

NUMERICAL MODELLING OF SLOPE UNCERTAINTY DUE TO ROCK MASS JOINTING

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ABSTRACT: Analysis of the stability of slopes in jointed rock masses is not trivial and is fraught with uncertainty and risk. This paper evaluates the ability of different probabilistic methods to model slope stability uncertainty caused by randomness in the geometry of joint networks. The stability of slopes is modelled with a Finite Element-based, shear strength reduction method. The probabilistic techniques considered are the point estimate, response surface, reliability, Monte Carlo and Latin Hypercube methods. The intent of the probabilistic analysis is to calculate the statistical moments of the distribution of factors of safety.

The paper establishes that Monte Carlo simulation is the most appropriate method for analyzing uncertainty caused by joint network randomness. Because of the diversity in factors of safety and failure modes that stem from joint network randomness, it is concluded that probabilistic analysis must be applied more regularly to improve understanding and the robustness of design.

Keywords: Rock Slopes; Probabilistic Analysis; Risk Analysis; Probability of Failure; Joint Networks; Discrete Fracture Networks; Jointed Rock Masses, Finite Element Method; Shear Strength Reduction Analysis;

1. INTRODUCTION

Rock masses are formed under wide-ranging complex physical conditions. As a result, the engineering of excavations in rock involves uncertainty. In such an environment, predictions based on single evaluations (typically using averaged values) have practically zero probability of ever being realized, and conclusions based on them are therefore open to question. The risks (which we will define as the probability of unpleasant circumstances) associated with this uncertainty must be managed.

Uncertainty can be dealt with in different ways. Parametric and scenario analysis – the assessment of possible ranges of behaviours through variation of input properties and consideration of different conditions – can be applied. However statistical simulation is even more powerful. In addition to identify possible outcomes, it can quantify uncertainty and estimate the likelihoods of outcomes occurring. As a result probabilistic simulation helps engineers to develop more robust and economic designs and solutions.

The geometric layout of networks of joints and other types of discontinuities in a rock mass is a significant contributor to rock mass behaviour and uncertainty. This geometry causes rock mass strength and deformation properties to be directional, and influences the manner in which excavations fail.

Numerical methods such as the Finite Element Method (FEM) and the Discrete Element Method (DEM) have been successfully applied to slope stability analysis [1-4]. This is achieved through the Shear Strength Reduction (SSR) approach [5-11] for calculating factor of safety.

A primary advantage of these methods is their versatility. They can model a broad range of continuous and discontinuous rock mass behaviours without *a priori* assumptions on failure mechanisms.

Their capabilities have helped soften the boundaries between the classification of rock slope stability problems into categories such as wedge-type failures controlled discontinuities, step-path failures that combine slip along joints with shearing through

intact material, and rotational-type failures in which rock masses essentially behave as continua.

This paper intends to evaluate the impact joint network geometry randomness has on the stability of slopes in blocky rock masses. It will analyze a slope example using the FEM-based shear strength reduction approach, which in the rest of this paper this approach will be referred to as FEM-SSR analysis.

The paper will provide overviews of methods for performing engineering probabilistic analysis. It will then examine their applicability to the analysis of joint network geometry induced slope risks.

2. PROBABILISTIC METHODS FOR SLOPE STABILITY ANALYSIS

The ultimate goal of a probabilistic slope stability analysis is to obtain the complete distribution of factor of safety values given a set of random (uncertain) input variables with specified statistical properties. In this case, factor of safety is known as the response variable, the functional form that relates input to factor of safety termed the response function. From the distribution of factor of safety values, probability of failure can be determined.

It is generally difficult to obtain an output probability distribution when the response function is complex or implicit. In such cases, attempts are made to only determine statistical moments of the output distribution, and not the distribution itself.

