyses.

3.3 The RLEM

Figure 6 summarizes the mean FS results from composite circular and non-circular RLEM analyses. In these analyses the soil properties were expressed as anisotropic random fields with spatial variability in the vertical direction only. The vertical correlation length was taken as 1 m as noted earlier. For these conditions the mean FS values are lower using the non-circular LEM approach, as expected.

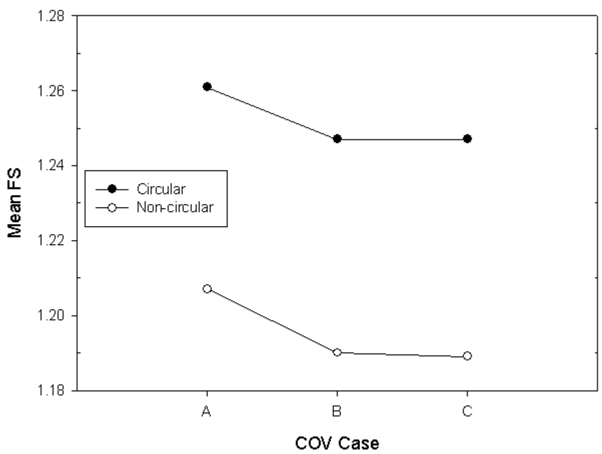


Figure 6. Mean FS results plotted against COV for composite circular and non-circular analyses using RLEM.

Figure 7 shows the non-circular results from SRV analyses and non-circular results using RLEM. The RLEM analyses give smaller PF values, as expected. Javankhoshdel et al. (2017) considered spatial variability of soil properties in RLEM analyses using circular slip geometry. For the same variable soil properties but with random spatial variability he showed that the probability of failure was less.

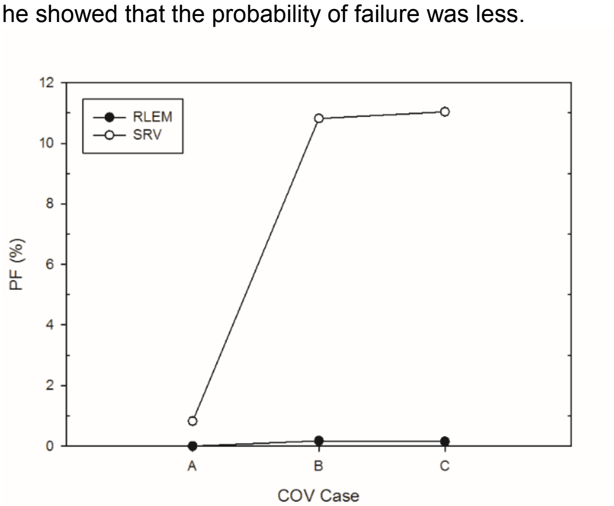


Figure 7. Probability of failure (PF) results plotted against COV for non-circular analyses using RLEM and SRV approaches.

In Table 3, an example of the results of the three methods are summarized for the non-circular LEM search, using COV case B.

Table 3. Non-circular LEM results for the three methods, for COV case B assuming vertical spatial variability of soil properties.

Deterministic	FS = 1.27
SRV	PF = 10.82%
RLEM	PF = 0.17%

4 DISCUSSION

The deterministic non-circular LEM search results were in agreement with the SSR results, both in terms of location and when comparing the SRF to the FS, while the deterministic composite circular LEM search results located a slip surface with a higher FS. This suggests that a non-circular LEM computation can locate the same surface as a SSR computation.

In all cases, the non-circular LEM search resulted in higher PF values (or lower mean FS) values when compared to the composite circular LEM. This is expected because the non-circular LEM search starts with the composite circular surface and adjusts it to find a potential failure mechanism with lower FS value.

The probability of failure increased with increasing COV of soil properties (i.e. increasing variability in soil parameters) as expected for both SRV and RLEM.

By considering spatial variability, the RLEM approach is judged to give more realistic results when compared to the other two methods for the tailings dam case used here. In most cases, this means a less conservative PF value. As seen in Table 3, the probability of failure (PF) using the SRV approach is relatively conservative, resulting in a different appreciation of the margin of safety against collapse when compared to the other two results. Using the SRV results could lead to an unnecessarily conservative slope design for similar tailings dam projects.

5 CONCLUSION

The purpose of this study is to compare different results from stability analysis of a tailing dam slope using three different limit equilibrium methods: deterministic analysis, SRV analysis, and RLEM.

This study is a first attempt to consider spatial variability of soil properties for the case of a very complicated slope geometry. Further studies are required to achieve a better understanding of the advantages and limitations of composite circular and non-circular approaches to investigate spatial variability of soil properties.

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