Probabilistic analysis of cohesive-frictional slopes using the RLEM (circular and con-circular) and the RFEM

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ABSTRACT
The random finite element method (RFEM), the non-circular random limit equilibrium method (non-circular RLEM), and the circular random limit equilibrium method (circular RLEM) are used to investigate the influence of isotropic spatial variability of soil strength parameters on probability of failure of cohesive-frictional soil slopes. The RFEM uses a combination of 2D random field theory, FEM, shear strength reduction, and Monte Carlo simulation. The non-circular RLEM uses a combination of 2D random field theory, non-circular slip method, and Latin Hypercube simulation (or Monte Carlo simulation). The circular RLEM uses a combination of 2D random field theory, classical circular slip LEM of slices, and Latin Hypercube simulation (or Monte Carlo simulation). In this paper, all three methods are used to investigate the influence of isotropic spatial variability of soil strength parameters on probability of failure and the results are compared. It is shown that in cases where the failure mechanism is close to a circular shape, the outcomes of all three methods are in good agreement. In other cases, RFEM can predict more complicated failure mechanisms than circular failure type. In these cases, non-circular RLEM is in good agreement with RFEM results. Cross-correlation between the random fields (negative correlation between cohesion and friction angle; positive correlation between cohesion and unit weight; positive correlation between friction angle and unit weight) is also considered. It is shown that considering possible practical correlations between all soil input parameters reduces the probability of failure. Computation times between the three methods are recorded. It is shown that non-circular RLEM results that are in good agreement with RFEM results are generated with much shorter computation times.

RÉSUMÉ
French version of abstract will be supplied at time of final paper submission.

1 INTRODUCTION

The influence of spatial variability of soil properties in slope stability analyses has been the subject of investigation by a number of researchers including Cho (2007), Low et al. (2007), Srivastava and Sivakumar Babu (2009), Cho (2010), Srivastava et al. (2010), Wang et al. (2011), Ji et al. (2012), Li et al. (2014), Zhu and Zhang (2013) and Javankhoshdel et al. (2017).

Probabilistic stability analyses considering spatial variability of soil properties have been carried out using limit equilibrium method (LEM) and finite element method (FEM) approaches. Studies adopting the circular LEM in deterministic analyses include the work of Li and Lumb (1987), El-Ramly et al. (2002), Low (2003), Babu and Mukesh (2004), Cho (2007), Hong and Roh (2008), Wang et al. (2011), Ji et al. (2012), Li et al. (2014), Javankhoshdel and Bathurst (2014) and Javankhoshdel et al. (2017).

Other researchers have studied the influence of spatial variation on slope reliability using the random finite element method (RFEM) which combines the shear strength reduction method with random fields that are generated using the local average subdivision (LAS) method (Griffiths and Fenton 2004; Griffiths et al. 2009; Hicks and Spencer 2010).

Tabarroki et al. (2013) compared the mean factor of safety results of the non-circular RLEM and RFEM. They showed that there is good agreement between results using non-circular RLEM and RFEM with the Morgenstern-Price method.

Javankhoshdel et al. (2017) presented the results of a comparison between the circular RLEM and the RFEM. They showed that, for the case of cohesive slopes with isotropic spatial variability, there is good agreement between the results of both methods for different values of spatial correlation length and different values of cross-correlation between soil properties. However, they reported differences between the results of the circular RLEM and the RFEM for the case of anisotropic spatial variability with and without cross-correlation between soil properties.

There are only a limited number of software packages available to geotechnical practitioners to carry out probabilistic slope stability analyses. There are fewer still that can also consider spatial variability of soil input parameters.

