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On Using Spatial Methods for Heterogeneous Slope Stability Analysis

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ABSTRACT: Kriging is a well-documented spatial interpolation technique used in geotechnical engineering problems where a material property is governed by an unknown spatial distribution. This paper studies the effect of kriging on slope stability analysis and compares it to other interpolation methods. Statistical distributions describing the factor of safety (FS) of a trial slope are generated. For reference, purely random Gaussian fields with increasing variance are examined first. Kriging and other spatial interpolation methods are then introduced using subdomains of the random field as input points. The number of known points is found to have a significant effect on the FS distribution, underscoring the importance of good sampling methods. Kriging has a smoothing effect on the input data and kriged predictions revert to the mean of the input points when no input points are nearby. The interpolated fields it produces tend quickly to the reference value of the FS as the number of input points becomes larger. In this respect, kriging outperforms other interpolation methods by supporting the results of homogeneous analyses while accommodating measured deviations from the mean rock properties. However, the smoothed field generated by kriging does not reproduce the statistical features of the original data. It may omit potential failure mechanisms due to localized, probabilistic weakness in the rock mass. Several representative examples of rock slopes are presented in this paper to illustrate the effects of using kriged estimates to calculate the overall FS for a slope stability problem.

1. INTRODUCTION

Recent trends in geotechnical design and analysis have emphasized the importance of spatial variability of rock and soil properties. The properties of in-situ rock masses are governed by diverse depositional processes, stress history, groundwater behaviour, and differing mineralogical makeup. Because of these natural factors, properties are difficult to measure and exhibit considerable spatial variation even within small fields of study. The cost and effort required for exhaustive sampling and study force the geotechnical engineer to make design decisions based on sparse data. Numerical methods for continuum models such as the Finite Element Method (FEM) and the Finite Difference Method (FDM) require good knowledge of material properties if their results are to be of practical use. Traditionally, numerical analysis has been dependent on average or representative rock mass properties, and has compensated for inherent variability using healthy factors of safety (FS). Neglecting the effect of spatial variability can lead to underestimation of the true risk associated with a design.

The strength of spatially variable materials has been studied extensively by various authors [1-4] using the principles of geostatistics (see [5] for an overview). The geostatistical approach assumes that a spatially variable rock property such as shear strength or hydraulic conductivity is a realization of a random field, denoted by a random variable $Z(\boldsymbol{x})$ that follows some known statistical distribution. The degree of spatial correlation exhibited by $Z(\boldsymbol{x})$ is quantified by the variogram function $\gamma(\boldsymbol{h})$, where $\boldsymbol{h}$ is a vector separating two points in the field.

Kriging is a linear interpolation method that estimates unknown values based on a weighted linear sum of unknown values (see e.g. [6]). A kriged point $Z(\boldsymbol{x_0})$ is calculated as:

$$Z(\boldsymbol{x_0}) = \sum_{i=1}^{n} w_i Z(\boldsymbol{x_i})$$

where $\{w_1, w_2 \dots w_n\}$ is the vector of weights that minimizes the variance of the kriged estimate based on a known spatial correlation structure. The simplest form of kriging is ordinary kriging, which assumes that a field has an unknown but spatially constant mean.

The major drawback to kriging is the difficulty of developing a reliable variogram $\gamma(\boldsymbol{h})$ – experimental variograms are grid-dependent, site-specific, and require large sets of data. Journal and Huijbregts [7] found that the results of kriging are resilient with respect to the choice of variogram. From a design standpoint, a desirable variogram is not necessarily an accurate depiction of the true field of rock properties, but one that minimizes the effect of this uncertainty on the predicted factor of safety. Based on this premise, only analytical variograms are used for kriging in this paper. For most simple variogram models, the specification of a suitable correlation length is all that is required; the nugget parameter is assumed to be zero and the sill equal to the field variance.

This paper tests the application of kriging to rock slope stability problems, with the goal of assessing the robustness and accuracy of kriging as an interpolation method to map the shear strength parameters to any point in a continuous rock slope. Kriged values of rock shear strength parameters are mapped to individual nodes on a FEM mesh.

2. SPATIAL METHODS AS APPLIED TO HETEROGENEOUS SLOPE ANALYSIS

2.1. Spatially Variable Materials

It has been observed [8] that increasing the variability of a soil, as quantified by the coefficient of variation (COV) of its strength properties, lowers the mean FS of a slope stability problem, as well as increasing the COV of the output FS distribution. Effectively, this implies that large degrees of rock heterogeneity produce designs that are both statistically weaker and more susceptible to outlying values of the FS.

