Analysis of Blocky Rock Slopes with Finite Element Shear Strength Reduction Analysis

R.E. Hammah, T. Yacoub, B. Corkum & F. Wibowo
Rocscience Inc., Toronto, Canada

J.H. Curran
Department of Civil Engineering & Lassonde Institute, University of Toronto, Toronto, Canada

ABSTRACT: This paper will discuss application of the Shear Strength Reduction (SSR) method, using finite element (FE) analysis, to determine the stability of slopes in blocky/jointed rock masses. It will demonstrate the method's versatility and reliability through analysis of five simple examples. FE- SSR results are verified through comparison to the results of UDEC, a discrete element program. The paper provides arguments for incorporating SSR analysis into the standard solution toolkit of rock slope engineers.

1 INTRODUCTION

The efficiency of the Shear Strength Reduction (SSR) method, based on finite element (FE) analysis, has been well demonstrated for slopes in soil or rock masses, which can be treated as continua (Hammah et al 2006, 2005 a, b, 2004). (In the rest of the paper, this method of slope stability analysis will be labelled the FE-SSR method.) This paper intends to show that the FE-SSR method can be reliably extended to the analysis of slopes in blocky rock masses. The paper will analyze five slope examples. In all the cases the method determines critical failure mechanisms and factors of safety that compare very well with those obtaining from analysis with UDEC (Itasca 2004), a commercially available numerical analysis program that performs SSR analysis based on the Discrete Element Method (DEM).

The FE-SSR approach offers several key benefits. It is very general and can be used with any geotechnical FE software. The widespread availability of such software today makes the approach readily accessible and constitutes the primary motivation for this paper.

The paper will briefly describe unique aspects of the mechanical behaviour of blocky rock masses and why DEM tools have been traditionally applied to this class of geotechnical problems. It will then give an overview of the SSR method and show how the approach can be used with any FE program. This discussion establishes the groundwork for explaining why the FE-SSR can be reliably used to analyze slope stability problems in blocky rock masses. Lastly five examples, which illustrate the capabilities of the FE-SSR on a range of blocky rock mass slope problems, will be described.

2 MECHANICAL BEHAVIOUR OF BLOCKY ROCK MASSES

Behaviour in blocky rock masses is controlled by discontinuities. Such rock masses respond in non-linear and anisotropic fashion to loads and excavations. The non-linear response is caused by a combination of the characteristics of intact rock material and relative movements of blocks (sliding along discontinuities, opening and closing of discontinuities, rotations of blocks), which are accompanied by local failures along discontinuities. The response may also involve shearing through intact material.

In most cases, slopes in blocky rock masses have been modelled with the DEM, a method which treats block rock masses as assemblages of discrete blocks (rigid or deformable) bounded by discontinuities. The DEM tracks the movements of blocks (using the equations of motion) and the accompanying changes in block contacts. As a result, the method can model large displacements of blocks including complete separation of blocks.

3 OVERVIEW OF THE SSR METHOD

The Shear Strength Reduction technique (Dawson et al 1999, Griffith & Lane 1999, Hammah et al 2006, 2005 a, b, 2004, Matsui & San 1992) enables slope factor of safety to be calculated using numerical modelling methods such as FE analysis. In the approach, FE analysis is systematically used to search for a stress reduction factor (factor of safety value) that brings a slope to the very limits of failure. The approach is best explained for slope material of Mohr-Coulomb strength.

The factored or reduced shear strength of a Mohr-Coulomb material is described by the equation

$$\frac{\tau}{F}=\frac{c'}{F}+\frac{\tan\phi'}{F}$$, where F is the reduction factor.

This equation can be re-written as

$$\frac{\tau}{F}=c^*+\tan\phi^*,$$

where $c^*=\frac{c'}{F}$ and $\phi^*=\arctan\left(\frac{\tan\phi'}{F}\right)$ are factored Mohr-Coulomb shear strength parameters.