A new spatial random probabilistic algorithm is developed for this research and it will be implemented in the next version of the commercial software, Slide. v.8 (Rocscience Inc. 2017) was used to carry out probabilistic analysis considering spatial variability of soil properties, and cross-correlation between random fields using the circular and non-circular RLEM. For the RFEM analysis in this paper, the open-source FEM code (mrslope2d) described by Fenton and Griffiths (2004) (http://courses.engmath.dal.ca/rfem/) was used. This paper presents: 1) An overview of the three methods used; 2) software validation: the results generated using spatial variability in *Slide* are compared to the results presented by Javankhoshdel et al. (2017); 3) a sensitivity analysis to determine the number of slices and number of simulations necessary to generate a confident estimate of probability of failure; 4) the effect of COV values on numerical outcomes using the three methods; 5) the effect of slope angle on numerical outcomes using the three methods; and 6) discussion about computation times. It is shown that the RFEM generally results in higher values of probability of failure compared to the RLEM approach. The non-circular RLEM generally results in higher values of probability of failure compared to the circular RLEM.

2 METHODS OF ACCOUNTING FOR SPATIAL VARIABILITY OF SOIL PROPERTIES

2.1 Random finite element method (RFEM)

Griffiths et al. (2009) applied the RFEM to undrained cohesive and cohesive-frictional soil slopes. A random field of each shear strength parameter (cohesion and friction angle) was generated using the local average subdivision method (LAS) developed by Fenton and Vanmarcke (1990) and mapped onto the finite element mesh. Each node has different values of the soil property assigned to it, but nodes close to each other are correlated using horizontal and vertical correlation lengths. Theoretically, the correlation structures of the underlying Gaussian random field can be determined using the Markov correlation coefficient function:

$$R\left(\tau_x, \tau_y\right)=\exp\left\{-\sqrt{\left(\frac{2\tau_x}{\theta_x}\right)^2+\left(\frac{2\tau_y}{\theta_y}\right)^2}\right\} \qquad [1]$$

where, $R(\tau_x, \tau_y)$ is the autocorrelation coefficient, τ_x and τ_y are the absolute distances between two points in horizontal and vertical directions, respectively. θ_x and θ_y are the spatial correlation lengths in horizontal and vertical directions, respectively. For the isotropic case where $\theta_x = \theta_y = \theta$, Equation 1 can be simplified to:

$$R\left(\tau\right)=\exp\left\{-\frac{2\tau}{\theta}\right\} \qquad [2]$$

where τ is the absolute distance between two points in the isotropic field. In the remainder of the paper, the spatial correlation length is normalized to the height of the slope (H).

2.2 Circular random limit equilibrium method (RLEM)

The circular RLEM is a combination of LEM as a deterministic method of analysis together with the same random field generated for the RFEM analysis explained above, and Monte Carlo simulation. In the verification section of this paper, 5000 Monte Carlo simulations were used to be consistent with the study of Javankhoshdel et al. (2017) who used the same number of realizations. A sensitivity analysis was then carried out using different numbers of Latin Hypercube Samples (LHS).

In the RLEM, a random field is first generated using the local average subdivision (LAS) method and then mapped onto a grid mesh, similar to the FEM mesh in the RFEM analyses. Each mesh cell in the random field has different values of soil properties, and cells close to one another have similar values, based on the value of the spatial correlation length. Then, the circular slip LEM analysis is carried out in each Monte Carlo realization to calculate factor of safety. The combination of the random field and circular failure mechanism in the LEM is shown in Figure 1. In each Monte Carlo realization, a search is carried out to find the mesh elements intersected by the circular slip surface. Random soil property values are assigned to all slices whose base mid-point falls within that element. A limit equilibrium approach (the Simplified Bishop's method or the Morgenstern-Price method) is then used to calculate factor of safety for each Monte Carlo realization. The probability of failure is defined as the ratio of the realizations that failed (F_S<1), to the total number of realizations.

2.3 Non-circular RLEM

The non-circular RLEM used in this study is a combination of a refined search and a LEM approach (the Morgenstern-Price method). The refined search is based on circular surfaces that are converted to piece-wise linear surfaces. The search for the lowest safety factor is refined as the search progresses. An iterative approach is used, so that the results of one iteration, are used to narrow the search area on the slope in the next iteration.

In many cases, for the same number of surfaces, a larger number of slip surface with lower factors of safety were detected than the number determined from conventional grid or slope search techniques.