A major consideration in slope stability analysis is the choice of correlation length (also referred to as the scale of fluctuation or the range of correlation). Correlation length is mathematically defined as the area under the correlation function, and can be thought of as a measure of the "roughness" of a rock property. A material with zero correlation length is purely random – unknown properties cannot be predicted even in very close proximity to known points. A material tends towards homogeneity as its correlation length approaches infinity.

The effect of correlation length on the strength of spatially random materials is an open question. For some classes of problems, a "worst-case" normalized correlation length of approximately 1 exists, i.e the probability of failure (PF) reaches a maximum when the scale of fluctuation is approximately equal to the nominal size of the problem. This can be explained in the context of slope stability problems by the tendency of the failure surface to seek areas of localized weakness. For very short correlation lengths, a potential failure path connecting adjacent weak zones becomes prohibitively long and tortuous. For very high correlation lengths, the soil approaches homogeneity and no preferred failure paths exist. For correlation lengths on approximately the same scale of the problem, areas of localized weakness can lead to preferential, local failure paths and a lower FS. Spatially random materials can also introduce asymmetry to normally symmetrical problems such as footings.

2.2. Interpolation of Spatial Variables

The focus of this paper is the interpolation of spatially variable rock properties for slope stability problems. Both kriging and inverse distance weighting (IDW) are applied to test slopes and the resultant factors of safety are compared. IDW is a conceptually similar linear sum method that assigns weights to know points based on absolute distance from the unknown point. The two methods are usually compared using jackknifing or cross-validation. Results have been mixed – some authors [9] found that kriging reproduced known measurements with greater accuracy than IDW, while others [10] have found IDW to be superior or that neither is more effective to a practical degree. Both interpolation schemes address rapid, discrete fluctuations in soil properties poorly. This had led to some hybrid methods [11] that use *a priori* knowledge regarding material constitution to categorize data and krige within local regions.

It stands to reason that kriging produces more accurate maps of unknown variables when there is significant spatial correlation within a rock mass. With this in mind, this paper will consider purely random slopes where rock properties have a known mean and variance but no spatial correlation (zero correlation length). With regards to the accuracy of kriging in predicting the random field, this is a worst-case scenario. Essentially, the sampled values randomly seed the kriging process, which then produces a smoothed field of rock properties with some degree of spatial correlation. In this way, a correlation length is artificially induced by the interpolation method. It should be noted that the smoothing effect produced by kriging is undesirable – kriged fields underestimate the true variability of rock masses and leading to an inherently unconservative approach.

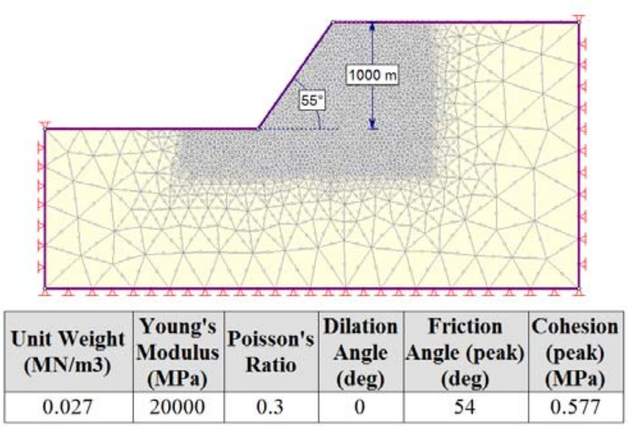
IDW also performs relatively poorly when no spatial correlation exists. In this respect, the cases studied will put both methods on a level playing field. It should be noted, however, that there is some local averaging on an element-by-element basis required for the FEM. This artificially induces some spatial correlation, but with a suitably fine mesh this is negligible with respect to the correlation length studied.

Ordinary kriging was used in all cases. The correlation length chosen for kriging was a fixed multiple of the two most distant points within the basis, a reasonable proxy for the characteristic size of the problem. This is similar to the scaled ranges used in literature [1]. The default scaled correlation length was 1.5, and the effect of modifying this parameter was also tested. The nugget parameter has been set to zero; the sill is equal to the field variance.

A third interpolation method first developed by Chugh [12] was also tested. This method makes no assumptions regarding the spatial structure of the rock and performs extrapolation very poorly.

2.3. *Slopes with Random Mohr-Coulomb Strength Properties*

A rock slope subject to the Mohr-Coulomb failure criterion was modelled in the two-dimensional plane-strain FEM program *Phase*^{2} [13]. Figure 1 shows the model geometry, material properties and boundary conditions.



Unit Weight (MN/m3)	Young's Modulus (MPa)	Poisson's Ratio	Dilation Angle (deg)	Friction Angle (peak) (deg)	Cohesion (peak) (MPa)
0.027	20000	0.3	0	54	0.577

Figure 1: Two-dimensional rock slope model.