The steps for systematically searching for the critical factor of safety value F that brings a previously stable slope ($F\geq1$) to the verge of failure are as follow:

Step 1: Develop an FE model of a slope, using the appropriate material deformation and strength properties. Compute the model and record the maximum total deformation.

Step 2: Increase the value of F (or strength reduction factor) and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total deformation.

Step 3: Repeat Step 2, using systematic increments of F, until the FE model does not converge to a solution, i.e. continue to reduce material strength until the slope fails. The critical F value at which failure occurs is the factor of safety.

For a slope with a factor of safety less than 1, the procedure remains the same except for use of systematically reduced F values (this is the equivalent of increasing the factored strength of materials) until the slope becomes stable.

Due to the use of factored strength parameters as input into models, the SSR technique can be used with any existing FE software. The only task required of the slope analyst is computation of factored Mohr-Coulomb strength parameters to be input into an FE slope model. (A few commercial programs (Plaxis 2006, Rocscience 2005) offer tools that automate the process.)

Non-convergence within suitably specified number of iterations and tolerance is an appropriate indicator of slope failure because it arises in the absence of equilibrium. Lack of convergence indicates that stress and displacement distributions that satisfy the equations of equilibrium cannot be established for a given set of slope material strength. This state of collapse is precisely what occurs during real slope failure. In real slope collapses, and in the numerical models described, failure is often characterized by a sudden increase in slope displacements.

A major advantage of the SSR method is that it does not require *a priori* assumptions on the nature of failure mechanisms; it can find a broad range of mechanisms including complex ones. For example, for blocky rock mass problems, the method can detect composite mechanisms that combine failure along discrete discontinuities with shearing through intact material.

4 APPLICATION OF THE FINITE ELEMENT METHOD TO BLOCKY ROCK MASS MODELLING

The Finite Element Method (FEM) is the most widespread numerical analysis method. It is a continuum analysis method. Its popularity can be primarily attributed to its ability to:

1. Handle multiple materials in a single model (material heterogeneity)
2. Readily accommodate non-linear material responses, and
3. Model complex boundary conditions.

Although the FEM is a continuum method, special elements – joint (interface) elements – have been developed to directly represent the discontinuous behaviour characteristic of joints and interfaces between adjacent blocks of material. These elements can have either zero thickness or thin, finite thickness. They can assume linear elastic behaviour or plastic response when stresses exceed the strengths of discontinuities.

Unlike the DEM, which can readily model large displacements of blocks, most FEM codes only model small strains. Due to the fundamental continuum analysis condition of displacement compatibility at element nodes, FEM programs do not allow the detachment of individual blocks (Jing 2003).

Several joint elements can be included in an FE model. However, incorporating joint elements into a finite element model substantially increases the degrees of freedom. Joint elements in a model can also lead to ill-conditioned stiffness matrices, the solutions of which may be numerically unstable. These two factors can combine to restrict the sizes of slope problems that can be solved with FE-SSR analysis.

There are compelling reasons, however, to apply FE-SSR analysis whenever possible (and there are several cases in which the method is applicable). These will be discussed next.

Why use FE-SSR analysis for slope stability problems in blocky rock masses?

Due to the popularity of the FEM, there are many easy-to-use, well-tested FE programs commercially available. Most geotechnical engineering practices own FEM software. As a result the power of the SSR method can be much more readily harnessed with the FEM than with any other method of numerical analysis.

We believe that the FE-SSR method works very well for blocky rock mass slope problems because up until failure, displacements are generally small. Large displacements occur after the full failure process has been initiated. In studying the stability of a slope, determination of the factor of safety and understanding of possible failure mechanisms and how they develop constitute the primary interest of geotechnical engineers. Seldom is the interest in the ultimate geometric configuration of the failed mass. The FE-SSR method fully answers these questions and thus solves the engineer's slope stability problems.

