Probabilistic analysis of layered slopes with linearly increasing cohesive strength and 2D spatial variability of soil strength parameters using non-circular RLEM approach

Sina Javankhoshdel, Ph.D.[1] Brigid Cami, B.Eng.,[2] Richard J. Bathurst, Ph.D.,[3] and Brent Corkum, Ph.D. [4]

[1]Rocscience Inc., 54 Saint Patrick St., Toronto, Ontario, M5T 1V1;
e-mail: sina.javankhoshdel@rocscience.com
[2]Rocscience Inc., 54 Saint Patrick St., Toronto, Ontario, M5T 1V1;
e-mail: brigid.cami@rocscience.com
[3]GeoEngineering Centre at Queen's-RMC, Department of Civil Engineering, Royal Military College of Canada, 13 General Crerar, Kingston, Ontario, K7K 7B4;
e-mail: bathurst-r@rmc.ca
[4]Rocscience Inc., 54 Saint Patrick St., Toronto, Ontario, M5T 1V1;
e-mail: brent.corkum@rocscience.com

ABSTRACT

The results of probabilistic analysis of simple and layered slopes with linearly increasing (mean) undrained shear strength with depth, and spatial variability using the 2D non-circular Random Limit Equilibrium Method (RLEM) are presented. For the case of simple slopes, the results of the circular RLEM approach and the Random Finite Element Method (RFEM) are also presented and are compared to the results of the non-circular RLEM approach. For the case of simple slopes, it is shown that the non-circular RLEM approach gives higher values of probability of failure compared to circular RLEM and RFEM. For the cases with mean value of factor of safety greater than one, considering spatial variability reduces probability of failure.

INTRODUCTION

Stability analyses of slopes with linearly increasing undrained shear strength with depth and simple geometry have been by several researchers. The most recent series of design charts were presented by Griffiths and Yu (2015). They generated stability charts based on moment limit equilibrium for the case of linearly increasing undrained shear strength and different values of depth ratio D representing the ratio of height of slope above a firm stratum to height of slope H above the toe.

The influence of spatial variability of soil properties on probability of failure using conventional limit equilibrium slope stability analyses and random field generation techniques has been investigated by El-Ramly et al. (2002), Low (2003), Babu and Mukesh (2004), Low et al. (2007), Hong and Roh (2008), Cho (2010), Wang et al. (2011), Tabarroki et al. (2013), Javankhoshdel and Bathurst (2014) and Javankhoshdel et al. (2017). Important contributions to the influence of spatial variability of soil properties on stability of slopes have been made by Griffiths and Fenton (2004) using the two-dimensional (2D) random finite element method (RFEM), which combines the finite element method, strength reduction method and Monte Carlo (MC) sampling of soil properties from random fields that are generated using the local average subdivision (LAS) method.

Javankhoshdel et al. (2017) investigated the influence of spatial variability of soil strength parameters on probability of failure for simple slopes with stationary mean undrained shear strength. However, the main focus of the paper was on the influence of method of analysis on numerical outcomes. They compared results using (one dimensional) 1D circular RLEM, 2D circular RLEM, and 2D RFEM.

Griffiths et al. (2015) extended the results of prior RFEM analyses of simple slopes with constant mean (stationary) undrained shear strength to the case of linearly increasing (non-stationary) strength. The influence of possible correlations between soil properties on probabilities of failure was not considered in their study. Li et al. (2014) investigated the influence of linearly increasing (non-stationary) undrained shear strength on stability of infinite slopes. They concluded that probability of failure will be over-estimated if statistical characteristics describing linearly increasing soil shear strength with depth are ignored.

In this study, probabilistic analysis of simple and layered slopes with linearly increasing (non-stationary) mean undrained shear strength with depth are investigated. Included in analyses is the influence of spatial variability of soil properties with and without cross-correlation between soil input parameters. Analyses are carried out using the non-circular 2D RLEM approach. Some comparisons between the results of the 2D non-circular RLEM, the 2D circular RLEM, and the 2D RFEM analyses are also presented.