These properties have been drawn from a slope studied using the FEM in previous work [14]. Both *c* and φ were assumed to be purely random spatial variables with zero spatial correlation.

Three levels of soil variability were considered. Table 1 summarizes the properties of each random field. Both Mohr-Coulomb parameters were assumed to follow normal distributions truncated at three standard deviations.

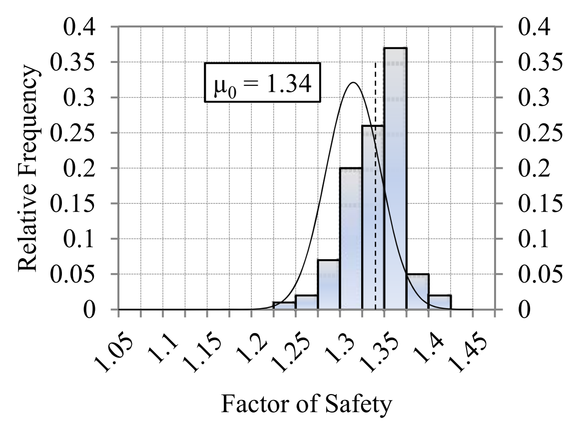
Table 1: Standard deviation (σ) and coefficient of variance of shear strength parameters for test rock masses.

Rock Type	Cohesion (*c*) (MPa)		Friction Angle (φ) (°)	
	σ	COV	σ	COV
I	0.025	4.3%	2	3.7%
II	0.05	8.7%	5	9.3%
III	0.10	17.3%	10	18.5%

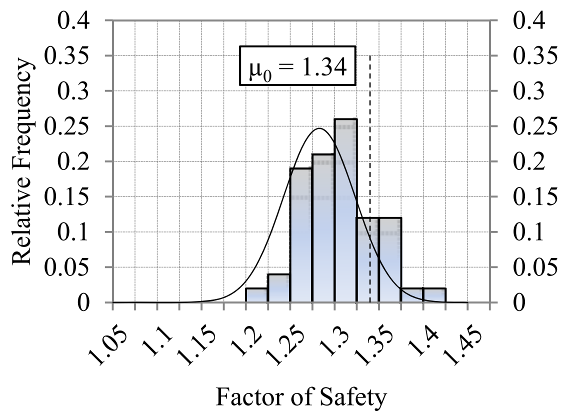
The FS of each test slope was calculated using the shear strength reduction (SSR) method [15,16]. The primary advantage of SSR is that it does not require assumptions about the shape and position of the failure surface. While the limit equilibrium method would be much less computationally demanding, it is probable that potential failure mechanisms due to complex slip surfaces through heterogeneous rock could be ignored.

As a preliminary check, 100 realizations of each random rock mass were tested. Figure 2 shows the resultant distributions of the FS. The homogeneous factor of safety μ_0 = 1.34 is shown for reference.

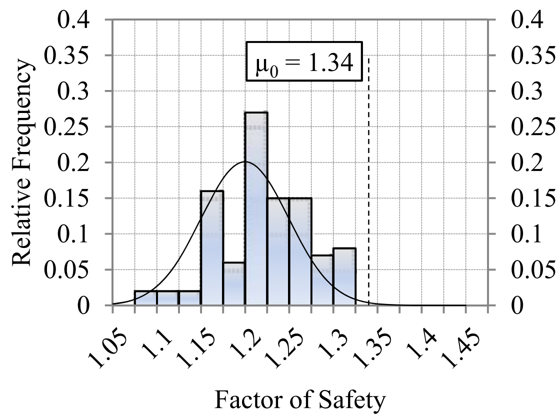
For all three spatially variable materials, the observed mean FS is well below the homogeneous FS of 1.34, as can be expected. Increasing rock COV both lowered the mean FS and increased the variance of the FS distribution. Figure 3 shows a sample random map of rock properties; Figure 4 shows two characteristic failure mechanisms



(a) Rock Type I



(b) Rock Type II



(c) Rock Type III

Figure 2: FS Distributions of randomized slopes and fitted normal distributions.

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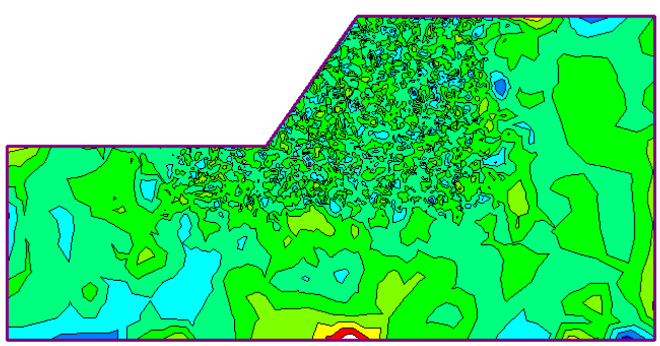


Figure 3: Friction angle contour map for a representative random slope of Type III.

