A TWO-DIMENSIONAL APPROACH FOR DESIGNING TUNNEL SUPPORT IN WEAK ROCK

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ABSTRACT

The design of support for tunnels in weak rock is an iterative process. A good starting point is essential to the process and facilitates safe and economic design. Support design for tunnels in weak rock is a clearly three-dimensional problem. However, currently there are very few practical three-dimensional software tools for carrying out the task. This paper provides an overview of a methodology being used by tunnelling experts, which captures the three-dimensional essentials of tunnel behaviour with two-dimensional analysis tools. The methodology allows support designers to start off with reasonable estimates of support requirements that can be quickly refined to meet requirements.

RÉSUMÉ

1. INTRODUCTION

The distribution of stresses around the advancing face of a tunnel is three-dimensional. At a section in a rock mass, which is at distance of two and a half tunnel diameters ahead of the face, the stress state is undisturbed and equal to the in-situ stress conditions. At the tunnel face, the rock mass provides a support pressure that is approximately 25% of the in-situ stress. The apparent support pressure provided by the face allows excavated sections to stand up long enough for support to be installed. Support pressure gradually reduces to zero at a distance of about four and one half tunnel diameters behind the advancing face.

Due to the three-dimensional stress distribution at tunnel faces, straightforward application of two-dimensional numerical analysis to the design of tunnel support systems is inaccurate. Most two-dimensional numerical formulations for excavation analysis assume plane strain conditions. However these conditions are only applicable to tunnel sections far from the advancing face.

In the numerical simulation of a tunnel, if the tunnel is first excavated and a passive support system installed thereafter, the support system will carry no loads. This is because all deformations would have taken place before the support is installed. On the other hand, if the supported is installed in the model before the tunnel is excavated, the support system will be exposed to the entire induced loading, a scenario that would arise only if the support were to be installed before any displacements whatsoever of the excavation boundary occurred. This would lead to conservative design since in reality some degree of stress relief always occurs by the time support is installed.

In order to use two-dimensional numerical tools to realistically design tunnel support therefore, one needs to estimate displacements of the excavation boundary that occur before support is installed. Using the two-dimensional finite element method, this paper outlines a practical approach for estimating these deformations, and designing realistic support. It provides an example of such design for a non-circular tunnel in non-hydrostatic stress conditions.

2. TWO-DIMENSIONAL METHODOLOGY FOR PRACTICAL TUNNEL SUPPORT DESIGN

The paper proposes a simple, yet practical, 5-stage methodology for designing realistic tunnel support using any two-dimensional numerical method such as the finite element or finite difference methods. The approach assumes that the axis of the tunnel is aligned with a principal stress direction. The paper uses $Phase^2$, a finite element program developed by Rocscience. However any other two-dimensional numerical programs such as Flac can be employed.

The proposed methodology is based on the softening behaviour of material with excavation. The three-dimensional stress distribution at the face of a tunnel can be interpreted in terms of material softening. To do so we shall examine the behaviour of a rockmass section before and after it is excavated. When the section is at such a distance from the advancing tunnel face that its stress state is undisturbed, the unexcavated material within the section is un-deformed and can be thought of as having *in-situ* deformation modulus.

As the face approaches the section this material begins to soften. At the time support is installed at the excavated section, it would have experienced deformations that can be modelled in two dimensions by reducing the deformation modulus of the excavated material by a ratio α. The section eventually attains plane strain conditions as the tunnel face advances further. If it were to be left unsupported, then its deformations under plane strain conditions could be modelled by assuming $\alpha = 0$.

The five steps of the methodology are as follow:

a) If the tunnel being designed is non-circular, determine an equivalent circle
b) Estimate the tunnel deformation that occurs prior to support installation
c) Determine the type of support (support system) required for the prevailing rockmass conditions
d) Estimate the ratio by which the modulus of elasticity of the rockmass must be reduced, and
e) Conduct two-stage modelling of the tunnel and support.