Slope Models

Figure 1 shows the slope model geometry where H = 10 m is slope height, $\beta = 15^o$ is slope angle, D = height ratio, s_{u0} = 20 kPa is undrained shear strength at crest level (z = 0) and r_{su} = 1 is the gradient of (mean) strength increase with depth z. The parameter H_0 is the height above the crest at which the extrapolated undrained strength becomes zero (Griffiths and Yu 2015). The unit weight γ is assumed to be constant. Undrained strength at depth z is computed as:

$$s_u(z) = s_{u0} + r_{su} z \qquad [1]$$

Two-layer slope case is shown in Figure 2 for D = 1.25, the case with D = 2 is also investigated in this study.

The top layer in these cases is the same as the slope shown in Figure 1 (s_{u0} = 20 kPa, r_{su} = 1 and β = 15^o). However, two different foundation heights are assumed (D = 1.25 and D = 2). In all cases r_{su} = 1 through the slope and foundation layers; therefore, assuming the value of the undrained cohesion at the top of the slope (s_{u0}) is 20 kPa, the value of cohesion at the top of the foundation layer (s_u) is 30 kPa. For each foundation height, two different foundation strengths are considered in this example: s_{uW0} = 15 kPa (weak foundation) and s_{uS0} = 35 kPa (strong foundation). In Figure 2, s_{uW0} and s_{uS0} are undrained shear strength at the top of the weak and strong foundation, respectively.

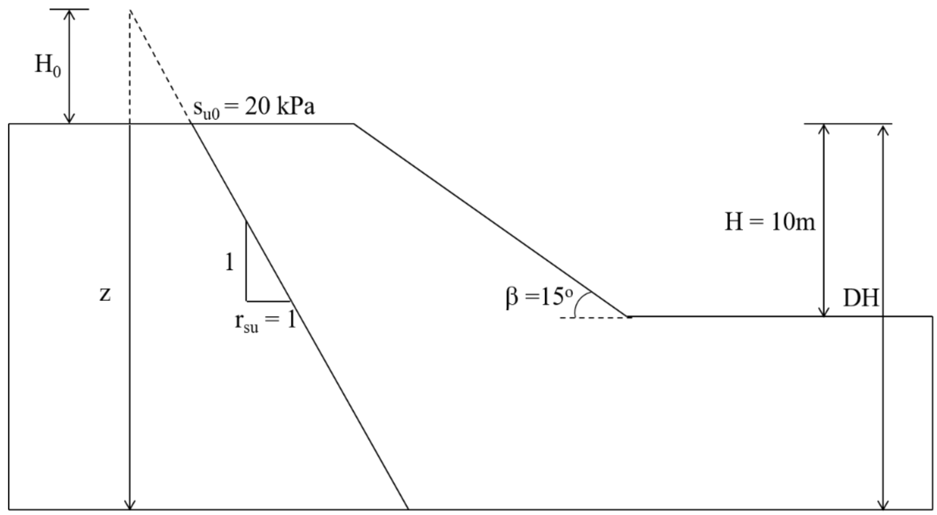
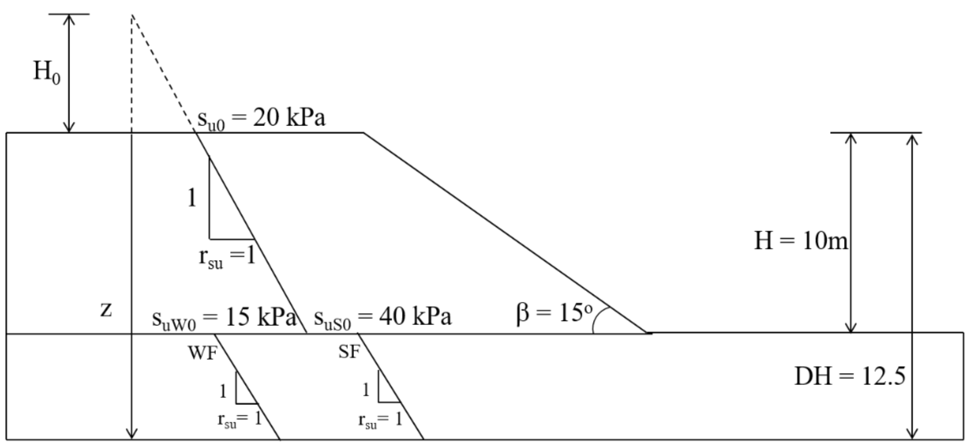