Statistical moments are quantities that capture both overall and in-depth information on the geometry (location and appearance) of a probability distribution function. The mean is the first statistical moment. It provides information on the location of a distribution. The other moments, which are of higher order, are commonly taken about the mean. The second moment of a statistical distribution is variance. It describes the spread or dispersion of the distribution about the mean. The third and fourth moments, skewness and kurtosis, respectively, provide further information on distribution shape.

We will investigate probabilistic methods used in engineering risk analysis, and assess how applicable each is to modelling rock slope uncertainty due to joint network geometry.

2.1. Reliability Methods

Given an explicit response function, it is relatively easy to use the first two moments – mean and

variance – to estimate the statistical moments of the response variable [12]. Methods based on this approach are termed first order second moment (FOSM) methods [12]. The most popular FOSM methods approximate the response function through partial derivatives or Taylor series expansion (about mean values) truncated to low order terms. They then compute statistical moments from these terms.

Reliability methods have the advantage of being fast and compact, because they use few evaluations of the response function to estimate moments [12]. Their main drawbacks are that they require partial derivatives, which may not be easily attainable. Also they cannot be applied to response functions that are not explicit.

Because the FEM, as well as the SSR, is an algorithm, and not an explicit response function, reliability methods cannot be applied to our rock slope problem.

2.2. Point Estimate Method (PEM)

The Point Estimate Method (PEM) was originally developed by Rosenblueth [13, 14]. As the name suggests, the PEM uses a series of point estimates – point-by-point evaluations of the response function at selected values (weighting points) of the input random variables – to compute the moments of the response variable. The method applies appropriate weights to each of the point estimates of the response variable to compute moments. The weights can differ for different points. Although the method is very simple, it can be very accurate [15, 16].

The PEM requires the mean and variance (and sometimes the third moment, skewness) as input variables. Unlike reliability methods, it can be used when partial derivatives are difficult to evaluate or obtain. It can be readily applied to response functions that are not closed-form or explicit, and to the results of existing deterministic programs. As a result of the latter benefit, it was considered for the analyses in this paper.

The PEM uses two weighting values – typically one standard deviation to each side of the mean – for each input random variable. Since the choice of weighting points is strictly dictated by the random inputs to a problem, the PEM different response variables (e.g. factor of safety of particular slope benches, displacements at specific locations, etc.) to be changed in an FEM analysis, without having to re-compute point estimates.

The main disadvantage of the PEM is that it suffers from the ‘curse of dimensionality’ – as the number of random variables increases the number of point evaluations increases exponentially, significantly increasing computational effort. Modifications have been made to the method [17, 18] that reduce the number of point evaluations. However, these modifications move weighing points farther from mean values as the number of dimensions increases, and as a result can lead to input values that go beyond valid domains [16].

2.3. Response Surface Method (RSM)

Like the PEM, the Response Surface Method (RSM) is based on point estimates of the response function. After calculating these point estimates, the RSM computes a polynomial response surface that approximates the original response function. This is usually accomplished through least-squares linear regression. The RSM then applies the conventional reliability method to this explicit polynomial approximation.

The manner in which the RSM selects points at which to estimate response functions differs from that employed by the PEM. The RSM uses design of experiment techniques to best locate the points at which experiments (response function evaluations) will be run so that the fit of the polynomial surface to the true response surface meets specified criteria [12].

The RSM becomes less accurate when the region of interest is expanded, unless more point estimates are employed [12].

2.4. Monte Carlo Method

The Monte Carlo method is very powerful and flexible, and can be applied to a very wide range of problems. It is also very simple to use and accurate. In the Monte Carlo method samples of probabilistic input variables are generated and then used to perform a number of deterministic computations. Information on the distribution and moments of the response variable is then obtained from the resulting simulations.

Because Monte Carlo simulations can be used on existing deterministic programs without modifications, they are popular for probabilistic FEM analysis [19]. Like the PEM it allows for multiple response functions in a single model. Unlike the PEM and RSM, it is not affected by the ‘curse of dimensionality,’ since the number of

simulations required is independent of the number of input random variables.