The refined search in this study was used together with an additional optimization technique. The optimization is based on a Monte Carlo technique, often referred to as "random walking" (Greco 1996). When used in conjunction with a non-circular search this optimization method can be very effective at locating slip surfaces with lower safety factors. Although the option is referred to as "optimization," it can also be considered an additional search method.

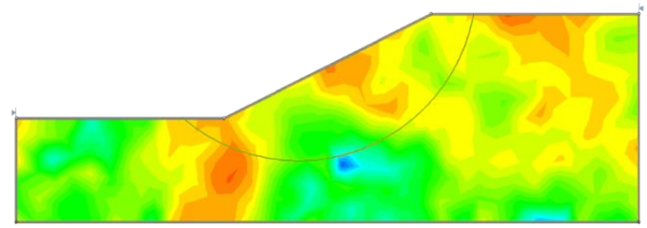

Figure 1. Example failure surface in circular RLEM approach. Cohesion RF: Warmer colors indicate lower cohesion.

This technique is known as "random walking," because a randomly generated number determines the direction that the vertices are moved. There is no complex underlying algorithm that is searching for the surface. The only data that is used to determine whether one surface is preferable to another, is the factor of safety. A detailed explanation of the refined search together with optimization is available in the *Slide* v.7 (Rocscience Inc. 2015) theory manual.

The combination of refined search with optimization and random fields generated using LAS helps to locate the critical slip surface in the spatially variable field. The disadvantage of the circular RLEM, as mentioned by Javankhoshdel et al. (2017) is that the circular RLEM cannot capture irregular shapes of failure. This is especially noticeable in cases with highly fluctuating random fields. However, the optimization technique in the non-circular RLEM, moves the vertices along the slip surface to find the lowest factor of safety. Moving the vertices allows cells with lower values of soil strength in the random field mesh to be found and therefore weaker (more critical) failure paths are located.

Figure 2 shows an example failure mechanism using non-circular RLEM.

It should be mentioned that only isotropic spatial variability is considered in this paper.

3 SOFTWARE VALIDATION

Javankhoshdel et al. (2017) presented comparisons between the results of circular RLEM and RFEM. They showed that there are several cases where the circular RLEM and RFEM

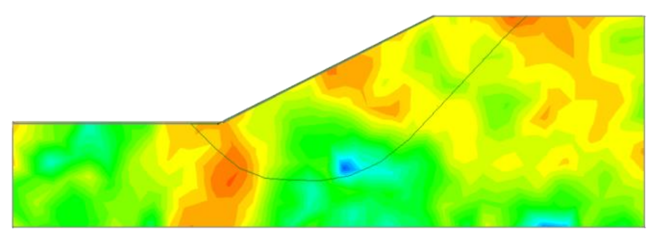

Figure 2. Example failure surface in non-circular RLEM approach. Cohesion RF: Warmer colors indicate lower cohesion.

give the same value of probability of failure for the case of cohesive soil slopes with and without cross-correlation between cohesion and unit weight. In this study, the same example presented by Javankhoshdel et al. (2017) and Griffiths and Fenton (2004) (slope angle β = 27°, coefficient of variation (COV) of soil cohesive strength COV_{su} = 0.5, slope height H = 10 m, depth of foundation soil D = 2 and soil unit weight γ = 17 kN/m^3) was investigated to validate the results generated by the software. As previously mentioned, 5000 Monte Carlo simulations were used for this part of the study. The Simplified Bishop's method was used to calculate factor of safety in each realization

The results of the circular RLEM in the current study were compared to the results of the circular RLEM and RFEM analysis. Figure 3 shows the comparison. In this figure, probability of failure is plotted against normalized spatial correlation length (θ/H) for different values of F_s to examine the differences (if any) between results using the two methods. It can be seen in Figure 3 that there is a detectable but negligible difference between the results of the circular RLEM and the RFEM using the current software and the results presented by Javankhoshdel et al. (2017).

Figure 4 shows the same comparison, while also considering a cross-correlation coefficient between cohesion and unit weight of ρ = 0.7. It can be seen that, similar to Figure 3, there is good agreement between the results using different approaches considering cross-correlation between soil properties. However, probability of failure calculated using the circular RLEM in this study is the highest when compared to the probabilities of failure presented by Javankhoshdel et al. (2017) for F_s = 0.9 and is the lowest for $F_s \geq$ 1.15.