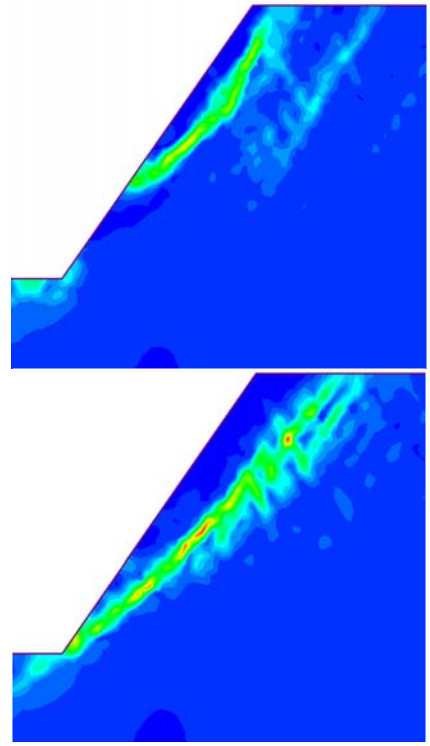


Figure 4: Shear strain contour map at failure for two sample slopes of rock type III.

Generally, there was little variation in the position of the mechanism; most failure surfaces were nearly circular and fairly shallow. Most passed through the toe of the slope. The left slope in Figure 4 is an example of shallower failure due to local weakness.

Findings in literature regarding the strength of spatially variable materials indicate that the cases tested – which have only short, contrived correlation distances due to FEM mesh local averaging – may not represent the worst-case effect on the factor of safety. However, the random slopes are significantly weaker than the homogeneous, deterministic slope and provide a good basis for comparison with interpolated slopes.

2.4. Randomly Seeded Interpolated Rock Slopes

For the purposes of this paper, the fully randomized slopes such as Figure 3 are assumed to be representative of worst-case (from an interpolation standpoint) in-situ rock conditions. It is not expected that kriging or other methods will accurately predict the random field; this paper seeks instead to examine the effect of kriging on the output FS distribution and whether it is comparable or conservative relative to the fully random slopes.

Further experiments will operate under the premise that a random subset of known points has been drawn from the random slopes. The set of known points will be referred to as the basis for interpolation. The size of the basis was varied from 10 to 200 points, simulating different extents of sampling. Kriging, IDW and the modified Chugh's Method (MCM) were all applied to these subsets. For each realization, random bases were drawn from the random in-situ field. A total of 100 SSR realizations were conducted for each combination of rock type, basis size and interpolation method.

Figure 5 shows a representative test slope for rock Type III, as mapped using the three interpolation methods. The basis (set of ten known points) for all three cases is the same.

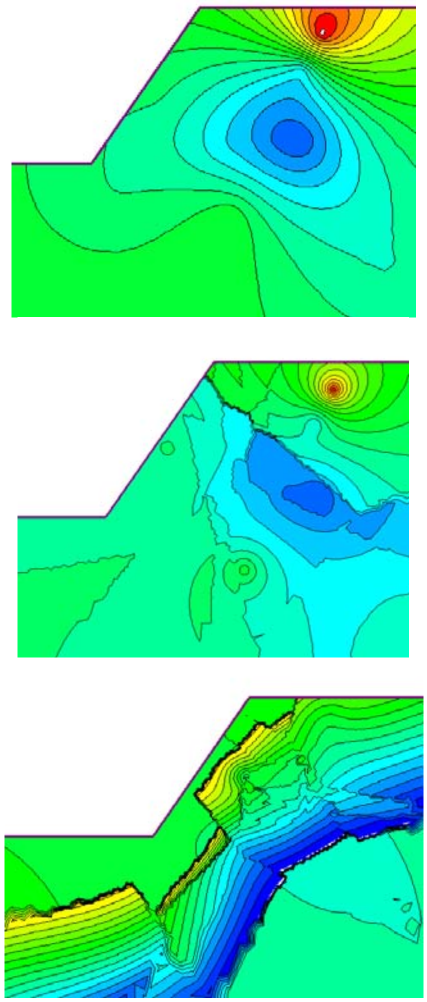


Figure 5: Friction angle contour maps for one randomly seeded slope interpolated using kriging, IDW, and MCM (top to bottom).

When either kriging or IDW is used, local maxima and minima correspond to known (and randomly generated) basis points. In areas that are relatively distant from known points, the interpolated estimate reverts to the mean, as shown by the leftmost two figures. This smoothing effect does not exist for MCM, resulting in the more poorly defined plot at bottom. Note that the lowermost plot has necessarily been drawn using a larger scale, and the real discrepancy between MCM and kriged maps is even larger than shown.

