Influence of mesh size, number of slices, and number of simulations in probabilistic analysis of slopes considering 2D spatial variability of soil properties

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ABSTRACT

The Random Limit Equilibrium Method (RLEM) is a relatively new method of probabilistic slope stability analysis which uses a combination of 2D random field theory, limit equilibrium methods, and Monte Carlo simulation. The Random Finite Element Method (RFEM) uses a combination of 2D random field theory, finite element method of analysis, strength reduction method, and Monte Carlo simulation. In this paper, the effects of mesh size, number of slices, and number of Monte Carlo simulations on computed probability of failure are investigated using both approaches. Computation times using both methods to solve the same slope problem are also compared. Recommendations for mesh size, number of slices, and number of Monte Carlo simulations, with respect to the spatial correlation length, using RLEM are presented.

INTRODUCTION

Two methods of probabilistic slope stability analysis which consider 2D spatial variability are examined in this study: the non-circular Random Limit Equilibrium Method (RLEM), and the Random Finite Element Method (RFEM). The RLEM is a relatively new method of probabilistic slope stability analysis which uses a combination of 2D random field theory, circular or non-circular limit equilibrium methods and Monte Carlo simulation. RFEM uses a combination of 2D random field theory, finite element method of analysis, strength reduction method, and Monte Carlo simulation. Two disadvantages of the RFEM method are the large computational effort required, and convergence problems for the case of slopes with very small mesh size. The purpose of this paper is to examine the effect of mesh size, number of slices, and number of Monte Carlo simulations (MC) on computed probability of failure (PF) using RLEM analysis, and to compare the results using RFEM.

RFEM. Griffiths et al. (2009) applied the RFEM to undrained cohesive and cohesive-frictional soil slopes. A random field of each shear strength parameter (cohesion and friction angle) was generated using the local average subdivision method (LAS) developed by Fenton and Vanmarcke (1990) and then mapped onto the finite element mesh. The elements are assigned different values of each soil property, but elements close to each other are correlated using horizontal and vertical correlation lengths (Θ). Theoretically, the correlation structures of the underlying Gaussian random field can be determined using the Markov correlation coefficient function:

$$R\left(\tau_x, \tau_y\right)=\exp\left\{-\sqrt{\left(\frac{2\tau_x}{\theta_x}\right)^2+\left(\frac{2\tau_y}{\theta_y}\right)^2}\right\} \quad [1]$$

where, $R(\tau_x, \tau_y)$ is the autocorrelation coefficient, τ_x and τ_y are the absolute distances between two points in horizontal and vertical directions, respectively. θ_x and θ_y are the spatial correlation lengths in horizontal and vertical directions, respectively. For the isotropic case where $\theta_x = \theta_y = \theta$, Equation 1 can be simplified to:

$$R\left(\tau\right)=\exp\left\{-\frac{2\tau}{\theta}\right\} \quad [2]$$

where τ is the absolute distance between two points in the isotropic field. In the remainder of the paper, the spatial correlation length is normalized to the height of the slope (H).

In this study, the open-source FEM code (mrslope2d) by Fenton and Griffiths (2008) was used to carry out the RFEM analyses.

RLEM. Probabilistic stability analyses results considering spatial variability of soil properties and using LEM have been reported in studies by Li and Lumb (1987), El-Ramly et al. (2001), Low (2003), Babu and Mukesh (2004), Cho (2007 and 2010), Tabbaroki et al. (2013), Li et al. (2014), Javankhoshdel and Bathurst (2014) and Javankhoshdel et al. (2017).

Javankhoshdel et al. (2017) used a circular slip limit equilibrium method and random field theory to investigate the influence of spatial variability of soil properties on probability of failure. Tabbaroki et al. (2013) used a non-circular limit equilibrium approach together with random field theory to consider spatial variability in their probabilistic analyses.

In the RLEM, a random field is first generated using the local average subdivision (LAS) method and then mapped onto a grid mesh, similar to the FEM mesh in RFEM analyses. Each mesh element in the random field has different values of soil properties, and cells close to one another have values that are different in magnitude, based on the value of the spatial correlation length. In each realization, a search is carried out to find the mesh elements intersected by the slip surface. Random soil property values are assigned to all slices whose base mid-point falls within that element. A limit equilibrium approach is then used to calculate factor of safety (FS) for each realization. The probability of failure is calculated as the ratio of the number of simulations resulting in FS < 1 to the total number of simulations.