5 EXAMPLES

To illustrate the capabilities of the FE-SSR method for blocky rock masses, five blocky rock mass slope problems described by Lorig & Varona (2001) were analyzed. The analyses were performed with the two-dimensional FE program Phase2. The overall slope geometry was the same for all examples: the slope had a height of 260 m and a slope face angle of 55° (Figure 1).

The slope material had the following properties: cohesion of 675 kPa, 43° friction angle, unit weight of 26.1 kN/m3, bulk and shear moduli of 6.3 GPa and 3.6 GPa, respectively, and zero tensile strength. Mohr-Coulomb cohesion of 100 kPa and friction angle of 40° was prescribed for the joints. (Equivalent Young's modulus and Poisson's ratio were calculated from the bulk and shear modulus pair and input into the FE program used for the analysis.)

Visualization of the failure mechanisms in the examples is accomplished through a combination of total displacement contours and (exaggerated) deformed outline of boundaries and joints.

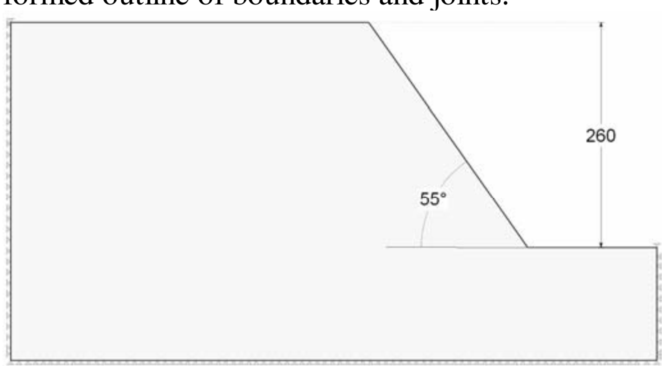

Figure 1: Basic slope geometry.

5.1 *Example 1 – plane failure of slope with daylighting discontinuities*

The first example involves a set of discontinuities spaced 10m apart, which dip out of the slope at 35° (see Figures 2a to 2c). The UDEC analysis performed by Lorig & Varona (2001) predicts a failure mechanism with a factor of safety of 1.27. This mechanism, shown on Figure 2a, combines sliding along a joint near to the slope toe with a curved tension crack that exits at the top of the slope. FE-SSR analysis produces a factor of safety value of 1.32 for a mechanism shown on Figure 2b. The FE-SSR results are very similar to those of UDEC.

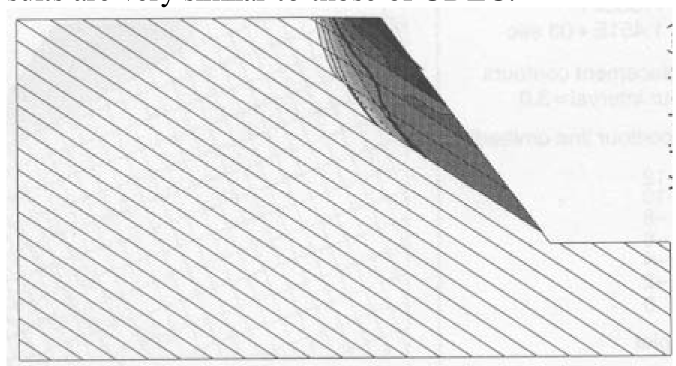

Figure 2a: Failure mechanism (sliding of wedge along joint with curved tension crack at slope top) for Example 1 predicted by UDEC.

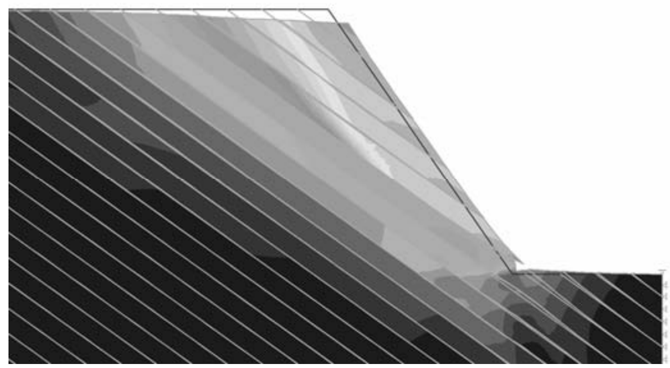

Figure 2b: Failure mechanism (as depicted by contours of total displacement) for Example 1 predicted by FE-SSR.