To illustrate the smoothing effect of kriging, consider the four representative surface plots shown below. All four are plots of randomly seeded friction angle fields that have been smoothed using kriging and mapped onto the finite element mesh.

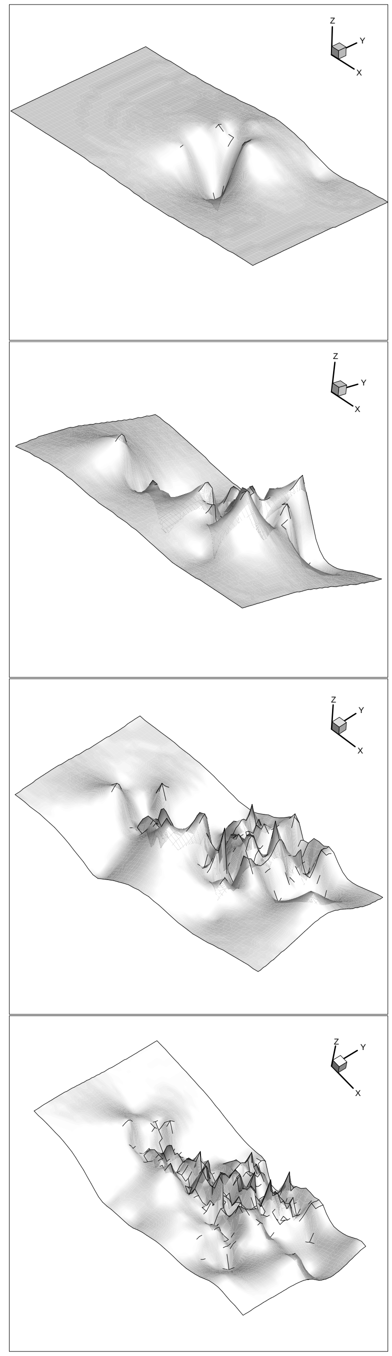
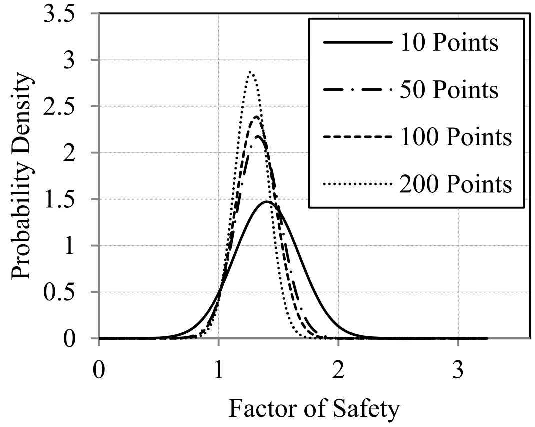


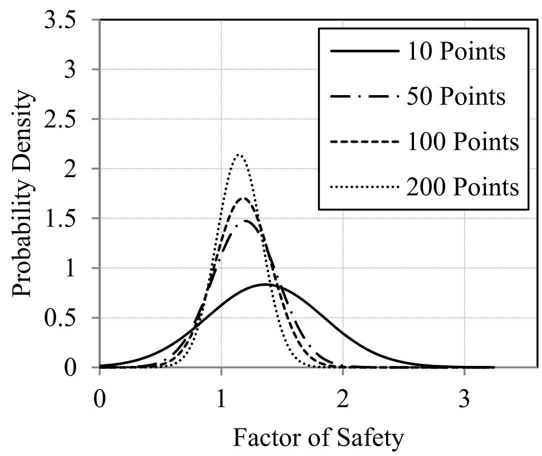
Figure 6: Surface plots of Type III rock cohesion for sample randomly seeded, kriged fields. Seeded (known) points appear as local extrema. Fields have been seeded with (top to bottom) 10, 50, 100, and 200 points.

For fields with small bases (few known points), the smoothing effect of kriging is very pronounced.

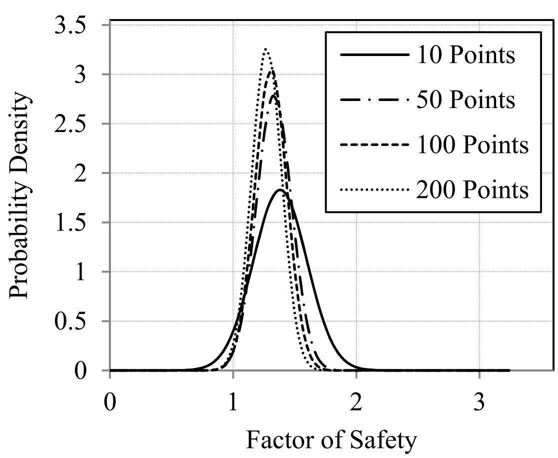
In general, it was found that increasing the size of the basis both lowered the mean FS and decreased its variance. Figure 7 shows normal distributions fitted to the output FS distribution for kriged Type III slopes using each interpolation method.