Figure 1. Simple slope model used in the study.



a)

Figure 2. Two-layer slope model: D =1.25 (WF = soil unit weight gradient and SF = soil cohesion gradient in the foundation)

SLOPE STABILITY ANALYSIS METHODS

RFEM

Griffiths et al. (2009) applied the RFEM to undrained cohesive and cohesive-frictional soil slopes. A random field of each shear strength parameter (cohesion and friction angle) was generated using the local average subdivision method (LAS) developed by Fenton and Vanmarcke (1990) and mapped onto the finite element mesh. Each node has different values of the soil property assigned to it and nodes close to each other are correlated using horizontal and vertical correlation lengths. Theoretically, the correlation structures of the underlying Gaussian random field can be determined using the Markov correlation coefficient function:

$$R(\tau_x, \tau_y) = \exp\left\{-\sqrt{\left(\frac{2\tau_x}{\theta_x}\right)^2 + \left(\frac{2\tau_y}{\theta_y}\right)^2}\right\} \qquad [2]$$

where, $R(\tau_x, \tau_y)$ is the autocorrelation coefficient, τ_x and τ_y are the absolute distances between two points in horizontal and vertical directions, respectively. θ_x and θ_y are the spatial correlation lengths in horizontal and vertical directions, respectively. For the isotropic case where $\theta_x = \theta_y = \theta$, Equation 2 can be simplified to:

$$R(\tau) = \exp\left\{-\frac{2\tau}{\theta}\right\} \qquad [3]$$

where τ is the absolute distance between two points in the isotropic field. In the remainder of the paper, the spatial correlation length is normalized to the height of the slope (H).

Circular RLEM

The circular RLEM is a combination of LEM as a deterministic method of analysis together with the same random field generated for the RFEM analysis explained above, and Monte Carlo simulation. In this study, 5000 Monte Carlo simulations were found to give a confident estimate of the probability of failure for $P_f > 0.02\%$ for all cases.

In the RLEM approach, a random field is first generated using the local average subdivision (LAS) method and then mapped onto a grid mesh, similar to the FEM mesh in the RFEM analyses. Each mesh cell in the random field has different values of soil properties, and cells close to one another have similar values, based on the value of the spatial correlation length. Then, the circular slip LEM analysis is carried out in each Monte Carlo realization to calculate factor of safety. In each Monte Carlo realization, a search is carried out to find the mesh elements intersected by the circular slip surface. Random soil property values are assigned to all slices whose base mid-point falls within that element. A limit equilibrium approach (the Morgenstern-Price method) is then used to calculate factor of safety for each Monte Carlo realization. The probability of failure is defined as the ratio of the realizations that failed ($F_s < 1$), to the total number of realizations.

Non-circular RLEM

The non-circular RLEM used in this study is a combination of a refined search and a LEM approach (the Morgenstern-Price method). The refined search is based on circular surfaces that are converted to piece-wise linear surfaces. The search for the lowest safety factor is refined as the search progresses. An iterative approach is used so that the results of one iteration are used to narrow the search area on the slope in the next iteration.

In many cases, for the same number of surfaces, a larger number of slip surfaces with lower factors of safety were detected than the number determined from conventional grid or slope search techniques.