5.2 *Example 2 - plane failure of slope with non-daylighting discontinuities*

In Example 2 the joints dip at 70° and have 20m spacing. The joints in this example have zero cohesion. UDEC predicts a failure mode with a 1.5 factor of safety.), which involves sliding along joints in the upper part of the slope and shearing through intact rock material in the lower part that exits near the toe. The mechanism is displayed on Figure 3a. FE-SSR analysis yields a factor of safety of 1.53 for a similar mechanism, which is shown on Figures 3b and 3c.

5.3 *Example 3 – failure involving forward toppling of blocks*

The slope in Example 3 has two perpendicular joint sets – one dipping 70° out of the slope with 20 m spacing, and the other dipping 20° into the slope face with 30m spacing. UDEC predicts that failure occurs in the form of forward toppling of blocks out of the slope (Figure 4a), and assigns a 1.13 factor of safety to the mechanism. FE-SSR produces a factor of safety of 1.23 for a very similar mechanism, which is shown on Figures 4b and 4c.

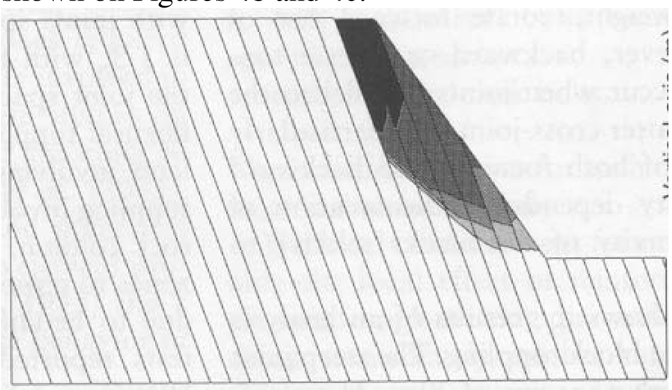

Figure 3a: Failure mechanism (sliding along joint in upper part of slope and shearing through intact rock in lower parts) for Example 2 predicted by UDEC.

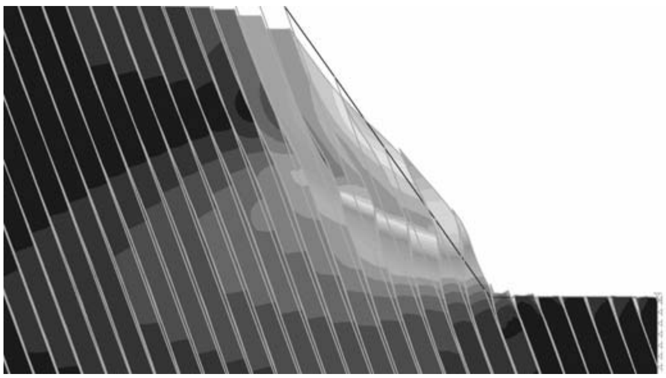

Figure 3b: Failure mechanism (as depicted by contours of total displacements) for Example 2 predicted by FE-SSR.