4 SENSITIVITY ANALYSIS

4.1 General

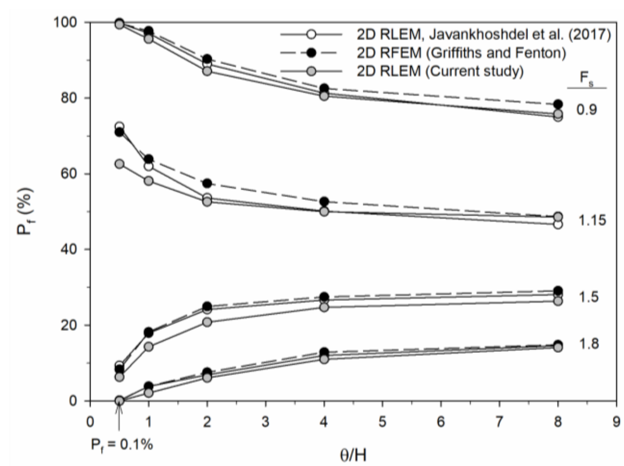

Figure 3. Influence of spatial variability of undrained cohesive soil strength on probability of failure using circular RLEM and RFEM approaches (β = 27°, COV_{su} = 0.5, ρ = 0).

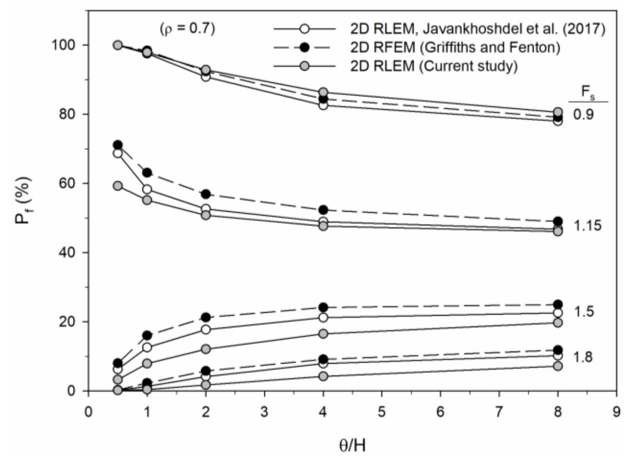

Figure 4. Influence of spatial variability of undrained cohesive soil strength on probability of failure using circular RLEM and RFEM approaches (β = 27°, COV_{su} = 0.5, ρ = 0.7).

A sensitivity analysis was carried out to find the required number of slices for the LEM analysis as well as the required number of Latin Hypercube simulations, for spatially variable cohesive frictional slopes. This sensitivity analysis is carried out in this study for non-circular LEM and circular LEM separately. The Morgenstern-Price approach was used to calculate factor of safety.

Figure 5 presents the results of the sensitivity analysis using the non-circular RLEM (refined search). The figure examines probability of failure values for 1) different values of spatial correlation length, 2) different numbers of slices, and 3) two different numbers of Latin Hypercube simulations. It can be seen that for larger values of spatial correlation length, there is negligible difference between the values of probability of failure after 100 slices. For θ/H = 2 and θ/H = 8, there is also a negligible difference between the results with different numbers of LH simulations. For θ/H = 0.5 and θ/H = 0.2, 200 slices together with 4000 LH simulations was determined to be sufficiently accurate.

Figure 6 presents the results of the sensitivity analysis using the circular RLEM and Morgenstern-Price to calculate factor of safety. The figure examines probability of failure values for 1) different values of spatial correlation length, 2) different numbers of slices, and 3) two different numbers of Latin Hypercube simulations. The results of sensitivity analysis show that for the circular RLEM, 100 slices and 4000 LHS provide sufficient accuracy. It should be noted that with the circular RLEM, the number of slices required to reach sufficient accuracy is less than with the non-circular RLEM. However, the circular RLEM in this study uses 200 slices and 4000 LHS in order to be consistent with the non-circular RLEM.