The refined search in this study was used together with an additional optimization technique. The optimization is based on a Monte Carlo technique, often referred to as "random walking" (Greco 1996) because a randomly generated number determines the direction that the vertices are moved. There is no complex underlying algorithm that searches for the surface. The only data that is used to determine whether one surface is preferable over another, is the factor of safety. A detailed explanation of the refined search together with optimization is available in the *Slide* v.7 (Rocscience Inc. 2015) theory manual.

The combination of refined search with optimization and random fields generated using LAS helps to locate the critical slip surface in the spatially variable field. The disadvantage of the circular RLEM is that the circular RLEM cannot capture irregular shapes of failure (Javankhoshdel et al. 2017). This is especially noticeable in cases with highly fluctuating random fields. However, the optimization technique in the non-circular RLEM, moves the vertices along the slip surface to find the lowest factor of safety. Moving the vertices allows cells with lower values of soil strength in the random field mesh to be found and therefore weaker (more critical) failure paths are located.

The number of slices in the LEM part of the non-circular 2D RLEM analysis can be expected to influence numerical outcomes. A sensitivity analysis was carried out in the current investigation that showed that 400 slices was sufficient to generate consistent numerical outcomes.

RESULTS

Effect of spatial variability of soil properties with non-stationary mean undrained shear strength on P_f ($r_{su} > 0$)

To obtain a non-stationary random field of undrained shear strength in simple slopes (or for each layer in layered slopes), a stationary random field is generated first based on mean and COV of undrained shear strength. This field is mapped onto a grid mesh in RLEM analyses that have the same geometry and element size as the finite element mesh used in the RFEM analyses. Next the strength of each soil element was scaled to depth using the following equation (Griffiths et al. 2015):

$$c_z = c_0 \frac{\mu_{su} + r_{su} z}{\mu_{su}} \qquad [4]$$

Here, c_0 is the soil strength at each element in the original stationary random field, μ_{su} is the mean strength in the original stationary random field, z is the depth of the soil element and c_z is the scaled strength in the non-stationary random field. In the probabilistic analyses in this paper, the COV values of s_u and γ are assumed as $COV_{su} = 0.5$ and $COV_\gamma = 0.1$ which are typical upper bound values found in the literature.

Simple slopes ($r_{su} > 0$)

For the non-circular RLEM analyses, random fields of undrained shear strength were generated for each Monte Carlo simulation and coupled with the Morgenstern Price method as the underlying limit equilibrium method of analysis.

Figure 3 shows the results of the influence of normalized isotropic spatial correlation length (θ/H) on mean F_s and its corresponding probability of failure (P_f) for different values of height factor, D. The plots in this figure show that for the same height ratio, the mean factor of safety increases but the probability of failure also increases as the spatial correlation length increases.

It can be seen in this figure that as the value of D decreases for the same value of spatial correlation length, the mean value of F_s increases and the corresponding probability of failure decreases. This effect shows the influence of foundation height on probability of failure and mean factor of safety. In slopes with very large foundation depths and undrained cohesive soil, the failure mechanism is deep. However, reducing the foundation height changes the failure geometry to a composite mechanism that has a larger factor of safety.