(a) Kriging



(b) MCM

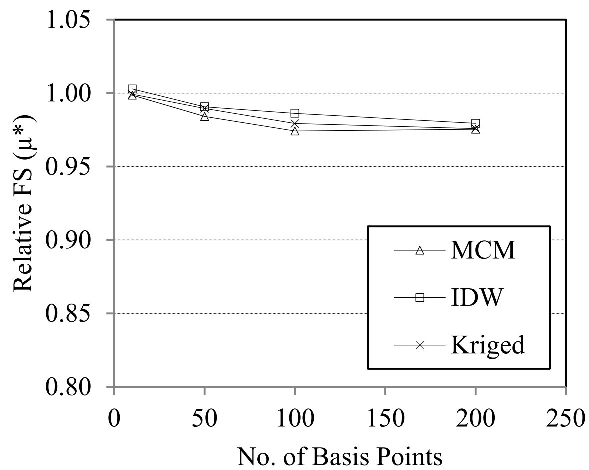


(c) IDW

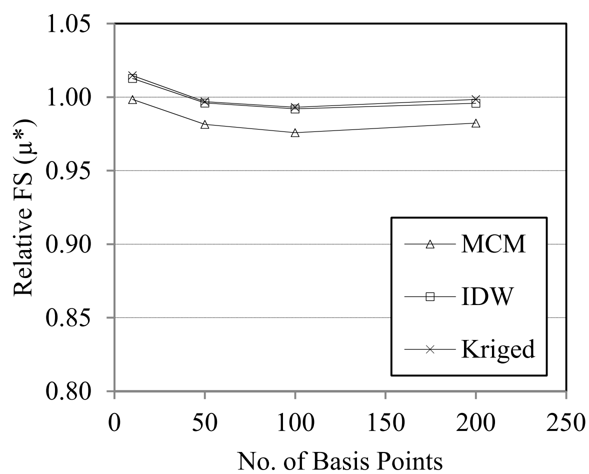
Figure 7: Fitted normal FS distribution for Type III slopes seeded with 10, 50, 100 and 200 random points and three interpolation methods.

These two trends are common to all interpolation methods and rock types tested and in accordance with previous literature. Because introducing more random points to the field counteracts the smoothing effect of interpolation, increasing the size of the basis increases the spatial variability of the soil. This is analogous to decreasing the correlation length in that it produces a rougher field, which has been shown to decrease the FS within a certain range of correlation lengths. The decrease in the variability can be attributed to the high dependence of the kriged FS on the mean of the basis points. Because the seeded points are a subset of a larger field with a known constant mean, larger bases will tend to have a sampled mean that approaches the global mean.

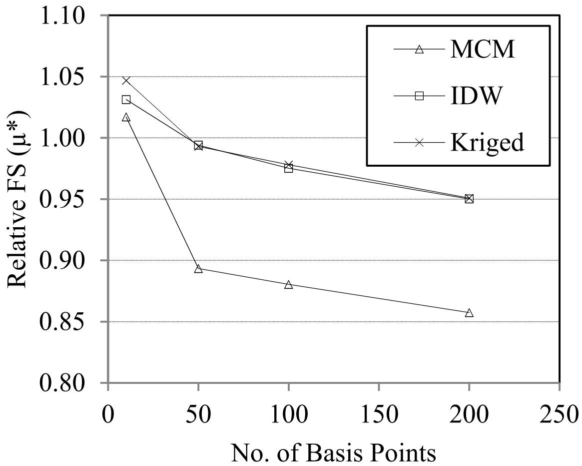
Figure 8 and Figure 9 show the normalized mean FS $\mu^* = \mu/\mu_0$ and coefficient of variation as a function of basis size for each rock type.