5 EFFECT OF COV VALUES

In this section, the comparison between the results of the RLEM (circular and non-circular) and the RFEM is carried out

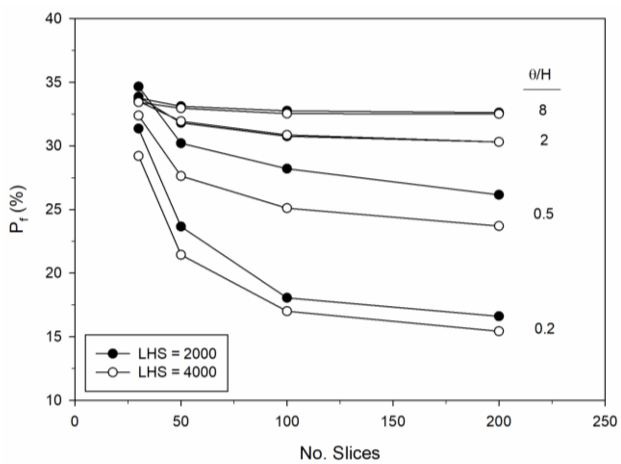

Figure 5. Sensitivity analysis for non-circular RLEM method.

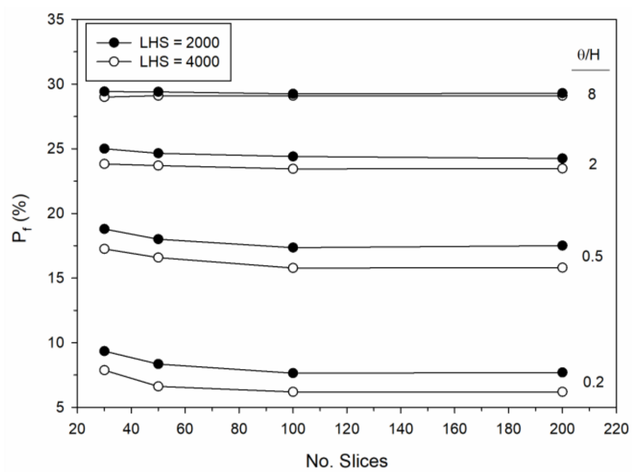

Figure 6. Sensitivity analysis for non-circular RLEM method.

for different values of COV of soil properties and for different values of spatial correlation length.

Figure 7 shows probabilities of failure for different values of spatial correlation length calculated using the circular and non-circular RLEM, and the RFEM. In this figure, $COV_c = COV_\phi = 0.2$ and $F_s = 1.1$. It can be seen that, increasing spatial correlation length increases probability of failure in all three approaches. Also, for the same values of spatial correlation length, the RFEM results in the highest probabilities of failure. The non-circular RLEM results in higher probabilities of failure compared to the circular method. In this figure $\beta = 27^o$.

Figure 8 compares the results of different combinations of coefficient of variation values of soil parameters: $COV_c = 0.2$ and $COV_\phi = 0.2$ (Figure 7), and $COV_c = 0.5$ and $COV_\phi = 0.2$. The non-circular RLEM is used in this figure. As expected, probability of failure increases with increasing spatial correlation length. For the same spatial correlation length, increasing COV_c increases probability of failure.

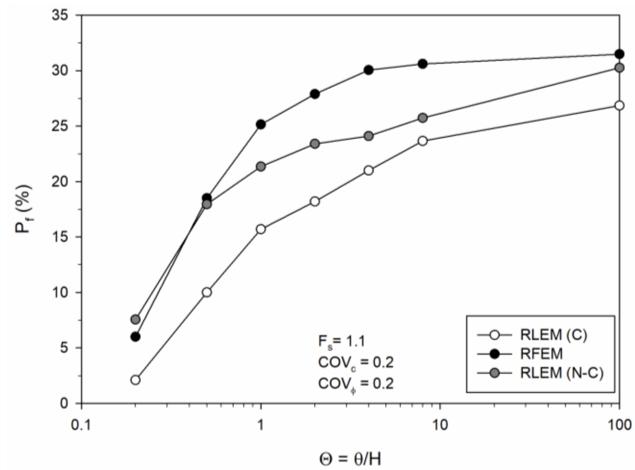

Figure 7. Comparison between the results of RLEM (non-circular and circular) and RFEM for different values of spatial correlation length. ($COV_c = 0.2$, $COV_\phi = 0.2$ and $F_s = 1.1$)