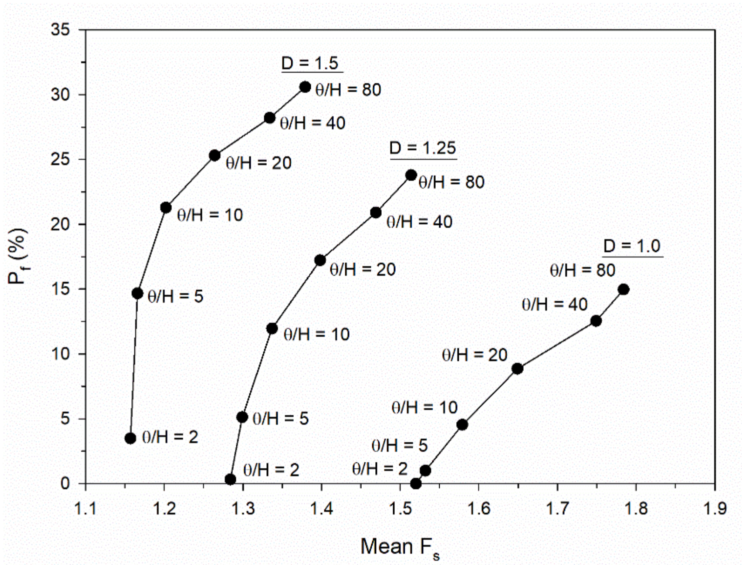


Figure 3. P_f vs. Mean F_s for different values of Θ/H using non-circular RLEM, for simple slopes with linearly increasing undrained cohesion.

Figure 4 shows probabilities of failure using circular and non-circular RLEM approaches with the same spatial correlation length. All solutions were carried out with a sufficient number of Monte Carlo realizations (5000) to achieve a consistent probability of failure for each problem case as noted earlier. It can be seen in this figure that, as the spatial correlation length increases, the probabilities of failure calculated using both methods become closer. However, there are noticeable differences between the values of probability of failure using circular and non-circular RLEM approaches for smaller values of spatial correlation length (P_f is larger using the non-circular method compared to the circular method) due to the ability of the non-circular method to find a weaker failure path. Finally, it can be seen that for both methods, as $\theta/H \rightarrow 0$, $P_f \rightarrow 0$.

The results of analyses for simple slopes with linearly increasing undrained cohesion using non-circular RLEM and RFEM approaches is shown in Figure 5. Again, to ensure fair comparisons, all solutions were carried out with 5000 Monte Carlo realizations to ensure a consistent probability of failure for the same problem conditions and the same method. It can be seen that for the same value of spatial correlation length, the non-circular RLEM approach gives higher values of probability of failure compared to the RFEM approach. This observation can be interpreted to mean that the optimization algorithm used in the refined search method for the non-circular RLEM approach is able to find a more critical (weaker) failure path than the RFEM approach. However, this conclusion may not be valid for all the slope cases and examples (stationary fields or slopes with more complicated geometries). This observation is presented in the literature for the very first time and further studies must be done in future to show the differences between the results of these two methods.

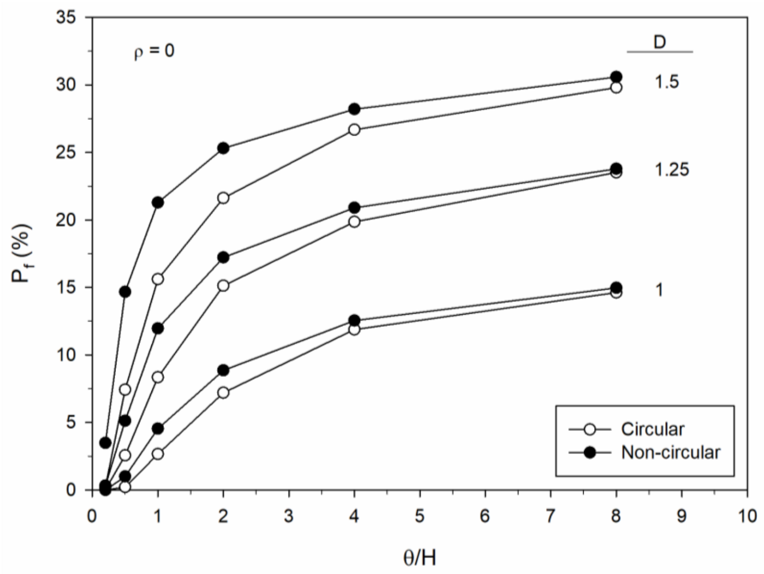


Figure 4. Results of circular and non-circular RLEM analyses of simple slopes with different D values and linearly increasing undrained cohesion.