Non-Circular RLEM

The non-circular RLEM used in this study is a combination of a refined search and the LEM approach (Morgenstern-Price method). The refined search is based on circular surfaces that are converted to piece-wise linear surfaces. The search for the lowest factor of safety is refined as the search progresses. An iterative approach is used so that the results of one iteration are used to narrow the search area for the most critical slope failure mechanism in the next iteration.

The refined search in this study was used together with an additional optimization technique. The optimization is based on a Monte Carlo technique, often referred to as "random walking" (Greco 1996). When used in conjunction with a non-circular search, this optimization method can be very effective at locating (searching out) slip surfaces with lower factors of safety.

In this study, a version of the program *Slide* v.8, which is currently in development (Rocscience Inc. 2017) was used to carry out the non-circular RLEM analyses.

The Slope Model. A simple 27 degree slope with a slope height of 10 m and a foundation depth of 10 m was used for the purpose of this study. The slope geometry and problem domain are shown in Figure 1. The Morgenstern-Price limit equilibrium method was used with the half sine interslice force function to calculate factor of safety.

Cohesion (c) and friction angle (ϕ) were considered to be random variables with typical coefficients of variation (COV) of 0.5 and 0.2, respectively. The mean values of these parameters were taken as: c = 5 kPa; ϕ = 20 degrees; unit weight (γ) = 20 kN/m^3. Lognormal distributions were assumed for all random variables. Only isotropic spatial variability is considered in this paper. ($\Theta_x = \Theta_y$).

Ching and Phoon (2012), Huang and Griffiths (2015), and Ching and Hu (2016) investigated the effect of mesh size used in finite element models that include soil properties with spatial variability. However, similar sensitivity analyses using non-circular RLEM have not been undertaken prior to this paper.

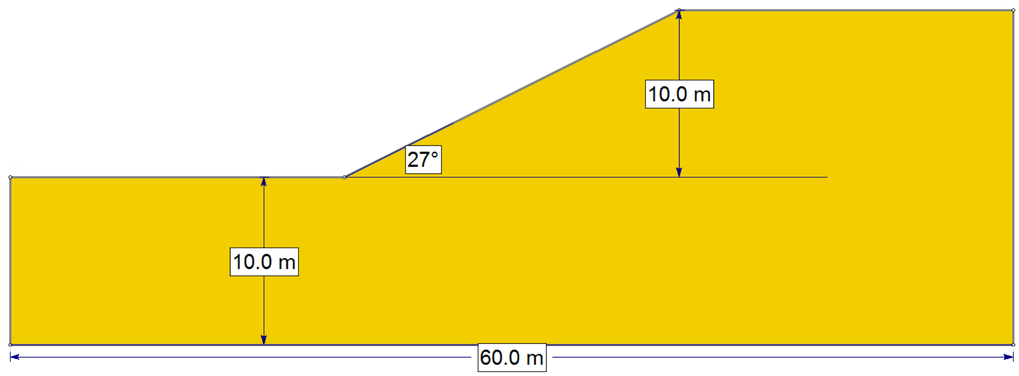

Figure 1. Slope model used in the study.

This study investigates the sensitivity of computed probability of failure using the RLEM approach to: 1) finite element mesh size for different values of correlation length; 2) different numbers of slices; and, 3) different numbers of Monte Carlo simulations. RLEM computation times and results are compared to the same slope problem using the RFEM approach.

MESH SIZE: SENSITIVITY ANALYSIS

RLEM. The initial computations used a conservatively large number of slices and number of Monte Carlo simulations of 500 and 5000, respectively. For a range of correlation lengths, the mesh size was decreased incrementally and PF computed. The PF was only computed for cases where the mesh size is at least half of the correlation length as recommended by Huang and Griffiths (2015) for RFEM analyses. This ensures that the correlation length is appropriately sampled by the mesh. The results of these computations are shown in Figure 2 where Θ/H is the ratio of isotropic correlation length to height of slope.

As may be expected, for the same mesh size, PF decreases with decreasing correlation length (Javankhoshdel et al. 2017). An interesting result in Figure 2 is the detection of a worst-case mesh size, where a peak value of PF occurs at a mesh size between 0.1 and 0.5 m, for Θ/H = 0.5, 0.2, and 0.1 cases.