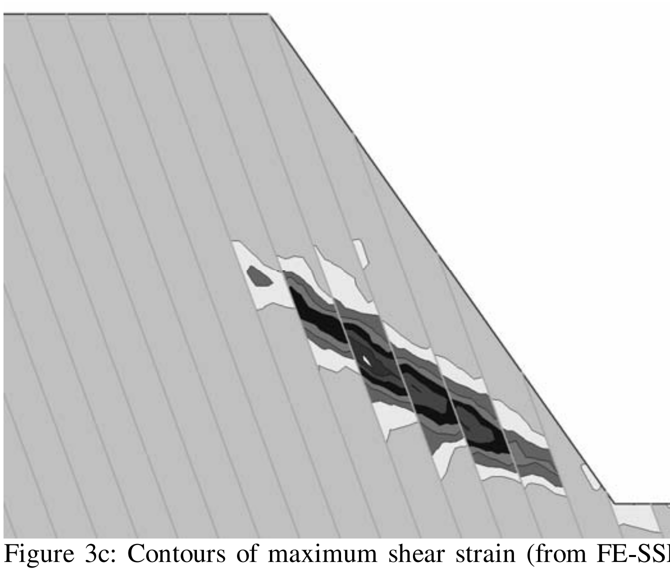

Figure 3c: Contours of maximum shear strain (from FE-SSR analysis) showing the component of the failure mechanism that involves shearing through intact rock material towards the slope toe.

5.4 *Example 4 - failure involving backward toppling of blocks*

In Example 4, the 20° cross joints are replaced by a set of horizontal joints with a 40m spacing, while the spacing of the 70° dipping joints changes to 10m. The UDEC predicted failure mechanism, which has a factor of safety of 1.7, is shown on Figure 5a. This failure mode comprises backward toppling, initiated by rotation of the block nearest to the slope toe. Again the FE-SSR identifies a similar backward toppling mechanism (Figures 5b and 5c) with a 1.54 factor of safety value.

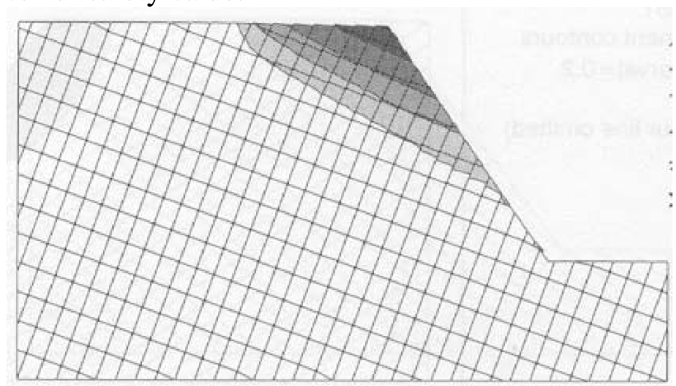

Figure 4a: Failure mechanism (forward toppling of individual blocks out of the slope through free rotations) for Example 3 predicted by UDEC.

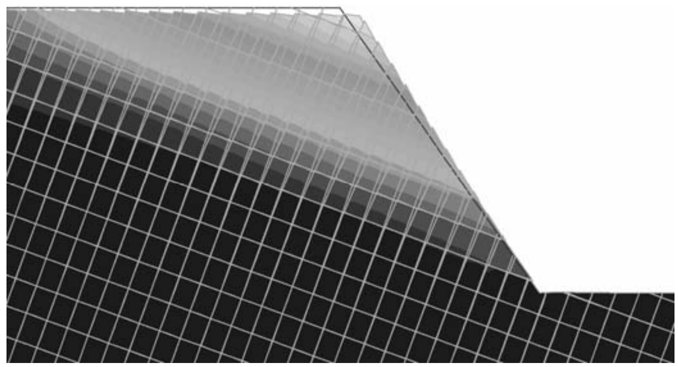

Figure 4b: Failure mechanism (as depicted by contours of total displacement) for Example 3 predicted by FE-SSR.

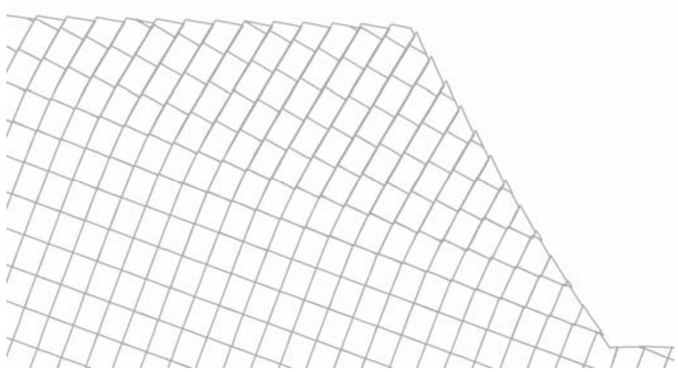

Figure 4c: Plot of deformed outlines of joints and slope face showing toppling movements of blocks through free rotations relative to each other (from FE-SSR analysis).