The primary disadvantage of the Monte Carlo method is that it can be computationally expensive, requiring many simulations in order to achieve desired accuracy [20].

2.5. Latin Hypercube Method (LHM)

Latin Hypercube sampling is a technique for improving the efficiency of Monte Carlo simulations [19]. It achieves this efficiency through better selection of input samples. Whereas the Monte Carlo method randomly selects samples from the valid domain of a variable, which results in an ensemble of numbers without guarantees, the Latin Hypercube method (LHM) adopts a more systematic sampling approach. It first divides the domain of an input variable into a number of equal sized bins. It then obtains a random sample from each of those bins. This ensures an ensemble of random numbers that more accurately conforms to the input probability distribution over the domain.

As a result of these improvements, the LHM produces accurate estimates of response variable distributions and their moments with fewer simulations. This speeds up computations.

3. APPLICABILITY OF STATISTICAL METHODS TO UNCERTAINTY INDUCED BY JOINT NETWORK GEOMETRY

There are challenges in applying most of the above-described methods to modelling uncertainty arising from joint network geometry. For blocky rock masses, the locations of the discontinuities relative to each other and relative to slope geometry have significant impact on how failure mechanisms form and propagate.

This is best illustrated with a simple example. Imagine a joint network consisting of two joint sets, each of equal, constant spacing. We will also assume that the joints are infinitely long. Lastly let us assume that we are analyzing a slope with height five times the spacing of the joints. Even for this very simple model, we can see that the location of joints and rock blocks relative to the slope toe would make a difference in how the slope fails.

Let us now assume that the spacing is stochastic, with a known mean and standard deviation (square root of variance). If we were to apply the PEM, for example, we would evaluate point estimates of

factor of safety at two spacing values: (mean – standard deviation) and (mean + standard deviation). As discussed above, although these spacing values are fixed, they each do not lead to unique values of factor of safety. Completely different factors of safety can arise depending on the locations of generated joints. This is not the situation in the PEM with non-geometric random properties such as strength and deformation parameters.

The randomness induced by joint network geometry even for constant network parameters makes it next to impossible to apply several of the probabilistic methods discussed above. It eliminates reliability methods and the RSM. Reliability methods are ruled out both by the inability to calculate partial derivatives with respect to joint network parameters (and by the use of the FEM).

It makes it difficult to implement the PEM and LHM. For the PEM, as illustrated above, point estimates at weighting points are not unique, but are dependent on geometry. The authors decided however to investigate whether there was any possibility of the PEM's ensemble adequately describing statistical moments of responses on an example to be described in the next section.

The difficulties in applying the Latin Hypercube method lie in the requirement to divide the domain of a variable into a number of bins. In many models for simulating joint networks, the number of times random variables such as spacing and orientation have to be sampled is not known *a priori* (not known ahead of time). Even if a number of Latin Hypercube samples larger than would be required in a simulation were to be generated, some of them would not be used, reducing the effectiveness of the method.

Monte Carlo simulation is the only method that can readily handle this problem without any special treatment or adaptation. However, it has the primary disadvantage (mentioned previously) of being computationally expensive, requiring many simulations in order to achieve desired accuracy. The authors found though that, if the goal is to use the method to estimate the moments of the response variable, and not to acquire detailed information on the response distribution, then it can give adequate results without too many simulations.

Some tests conducted for factor of safety computations with a limit equilibrium program,

Slide [21], indicated that sample sizes around 40-50 give adequate estimates of the mean. Such sample sizes generally overestimate the second moment (variance) slightly. A functional form can then be assumed for the response distribution and a probability of failure calculated based on this assumption [22]. The overestimated variance used in these functional forms lead to conservative estimates of probability of failure. This is not necessarily an unfavourable outcome.

4. EXAMPLE – ESTIMATION OF ROCK SLOPE PROBABILITY OF FAILURE

The performance of the PEM and Monte Carlo simulation on the evaluation of slope stability uncertainty due to rock mass jointing was evaluated on a simple, homogeneous rock slope with a network of joints. The FEM program *Phase²* [3], which performs SSR analysis and simulates joint networks, was used. The goal was to estimate the first two moments – mean and variance – of the distribution of factors of safety for the slope.