RFEM. A total of 5000 Monte Carlo simulations were carried out for each of the matching RLEM cases. For a range of correlation lengths, the mesh size was decreased incrementally and PF computed. The minimum mesh size that could be investigated using the compiled source code from Fenton and Griffiths (2008) was 0.5 m. As noted earlier, the other constraint on mesh size was that the mesh size must be at least half of the correlation length. This is necessary in order to ensure that the correlation length is appropriately sampled by the mesh.

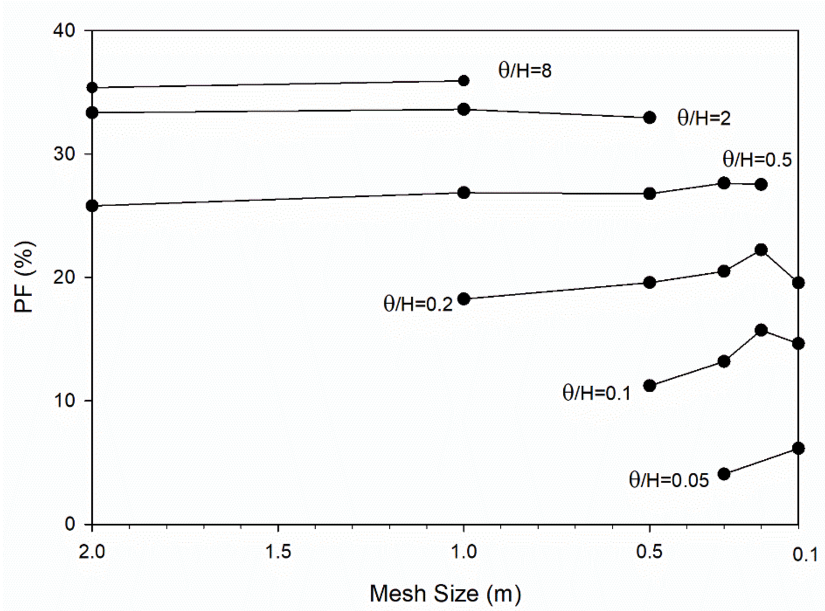

Figure 2. PF vs. mesh size (m) for different values of Θ/H using the RLEM.

The influence of mesh size on PF values using RFEM is presented in Figure 3. The trend of increasing PF with Θ/H for the same mesh size observed in the RLEM results appears also in Figure 3. However, the magnitude of PF can be seen to increase with decreasing mesh size for each normalized correlation length. To better compare results using each method, the data plots in Figure 2 and 3 appear together in Figure 4. The comparisons show that the RLEM analyses give higher probabilities of failure at the largest mesh size of 2 m and lower PF values at the smallest mesh size of 0.5 m.

NUMBER OF SLICES: SENSITIVITY ANALYSIS

Three of the normalized correlation lengths considered in the previous section were used to investigate the sensitivity of numerical outcomes to number of the slices: Θ/H = 8 (representing the homogeneous distribution of soil properties case), Θ/H = 0.5, and Θ/H = 0.1; based on Figure 2, three sufficiently accurate mesh sizes for non-circular RLEM were selected: 2 m, 1 m, and 0.2 m, respectively. PF values were computed for each of the three cases using the non-circular RLEM approach using different number of slices. The results of these computations are shown in Figure 5.

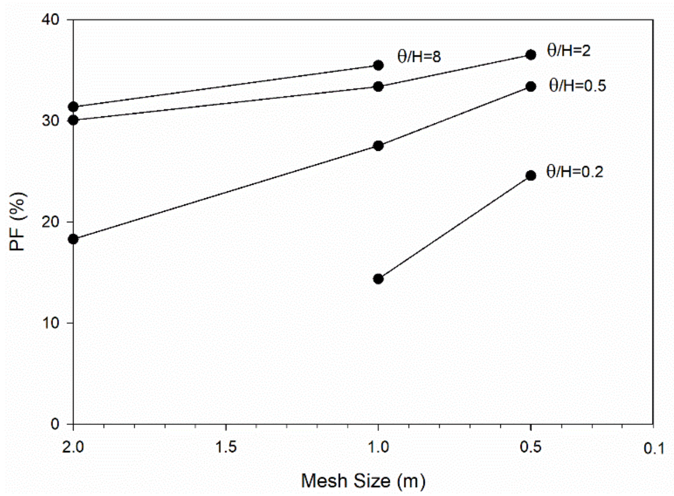

Figure 3. PF vs. mesh size (m) for different values of ϴ/H using the RFEM.