5.5 *Example 5 - failure involving flexural toppling*

The slope in the last example has a single set of joints of 20m spacing, which dip 70° into the slope face. UDEC analysis identifies a failure mode with a 1.3 factor of safety involving flexural bending of rock columns (Figure 6a). FE-SSR analysis identifies a very similar mechanism (Figures 6b and 6c) with a factor of safety is 1.4.

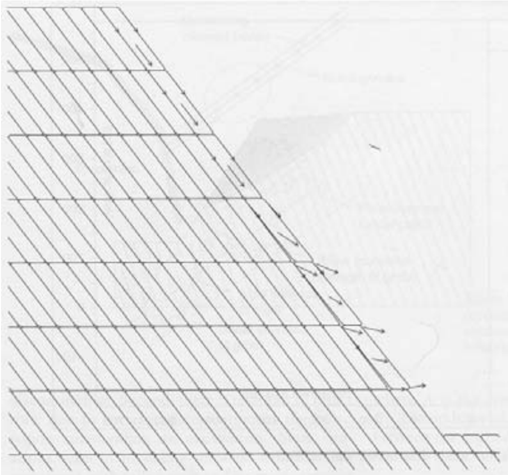

Figure 5a: Failure mechanism (reverse toppling – rotation of blocks starting with block nearest to slope toe) predicted by UDEC.

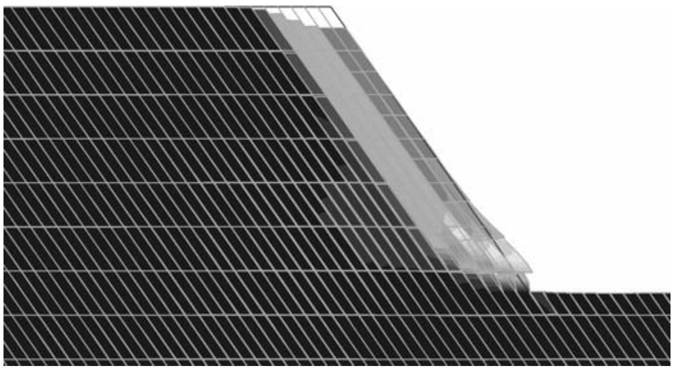

Figure 5b: Failure mechanism (as depicted by contours of total displacement) for Example 4 predicted by FE-SSR.

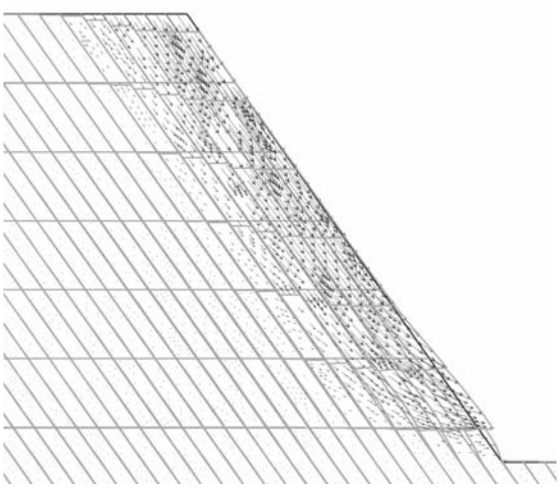

Figure 5c: Failure mechanism for Example 5 predicted by FE-SSR as shown by combination of deformed outlines of blocks and displacement arrows.