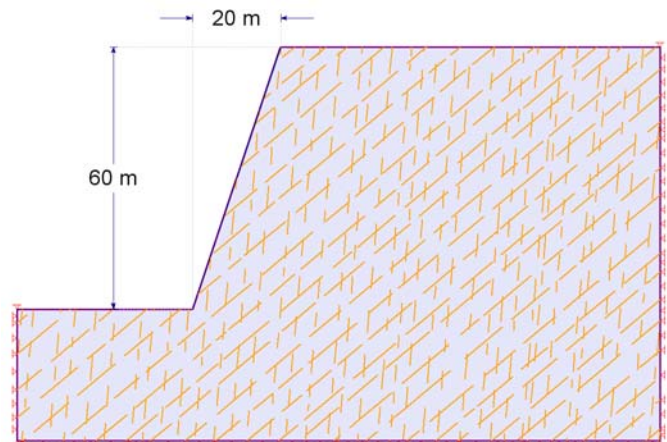


Fig. 1. Basic geometry of the slope. The figure also depicts a realization of the stochastic joint network. Notice the irregularity of joint spacing and length.

The rock mass was assumed to have two sets of parallel joints, each of which has stochastic spacing and joint lengths (in *Phase2* such joint sets are termed ‘parallel statistical’). The properties of these joint sets are shown in Table 1. All the joints had constant Mohr-Coulomb strength, with 0.01 MPa cohesion and 20° friction angle. They also had a constant normal stiffness of 100,000 MPa/m and shear stiffness of 10,000 MPa/m.

The intact rock was assigned Mohr-Coulomb strength as well. It had zero tensile strength, 1 MPa cohesion and 30° friction angle.

Table 1. Parameters of Joint Network

Joint Set	Parameters
Set 1	<i>Orientation</i> Inclination to Horizontal: 40°
	<i>Spacing</i> Distribution: Normal Mean (μ_1): 5m Standard deviation (σ_1): 1m Minimum: 1m Maximum: 9m
	<i>Length¹</i> Distribution: Normal Mean (μ_2): 10m Standard deviation (σ_2): 3m Minimum: 1m Maximum: 19m
	<i>Persistence²</i> Distribution: None Mean: 0.8
	<i>Orientation</i> Inclination to Horizontal: 85°
	<i>Spacing</i> Distribution: Normal Mean (μ_3): 4m Standard deviation (σ_3): 1m Minimum: 1m Maximum: 7m
	<i>Length</i> Distribution: Normal Mean (μ_4): 4m Standard deviation (σ_4): 1m Minimum: 1m Maximum: 7m
	<i>Persistence</i> Distribution: None Mean: 0.5

1. Parallel statistical joints are coplanar, separated from each other by intact rock bridges. Length refers to the total length of a joint the rock bridge to it.
2. The ratio of joint length to the sum of joint length and rock bridge length is the (length) persistence.

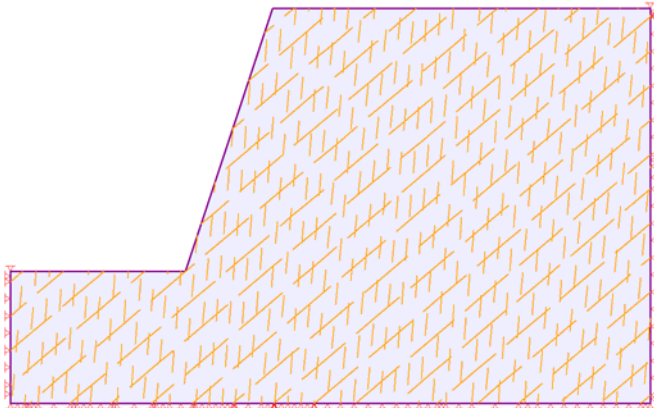


Fig. 2. Example of joint network generated for PEM analysis. Notice the regularity of joint spacing and length in this model.