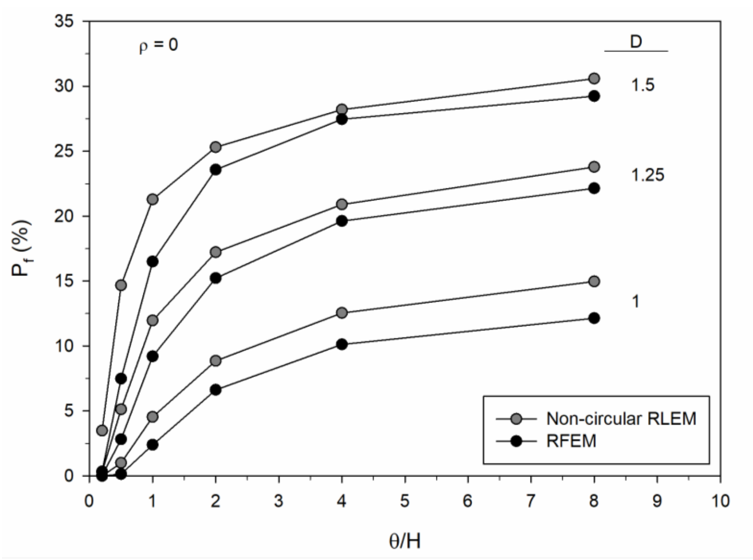


Figure 5. Results of non-circular RLEM and RFEM analyses of simple slopes with different D values and with linearly increasing undrained cohesion.

Layered slopes ($r_{su} > 0$)

Four different cases are investigated for a two-layer slope with linearly increasing undrained cohesion in both layers (i.e. bi-linear strength). The four cases are: 1) D = 1.25 with weak foundation; 2) D = 1.25 with strong foundation; 3) D = 2 with weak foundation; 4) D = 2 with strong foundation. Figure 6 shows values of mean F_s for different values of spatial correlation length for all cases. In this figure, case 2 (D = 1.25 with strong foundation) has the highest mean F_s and case 3 (D = 2 with weak foundation) has the lowest value of mean F_s for the same spatial correlation length. Case 3 also has mean F_s less than 1 for small values of spatial correlation length. For each case there is an overall trend of decreasing mean F_s value with decreasing spatial correlation. However, this rate of change is small and the mean F_s value for each plot in this figure varies over a small range and thus is not of practical concern.

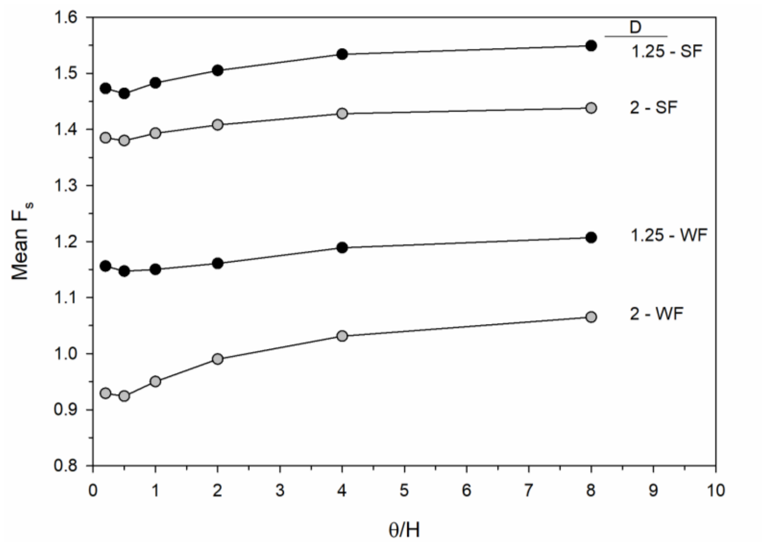


Figure 6. Influence of spatial variability of soil properties on mean F_s for four different two-layer slope cases (D =1.25 with weak and strong foundation, and D = 2 with weak and strong foundation) using non-circular RLEM approach. (WF = soil unit weight gradient and SF = soil cohesion gradient in the foundation)