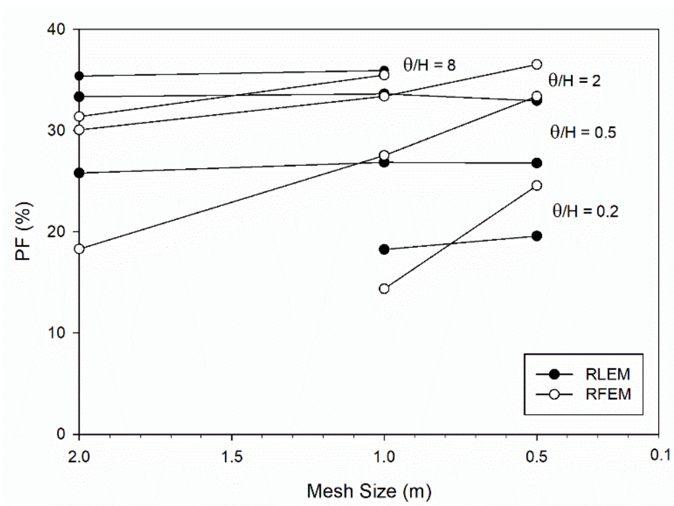

Figure 4. PF vs. mesh size (m) for different values of ϴ/H using the RFEM and the RLEM.

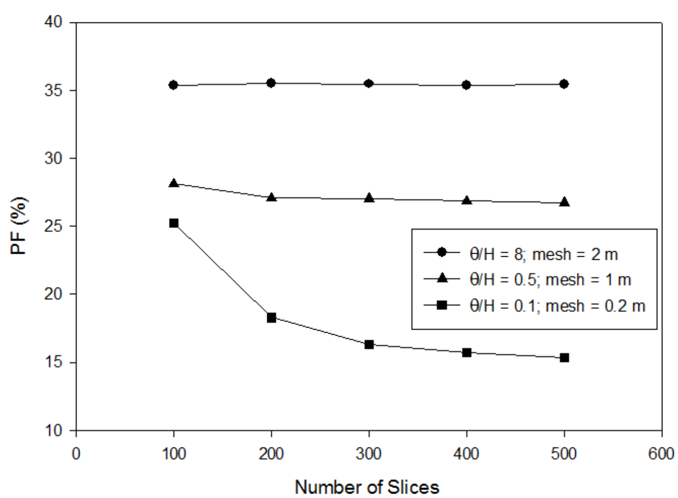

Figure 5. PF vs. number of slices for three different ϴ/H cases using the RLEM.

It can be seen in Figure 5 that the computed values of PF for cases with Θ/H = 8 are not sensitive to the number of slices; hence 100 slices are sufficient for this case in the calculations to follow in the next section. Similarly, 400 slices were selected as sufficiently accurate for the Θ/H = 0.5 case, and 500 slices for Θ/H = 0.1. It is also interesting to note that increasing the number of slices leads to small decreases in PF for the two smallest correlation lengths. However, both approach constant values with increasing number of slices. If less than 500 slices are used, then the computed values of FS and PF err on the safe side for design.

NUMBER OF MONTE CARLO SIMULATIONS: SENSITIVITY ANALYSIS

PF values were computed for each of the three correlation length cases noted above using different number of Monte Carlo simulations. The results of these computations are shown in Figure 6.

It can be seen that the PF is practically constant for each correlation case length and the range of MC realizations shown (e.g. maximum difference of PF = 2%); hence, 1000 simulations are found to be sufficient for these three cases.