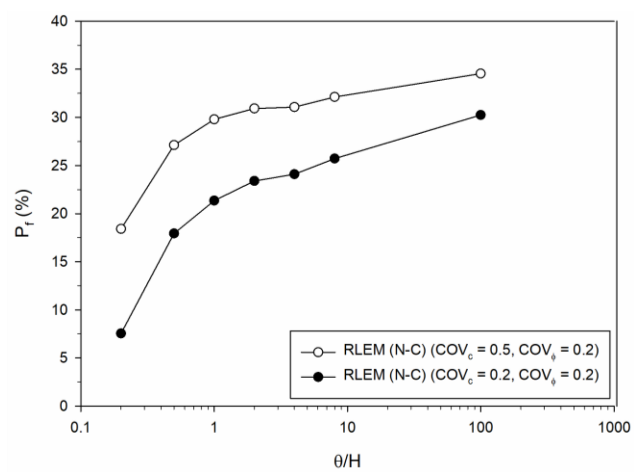

Figure 8. Influence of COV on probability of failure for different values of spatial correlation length using non-circular RLEM approach.

Figure 9 shows the comparison between the Mean F_s values for different combinations of COV: $COV_c = 0.2$ and $COV_\phi = 0.2$, and $COV_c = 0.5$ and $COV_\phi = 0.2$. The Mean F_s values in this figure correspond to probability of failure presented in Figure 8. It can be seen that increasing spatial correlation length increases Mean F_s. However, as the COV_c increases from 0.2 to 0.5 for the same spatial correlation length, Mean F_s decreases, which has the reverse trend compared to probability of failure in Figure 8. The deterministic factor of safety in this example is $F_s = 1.1$. It can be observed in this figure that for smaller values of spatial correlation length, Mean F_s is less than deterministic factor of safety. As the spatial correlation length increases, the Mean F_s approaches the deterministic factor of safety which corresponds to the probability of failure for infinity spatial correlation length (random probability of failure).

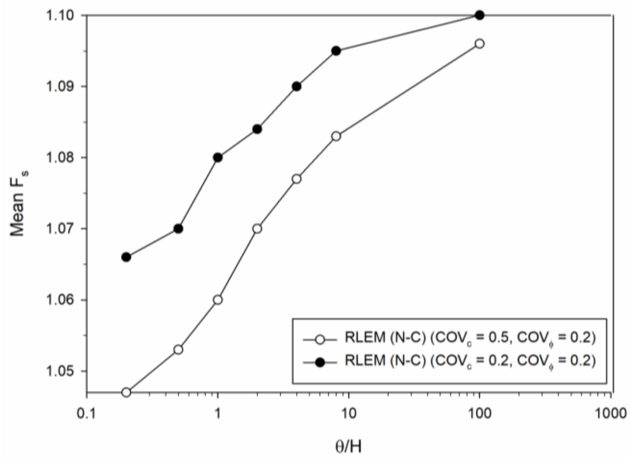

Figure 9. Influence of COV on Mean F_s values for different values of spatial correlation length using non-circular RLEM approach.

A negative cross-correlation between cohesion and friction angle, a positive cross-correlation between cohesion and unit weight, and a positive cross-correlation between friction angle and unit weight is reported in literature. Figure 10 shows the comparison between the results of the RFEM and the non-circular RLEM with and without cross-correlation between soil properties. In this figure $COV_c = COV_\phi = 0.2$ and $COV_\gamma = 0.1$. The curves without cross-correlation are the same curves presented in Figure 6. It can be seen in this figure, as in Figure 6, that increasing spatial correlation length increases probability of failure. For the same value of spatial correlation length, considering cross-correlation decreases probability of failure. Also, for the same spatial correlation length, the RFEM gives higher values of probability of failure compared to the non-circular RLEM with and without cross-correlation between soil input parameters.