(a) Type I Rock

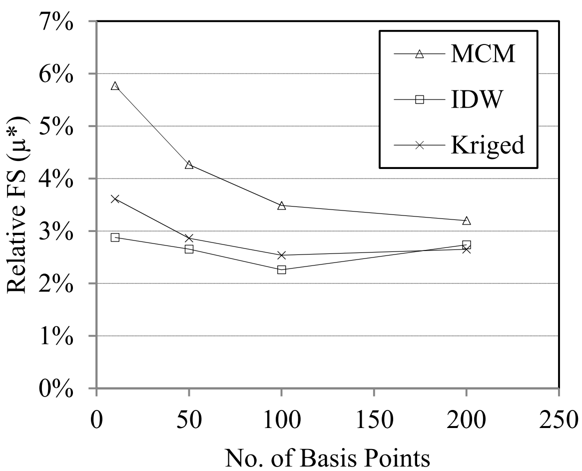


(b) Type II Rock

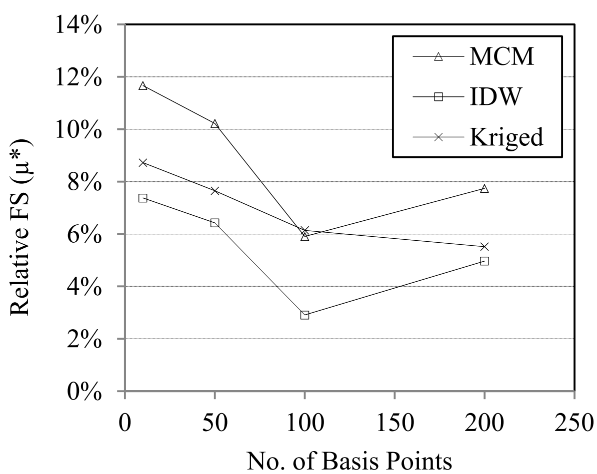


(c) Type III Rock

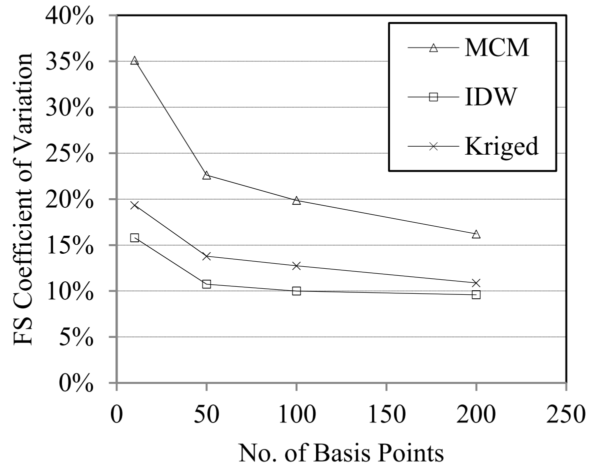
Figure 8: Normalized mean FS as a function of basis size for three interpolation methods.



(a) Type I Rock



Type II Rock



(b) Type III Rock

Figure 9: FS coefficient of variation as a function of basis size for three interpolation methods.

For rock with low variability, little deviation from the deterministic FS was observed and the FS variability was low, as would be expected. For rock with moderate variability, little FS deviation was again observed. Interestingly, the FS variability decreased for 100-point bases relative to 10-point bases before increasing again for 200 points. This can be explained with reference to the two opposing sources of variability for the FS distribution, being variability in the mean of the sampled basis and variability due to increasing the number of random local extrema. It would seem that for the Type II rock, the stabilizing effect of a larger basis is offset by the increasing roughness of the field.

Overall, both IDW and kriging performed much better than MCM in producing a consistent estimate of the FS. This is most pronounced for the third (high-variability) rock type. MCM performed more poorly for Type III rock, producing FS results that were both less accurate and less precise than kriged estimates. Differences in FS distributions using kriging and IDW were not significant; the two methods seem equally suited to this class of rock slope stability problem.

The differences between the FS distributions for each interpolation method decrease as the number and density of sampled points increases. Differences between interpolation methods are within the bounds of statistical sampling error when the number of sampled points is 100 or 200. From a geotechnical engineering standpoint, it is most useful to compare the performance of interpolation methods for slopes with the sparsest data.

Figure 10 compares the FS as calculated using kriging and IDW for 100 sample slopes, using a random basis of 10 points. Points between the two dotted lines correspond to slopes with an FS that diverges by less than 0.05 for the two methods. Neither method appears to be conservative relative to the other. For comparison, consider Figure 11, which plots the kriged FS against that of MCM.

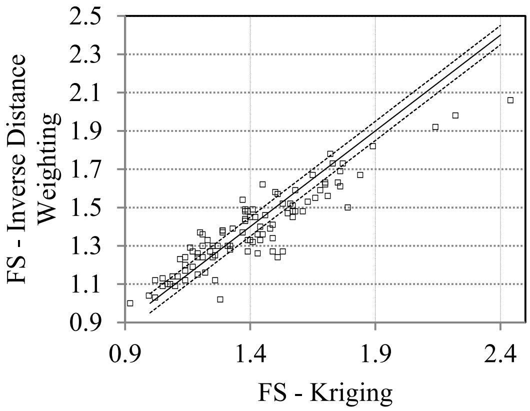


Figure 10: Comparison of calculated FS values from kriging and IDW for Type III rock slopes with bases of 10 points.