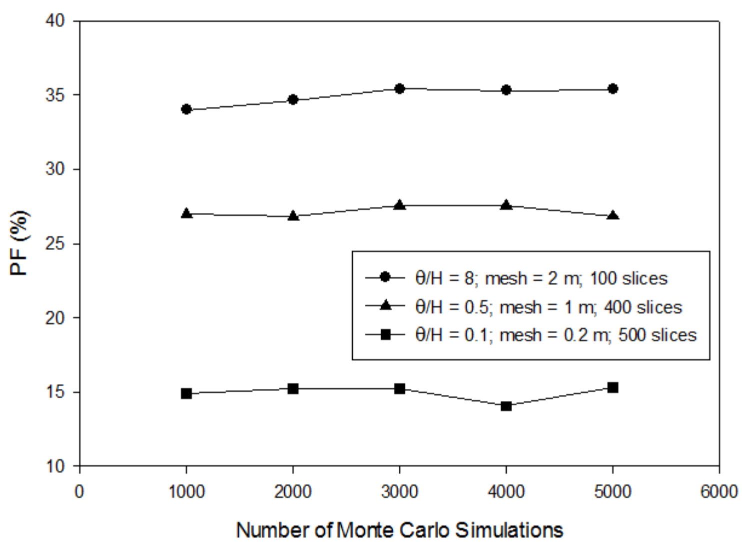

Figure 6. PF vs. number of MC simulations for three different Θ/H cases using the RLEM.

COMPARISON OF RLEM AND RFEM RESULTS

Results. Selected data in Figure 4 are replotted in Figures 7 and 8 to compare results for RLEM and RFEM using mesh element sizes of 1 m and 0.5 m, respectively. It can be seen that the results are in good visual agreement for the 1 m mesh case (Figure 7), with RLEM analyses giving slightly higher PF values for three of the correlation length cases. However, for the 0.5 m mesh case, the RFEM method gives PF outcomes that are consistently larger than the values using RLEM (difference of PF = 3-6%). Nevertheless, with the possible exception of the very lowest PF values that appear in these plots, most values are well beyond magnitudes which would be acceptable for design in a real-world scenario. Hence, it can be argued that any differences in PF values due to methodology in this study are of largely academic interest.

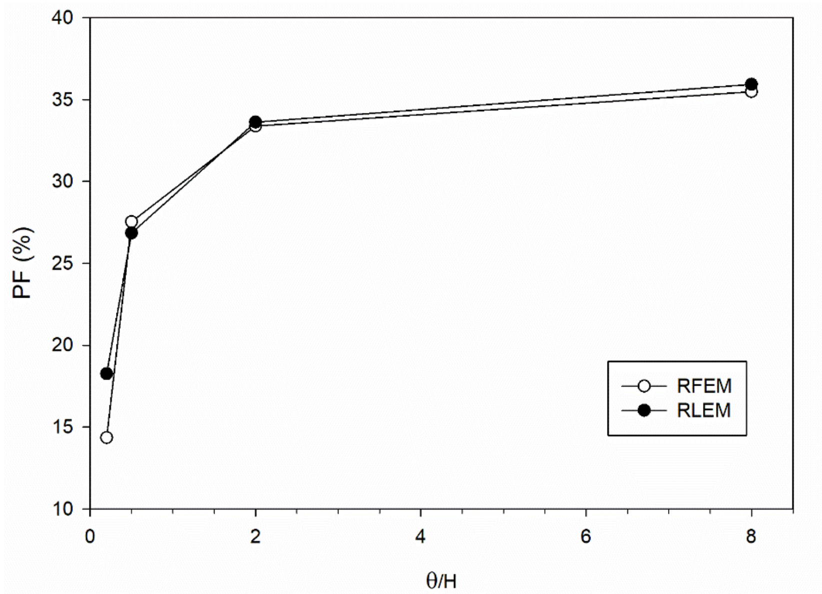

Figure 7. Probability of failure versus normalized correlation length using RFEM and RLEM with 5000 simulations and mesh size = 1 m.