Figure 7 shows values of probability of failure for different values of spatial correlation length for all cases. It can be seen in this figure that the trend in data for case 3 (top curve) is different from the other cases. Specifically, as spatial correlation length decreases, P_f increases and the largest correlation length in this figure ($\theta/H = 8$, which approximates the case of infinite spatial variability) gives a minimum probability of failure. For the other cases in the figure, the trend is the opposite and $\theta/H = 8$ corresponds to the maximum probability of failure when all other conditions remain the same.

As shown in Figure 6, for small values of spatial correlation length the mean F_s is less than 1. Thus, as reported by Javankhoshdel et al. (2017), decreasing spatial correlation length increases probability of failure in this case. However, in the three other cases, a correlation length approximating infinity (i.e. $\theta/H = 8$) has the highest probability of failure, and probability of failure decreases with decreasing correlation length.

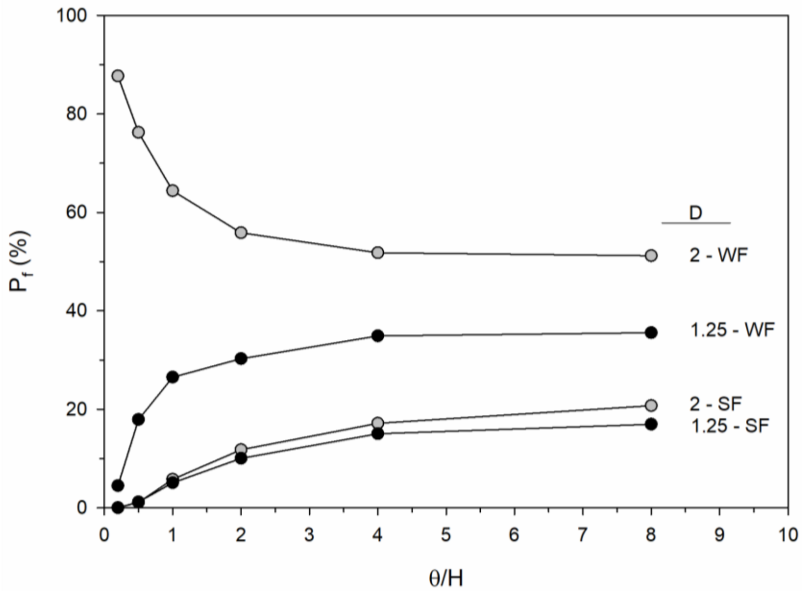


Figure 7. Influence of spatial variability of soil properties on probability of failure for four different two-layer slope cases (D =1.25 with weak and strong foundation, and D = 2 with weak and strong foundation) using non-circular RLEM approach.

CONCLUSIONS

This study presents the results of the influence of spatial variability of soil properties on probability of failure for simple single and double layer slopes with cohesive strength using the non-circular RLEM approach. Results are also compared to those using circular RLEM and RFEM for the same slope geometry and soil properties.

For the case of simple single layer slopes, the following conclusions can be made:

1) Considering spatial variability of soil properties reduces the probability of failure compared to slopes with random variable soil properties only.

2) Comparing the results of non-circular RLEM with circular RLEM and RFEM showed that non-circular RLEM gave the highest values of probability of failure for the same spatial correlation length and in slopes with different foundation heights. This is due to the greater freedom of the non-circular RLEM to examine potential failure paths that are not constrained to circular shape in combination with the optimization algorithm employed which together allow weaker failure paths in a slope to be discovered.

For the case of layered slopes, the case with D = 2 with weak foundation gave a mean F_s less than 1 for small values of spatial correlation length. Furthermore, for this case the probability of failure decreased with increasing spatial correlation length. For all other cases, the probability of failure increased with increasing spatial correlation length.

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