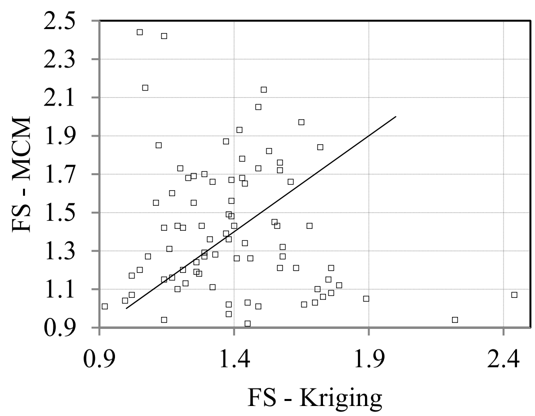


Figure 11: Comparison of calculated FS values from kriging and MCM for Type III rock slopes with bases of 10 points.

There is very poor correlation between the FS as estimated by kriging and MCM. This also holds true for IDW and MCM.

To this point, the effectiveness of each interpolation method has been defined relative to the homogeneous factor of safety. It is desirable that the FS calculated from a randomly sampled basis be robust with respect to the choice of basis. According to this criterion, the lower variances for the FS distributions observed from IDW and kriging are advantageous. However, relative to the purely random slopes previously considered, both methods overestimate the mean and underestimate the variance of the FS. This is a result of smoothing and suggests that interpolation based on small numbers of samples does not conservatively address rock variability. A kriged estimate of the FS can thus be thought to partially account for spatial variability, accounting for measured deviations from mean properties without fully capturing the heterogeneity-induced failure that can occur in spatially variable rock masses.

3. SUMMARY & CONCLUSIONS

For slope stability shear strength reduction (SSR) analysis, mapping random, spatially variable shear strength properties on to finite element method (FEM) meshes has a significant effect on the predicted factor of safety, producing a mean FS lower than the homogeneous value. This effect is more pronounced the greater the variability of the rock mass studied. From a design standpoint, this is advantageous in that it allows the designer to anticipate failure due to localized deviations from mean rock properties.

This paper tested only one example slope geometry, and assumed that some properties (e.g. elastic modulus, Poisson's ratio) were homogeneous. Only ordinary kriging with a fixed scaled correlation length of 1.5 was applied. The SSR method identifies only bulk rotational failure mechanisms with a well-defined shear surfaces. The coefficient of variation (COV) of rock properties ranged from 3% to 18%.

Kriging, inverse distance weighting (IDW) and the modified Chugh's Method (MCM) were used to interpolate fields of shear strength properties from sets of sample known points. The effect of rock COV, sampling density, and interpolation method on the output distribution of the factor of safety were examined. It was found that all methods performed reliably at large sampling densities and/or low rock mass variability. For sparse data and high rock variability, kriging and IDW produced output distributions with the lowest variability and most accurate mean. Both methods were judged to be effective, while MCM was deemed inadequate at low sampling densities. Direct comparison between IDW and kriging revealed that the factors of safety (FS) calculated for test slopes by each method were highly correlated.

Kriging and IDW were found to be relatively robust with respect to the sampling basis. While this is a desirable property, the consequent mean FS values remain higher than for the purely random slopes. This suggests that kriged rock property distributions may not fully account for the weakening effects of high spatial variability.

Under the conditions studied, kriging did not perform significantly better or worse than IDW. Kriging has the additional capabilities of estimating the uncertainty of predicted values, accommodating spatially varying means for rock properties, and capturing the spatial correlation of random fields. For these reasons, it is of the most use to the geotechnical engineer, especially relative to geometric methods such as MCM. The results of this paper support the use of kriging as a tool for probabilistic slope stability analysis.

The current paper did not explore the effect of modifying kriging or IDW parameters. Effectively, the random fields studied were a "worst-case" scenario for both methods. This was thought to be a necessary first step. Were spatially correlated slopes to be tested, it is expected that kriging and IDW would be more accurate relative to the purely randomized fields.

The number of SSR trials tested for each case (100) may not be enough for good statistical reproducibility. A convergence test and/or corroboration with simpler limit equilibrium methods would support the findings of this paper.

All interpolation methods suffer from an undesirable smoothing effect. Because higher degrees of heterogeneity tend to weaken rock masses, this makes for an unconservative process. Future work will examine the effect of more advanced probabilistic methods such as sequential Gaussian simulation (SGS), which preserves the statistical properties of measured natural fields. As with all shear strength reduction (SSR) and Monte Carlo (MC) studies, computational expense is a major difficulty. More refined sampling methods such as Latin Hypercube sampling (LHS) could mitigate this problem.

4. ACKNOWLEDGEMENTS

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