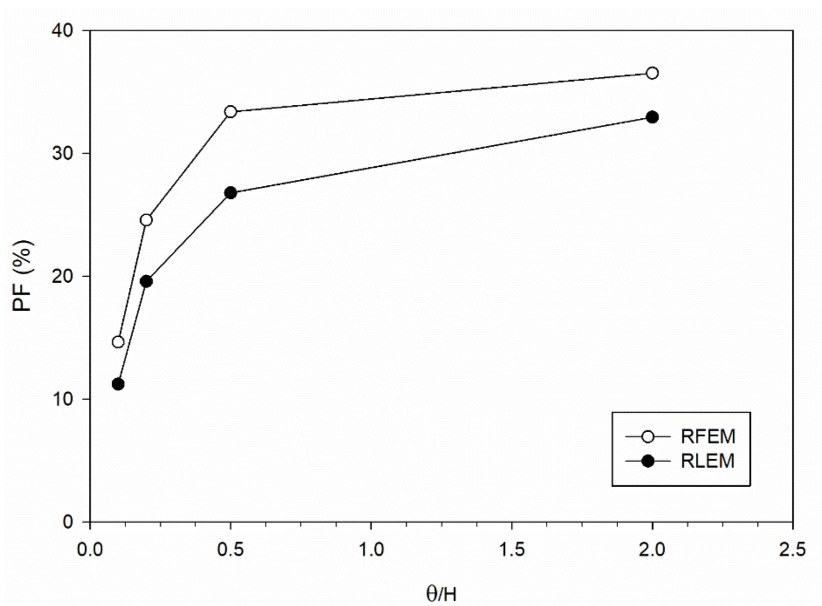

Figure 8. Probability of failure versus normalized correlation length using RFEM and RLEM with 5000 simulations and mesh size = 0.5 m.

Computation Time. An advantage of the RLEM compared to the RFEM is the faster execution time for the same problem conditions. Figure 9 shows computation time in minutes using RLEM and RFEM to execute 5000 MC using a mesh with 0.5 m size elements and different normalized correlation lengths. A total of 500 slices were used in the RLEM calculations. The plots show that the computation time using RLEM was 25% to 46% of the time required to complete the matching RFEM simulations.

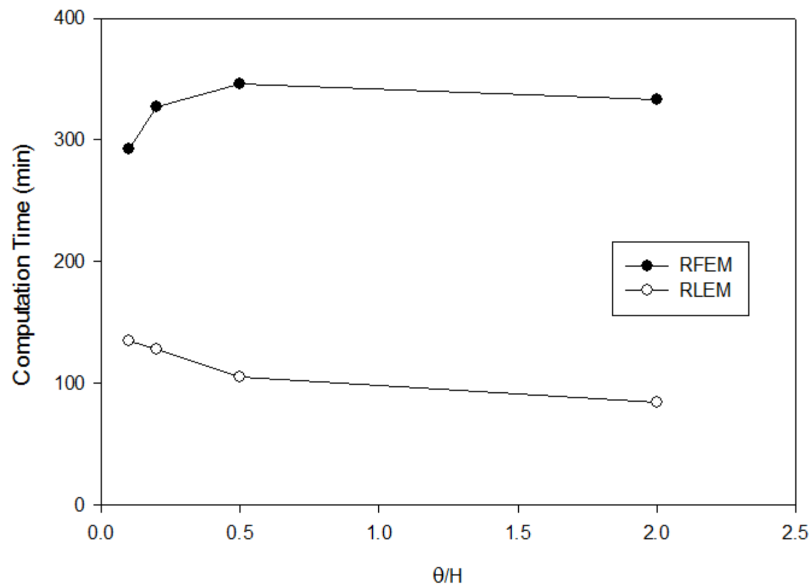

Figure 9. Computation times versus normalized correlation length for 5000 simulations using RFEM and RLEM and mesh size = 0.5 m.

CONCLUSIONS

The focus of this paper is on the results of sensitivity analyses that were carried out using the Random Limit Equilibrium Method (RLEM) for probabilistic slope stability analysis of slopes with homogenous and spatially variable random strength properties. The method uses a combination of 2D random field theory, circular or non-circular limit equilibrium methods, and Monte Carlo simulation. A range of element mesh sizes, number of slices, and spatial correlation lengths were investigated.

The RLEM mesh sensitivity analysis detected the existence of a worst-case mesh size for each correlation length. This observation warrants further investigation.

The slice sensitivity analysis found that for each of the three correlation length cases considered, increasing the number of slices led to lower probability of failure (PF) values but at a diminishing rate.

For the simple slope problem investigated and the range of two-parameter spatial correlation lengths considered, only 1000 Monte Carlo realizations were needed to generate a stable estimate of PF.

Comparison of RLEM and RFEM results were in practical good agreement. However, computation times for the same slope problem with 0.5 m-size elements using RLEM were 25% to 46% of the time required for RFEM.

This investigation is a preliminary and initial study with limited scope. Further investigation is required to examine the influence of different slope geometries, multiple soil layers, numerical model size, and COV of soil strength components on probability failure outcomes.

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