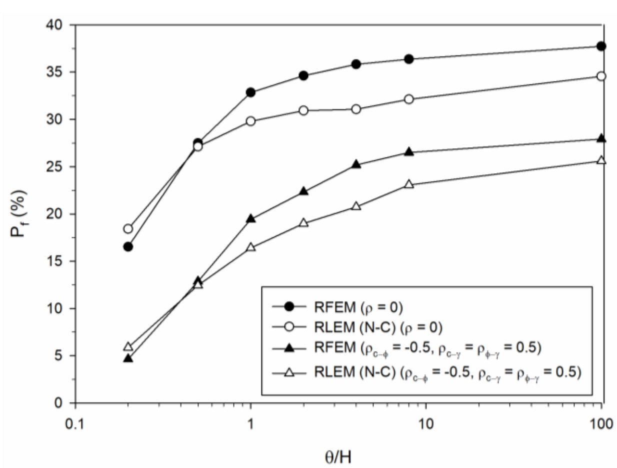

Figure 10. Comparison between the results of non-circular RLEM and RFEM for the case of $COV_c = 0.2$ and $COV_\phi = 0.2$ and $COV_\gamma = 0.1$ with and without cross-correlation between soil properties.

6 EFFECT OF SLOPE ANGLE

6.1 General

So far, only $\beta = 27°$ is investigated in this study. This section shows a comparison between the results of $\beta = 27°$ and $\beta = 63.5°$.

Figure 11 shows the comparison between probabilities of failure using the RFEM, the non-circular RLEM, and the circular RLEM for different values of spatial correlation length and for two different slope angles ($\beta = 63.5^o$ and $\beta = 27^o$). In this figure, $F_s = 1.2$. A different cohesion value was used for each slope angle in order to keep the factor of safety value constant at $F_s = 1.2$. It can be seen that as the slope becomes steeper probability of failure increases. The reason for this, as reported by Javankhoshdel and Bathurst (2014), is that to get the same factor of safety when increasing the slope angle, the ratio of $c/\gamma H\tan\phi$ should also increase. Increasing slope angle and $c/\gamma H\tan\phi$ increases probability of failure.

It can be seen in this figure that, for values of $\theta/H \geq 2$, the RFEM gives higher probabilities of failure compared to the non-circular RLEM. However, for $\beta = 63.5^o$ and $\theta/H < 2$, the non-circular RLEM gives higher probabilities of failure. On the other hand, the non-circular RLEM gives much closer values to the RFEM when compared to the circular RLEM.

It can also be seen that the difference between probabilities of failure using the RFEM and the circular RLEM increases by increasing the slope angle from $\beta = 27^o$ to $\beta = 63.5^o$. This can be due to the different failure mechanism for shallow and steep slopes, i.e. deep failure for $\beta = 27^o$ and toe failure for $\beta = 63.5^o$. This failure mechanism effect is better captured using the RFEM or the non-circular RLEM.

Figure 12 is similar to Figure 11, but with correlated soil parameters. The same trend as Figure 11 can be observed in this figure. As reported by Javankhoshdel et al. (2017), differences between probabilities of failure calculated using the RFEM and the circular RLEM become larger for the case with cross-correlated soil properties (compared to Figure 1 for $\beta = 63.5^o$). As with Figure 11, for $\theta/H \geq 2$, the RFEM gives higher probabilities of failure compared to the RLEM. However, for $\beta = 63.5^o$ and $\theta/H < 2$, the non-circular RLEM gives higher probabilities of failure.

7 COMPUTATION TIME

Observation of computation times for each simulation showed that the circular RLEM results in the shortest computation times, and should be used when the failure surface is known to be circular. The circular and non-circular RLEM both result in considerably shorter computation times when compared to the RFEM. It is concluded that the non-circular RLEM provides results that are in good agreement with the RFEM, with a 50% reduction in computational effort.