Since there are four stochastic variables in the example, the PEM evaluation of factor of safety moments involved 2^4 (=16) point estimates. For each combination of weighting variables, we generated parallel joints with constant (deterministic) spacing and joint length.

Table 2. Two sets of point evaluations of factor of safety

#	Weighting Point	Factor of Safety (1)	Factor of Safety (2)
1	++++	1.16	0.95
2	----	1.54	1.74
3	+++-	1.68	1.26
4	---+	1.71	1.5
5	++--	1.06	1.07
6	--++	2.02	2.53
7	+---	1.43	1.81
8	-+++	1.06	1.07
9	+++	1.65	1.47
10	---	0.92	1.17
11	+++	1.02	1.03
12	--+	2.09	2.05
13	+++	1.29	2.56
14	---	0.88	0.84
15	++-	1.93	2.06
16	+-+	0.99	0.93
	Mean Factor of Safety	1.402	1.503
	Variance	0.157	0.298

Because the location of joints in a network changes factor of safety results, and we wanted to check whether the ensemble the PEM adequately estimates the moments of the factor of safety distribution, we performed two sets of 16 point estimate calculations. (The networks realizations in the two sets differed completely from each other.) The results of the PEM computations are shown in Table 2.

We adopted the following widely-used naming rules for our PEM combinations listed in Table 2:

1. For any given stochastic variable, a weighting value of (mean + one standard deviation) is denoted with a “+”, while (mean – one standard deviation) is denoted with a “-”.
2. In all inscriptions for weighting points, the symbols for the stochastic variables appear in the same sequence in which the variables are listed in Table 1.

For example, ++++ refers to the weighting point $([\mu_1 + \sigma_1], [\mu_2 + \sigma_2], [\mu_3 - \sigma_3], [\mu_4 + \sigma_4])$.

Fig 2 illustrates a joint network realization corresponding to this weighting point. The first and second moments were calculated from the PEM [23] according to the following equations:

1. The mean $E(F) = \sum_{i=1}^{16} P_i F_i$, where the P_i s are the weights. For our example the weights have a constant value equal to $1/16$.
2. The variance $Var(F) = E(F^2) - [E(F)]^2$

Next, 40 Monte Carlo realizations of parallel statistical joint networks with the properties specified in Table 1 were generated. A factor of safety was then determined for each one of those models. The factors of safety ranged from a minimum of 0.93 to a maximum of 2.08. A histogram plot showing the distribution of the factors of safety is shown on Figure 3. The distribution is skewed to the right. The 40 factors of safety had a mean equal to 1.2803 and variance of 0.0883.

5. DISCUSSIONS

From the differences in the results of the two sets (ensembles) of PEM calculations, it is evident that the method does not reliably estimate the moments of the factor of safety distribution, especially the variance. This leaves Monte Carlo simulation as the only method that can estimate factor of safety uncertainty due to rock mass jointing with any consistency.

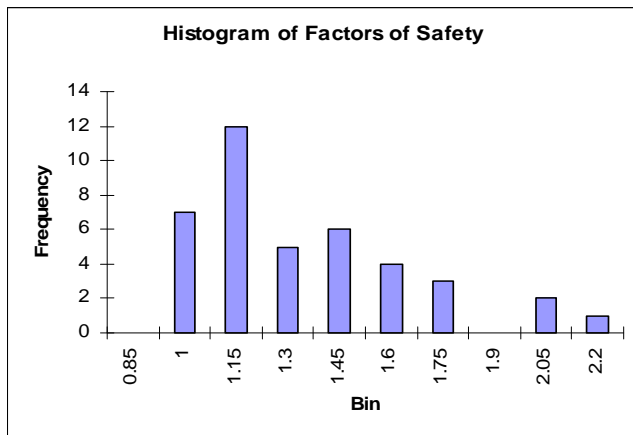


Fig. 3. Histogram plot of factors of safety from Monte Carlo analysis.