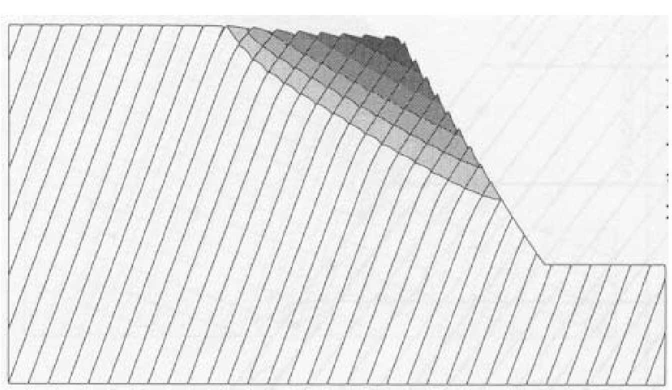

Figure 6a: Failure mechanism (flexural bending of rock columns) predicted by UDEC.

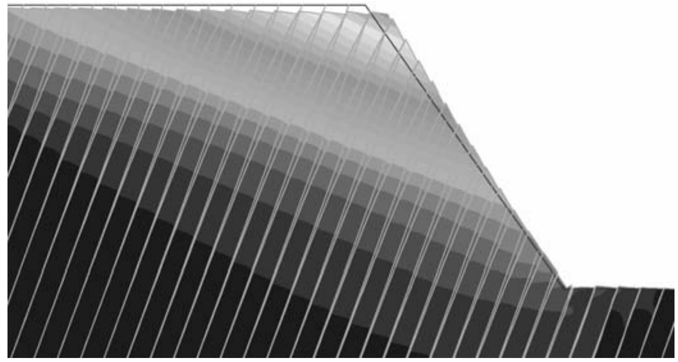

Figure 6b: Failure mechanism (as depicted by contours of total displacement) for Example 5 predicted by FE-SSR.

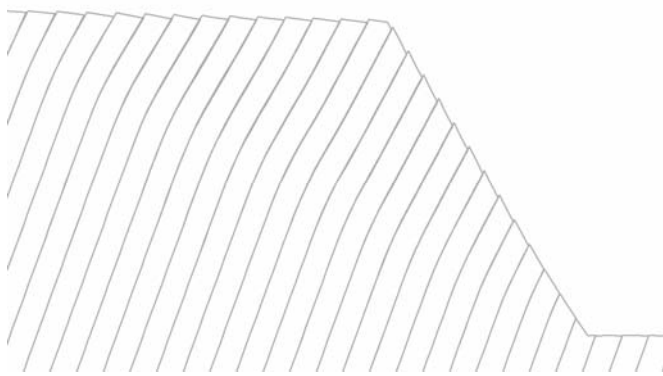

Figure 6c: Failure mechanism for Example 5 predicted by FE-SSR as shown by deformed outlines of columns.

6 CONCLUDING REMARKS

The results of the analyses in this paper demonstrate that even though the FE-SSR method is founded on continuum principles, it is a powerful, credible alternative to discrete modelling tools such as the DEM in modelling the stability of slopes in blocky rock masses.

Several previous publications have already confirmed the method's capabilities on a wide range of continuum slope problems. For example, in Hammah et al (2005b, 2006) it is shown that the FE-SSR methods accurately determines stability results for a wide range of unreinforced and reinforced slope problems. The current paper demonstrates the FE-SSR method's performance on blocky rock mass failure mechanisms. These mechanisms include planar wedge failure and different toppling regimes, some of which involve both block movements along discontinuities (sliding, opening and closing of joints) and shear failure of intact rock material. In all the cases, the FE-SSR method automatically determined critical failure mechanisms with absolutely no *a priori* assumptions on the modes, shapes or locations of these mechanisms.

Due to the widespread availability of FE analysis programs, the ability to apply SSR analysis with any FE program, and the demonstrated versatility of FE-SSR analysis, the authors encourage greater use of the approach in geotechnical slope problems. The profession only stands to benefit.

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