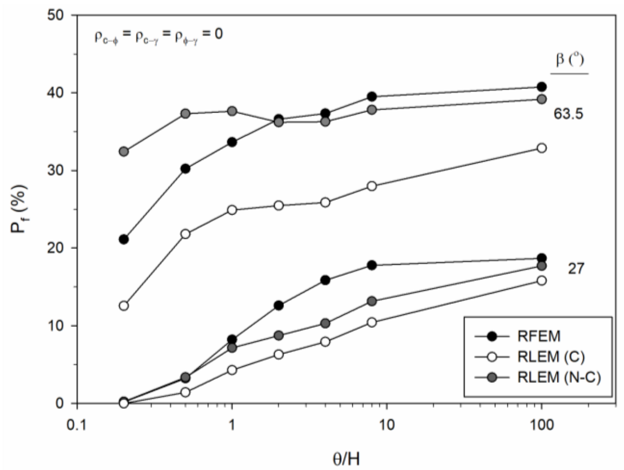

Figure 11. Comparison between the results of RLEM (circular and non-circular) and RFEM for different values of Spatial correlation length and for β = 27° and β = 63.5°.

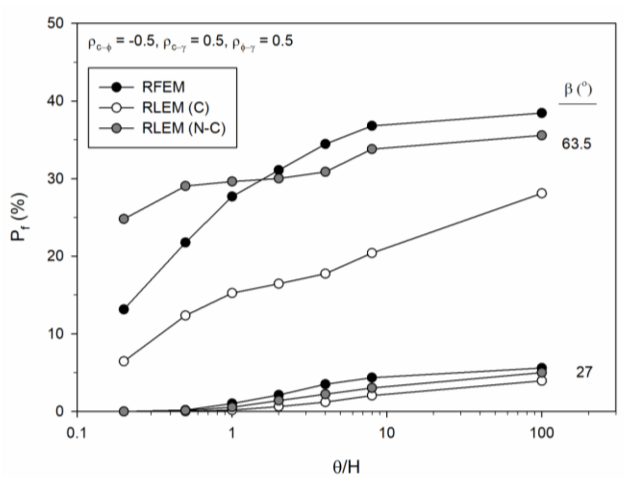

Figure 12. Comparison between the results of RLEM (circular and non-circular) and RFEM for different values of Spatial correlation length and for β = 27° and β = 63.5° (considering cross-correlation between soil properties).

8 CONCLUSION

In this study, the results of three different methods to characterize spatial variability of soil properties are examined. The non-circular and circular RLEM and the RFEM were compared for the same simple slopes with cohesive-frictional soil strength.

Comparison of RLEM and RFEM with 2D isotropic random fields showed that RFEM gives higher values of probability of failure when compared to the non-circular and circular RLEM approaches with and without considering cross-correlation between soil input parameters.

As the COV of soil properties increases, probability of failure also increases using the non-circular RLEM analysis. The corresponding Mean F_s decreases as expected.

The influence of slope angle is also investigated in this study. As the slope angle increases, to get the same factor of safety, the value of cohesion should increase and $c/\gamma H \tan\phi$ also increases. Numerical outcomes are in agreement with Javankhoshdel and Bathurst (2014) who showed that increasing the slope angle and also increasing $c/\gamma H \tan\phi$, the probability of failure increases.

For shallower slopes in cohesive frictional soil the results of the RFEM and the non-circular and circular RLEM are closer. As the slope becomes steeper the differences between the results of the circular RLEM, RFEM, and non-circular RLEM become larger. This is due to the difference in failure mechanisms for steep and shallow slopes with cohesive-frictional soil.

The differences in probabilities of failure using the circular RLEM with and without cross-correlation between soil properties is larger than the differences between probabilities of failure using the RFEM and the non-circular RLEM analyses with and without cross-correlation between soil properties.

The circular and non-circular RLEM result in much shorter computation times when compared to the RFEM.

Finally, it is expected that the non-circular RLEM gives similar results to the RFEM approach provided the slip surface is free to find the weakest path in the non-circular RLEM analysis. However, in a few examples, there are cases that result in insufficient agreement between the non-circular RLEM and the RFEM. Further study is required to investigate the reason for these differences which might be due to the influence of element size or number of Latin Hypercube simulations.

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