The moments calculated from the Monte Carlo analysis can be used to determine a probability of failure assuming either a normal or log-normal distribution for factor of safety using equations described in Wolff, 1999. (Since the factor of safety

distribution on Fig 3 is skewed, it would be more accurate to assume a log-normal distribution for factor of safety.)

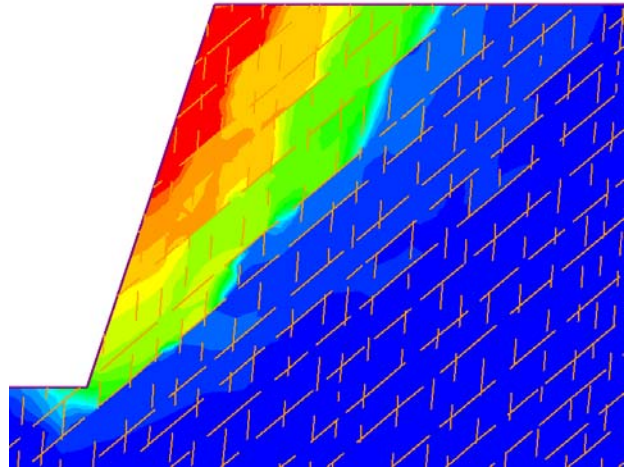


Fig. 4. Contours of total displacement for random joint network which gives factor of safety = 2.08.

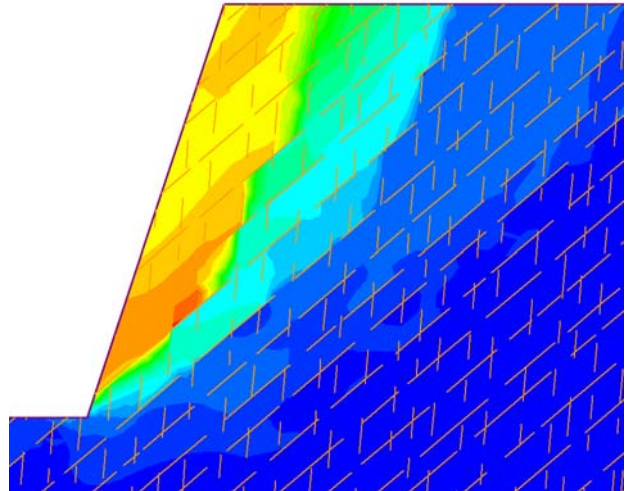


Fig. 5. Contours of total displacement for random joint network which gives factor of safety = 1.95.

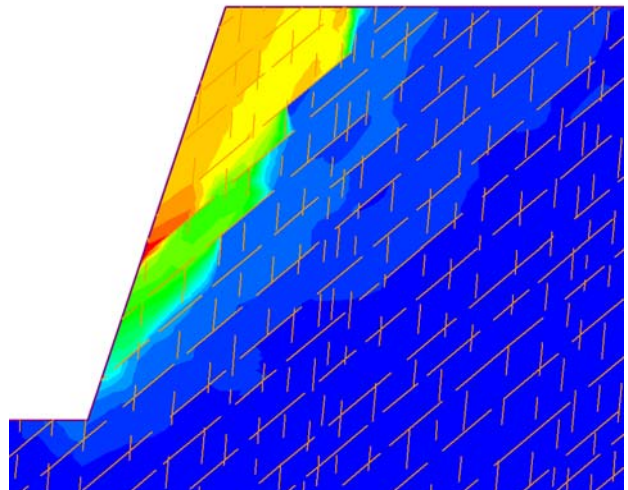


Fig. 6. Contours of total displacement for random joint network which gives factor of safety = 1.94.

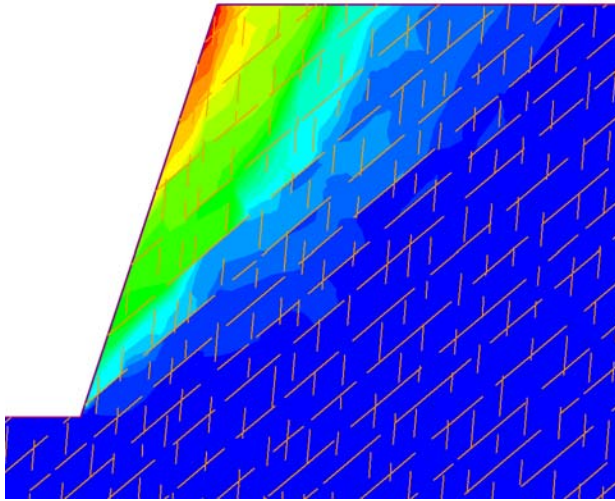


Fig. 7. Contours of total displacement for random joint network which gives factor of safety = 1.35.

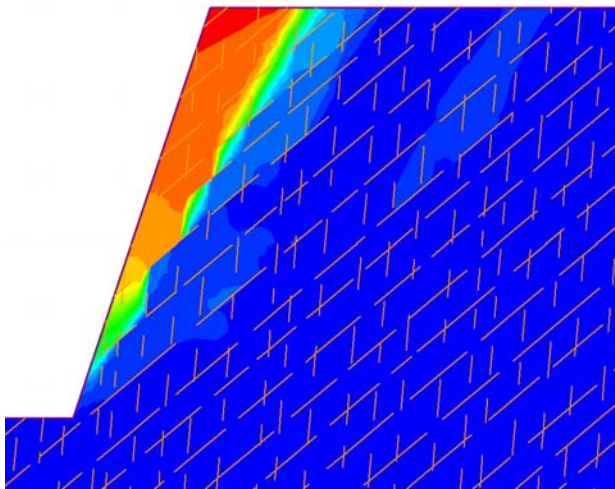


Fig. 8. Contours of total displacement for random joint network which gives factor of safety = 0.95.

The moments calculated from the Monte Carlo analysis can be used to determine a probability of failure assuming either a normal or log-normal distribution for factor of safety using equations described in Wolff, 1999. (Since the factor of safety distribution on Fig 3 is skewed, it would be more accurate to assume a log-normal distribution for factor of safety.)

The ability of FEM-SSR analysis to capture various types of failure without special treatments is powerful. The interplay between excavation geometry, joint network geometry, and joint and rock mass strength and stiffness properties is complex and almost impossible to anticipate prior to analysis. FEM-SSR yields results that at the very least can serve as the basis for further investigation. They make it possible to examine different design

ideas, and obtain meaningful results with a single tool.

Monte Carlo with FEM-SSR is computational expensive. However it is very feasible with today's multi-processor desktops. On a single desktop all the examples shown in this paper could be done in less than 48 hours. Because Monte Carlo simulations can be very easily computed in parallel, using a cluster of four desktops reduces computational time to only a few hours.

Plots of total displacement contours at failure for each computed Monte Carlo model yielded rich insights into the range of possible slope failure modes. The failure modes, shown on Figs 4 – 8, range from near surface wedge failure to deep-seated rotational-type mechanisms. Many of the failures exhibit step-path mechanisms that combine slip along joints with shearing through intact rock.

The diversity of failure modes is true even of situations with similar factors of safety. For example, the mechanisms shown on Figs 5 and 6 are very different in nature and yet have about the same factor of safety.

These analysis results indicate the need to perform probabilistic analysis on excavations in jointed rock masses. Such risk analysis promotes greater understanding of problems. It increases the chances of success through more robust design of excavations, stabilization measures and improved monitoring decisions.

Risk analysis permits assessment of relative increase (or decrease) in safety by different measures. The chances of successful design substantially improve when rock engineers can anticipate the range of different failures that can arise.

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