For the charts and example that will be provided in the paper, we model rock mass strength with the updated Generalized Hoek-Brown failure criterion (Hoek *et al*, 2002).

2.1 Step 1: Determine the Equivalent Circular Tunnel

Noncircular tunnel cross-sections are common in practice, but methods for generating ground reaction curves, including axisymmetric finite element analysis [Hoek 2002], assume circular tunnel shapes. To estimate the ground reaction curve for a noncircular tunnel, we propose approximation of the noncircular cross-section with an equivalent circular tunnel of the same cross-sectional area. If the noncircular tunnel shape differs greatly from a circle then this approximation may not be valid.

The area of an arbitrarily shaped cross-section can be calculated either using analytical equations for its different sub-areas, or using a numerical algorithm. One such simple algorithm is based on a discrete version of Green's theorem that relates the double integral over a closed region to a line integral over its boundary. If n vertices on the boundary of a tunnel cross-section labelled $(x_0, y_o), (x_1, y_1), ..., (x_{n-1}, y_{n-1})$ are known, the area of the cross-section is given by:

$$A = \frac{1}{2}\sum_{i=0}^{n-1}\left(x_i y_{i+1} - x_{i+1} y_i\right). \qquad (1)$$

The last vertex (x_n, y_n) in the formula must be set equal to the first vertex (x_0, y_0). The formula gives a positive area if the vertices are arranged counterclockwise, and a negative area if the vertices are arranged clockwise.

The diameter of the equivalent tunnel is easily calculated using the relationship:

$$D = \sqrt{4A/\pi}.$$

2.2 Step 2: Estimate Tunnel Displacements Prior to Support Installation

A plot of tunnel wall displacement versus distance from the advancing face, both normalized by tunnel diameter D, for the equivalent circular tunnel can be used to estimate the amount of displacement that will take place by the time support is installed. This curve can be generated using axisymmetric finite element analysis or empirical equations. Knowing the distance from the tunnel face at which support will be installed, amount of wall displacement that occurs prior to support installation can be determined from the curve.

Unlike the deformations around a circular tunnel in hydrostatic conditions, the deformations around a non-circular tunnel or around a circular tunnel in non-hydrostatic conditions are not evenly distributed. In this paper we suggest the maximum displacement of the tunnel boundary be used in generating the wall displacement – distance from face curve.

When creating the plot it is recommended to calculate displacements over a distance from the face that is at least five times the equivalent diameter. This allows the total convergence of the unsupported tunnel to be calculated, and used in the next step to estimate strain levels.

2.3 Step 3: Determine the Type of Support System Required

Tunnel support design is an iterative process that includes assumptions on support type installed and the support pressure it provides. There exists a very wide range of tunnel support systems. To obtain a good choice of support type, especially at the initial stages of design, one can use the guidelines provided in Hoek, 1998 and 2000.

The guidelines allow tunnel designers to anticipate the expected extents of squeezing problems and then estimate the type and amount of support required. The guidelines assume a circular tunnel in hydrostatic stress conditions with a closed ring of support (evenly distributed around the tunnel circumference).

The procedure starts off with the determination of the level of strain that will be experienced by a tunnel (Hoek, 2000 and Hoek and Marinos, 2000). This strain level is defined as the ratio of tunnel wall displacement to tunnel radius, expressed in percentage. It is dependent on the ratio of rockmass strength (for the Generalized Hoek-Brown criterion it is the uniaxial compressive strength of the rock mass σ_{cm}), to in situ stress, p_o (a hydrostatic stress state is assumed). Figure 1 is a chart from (Hoek, 2000 and Hoek and Marinos, 2000) that relates tunnel strain to the ratio σ_{cm}/p_o.

For non-hydrostatic stress conditions, we propose that p_o be set equal to the maximum principal perpendicular to the tunnel cross-section (remember it is assumed that the tunnel axis is parallel to a principal stress direction). From a known σ_{cm}/p_o ratio the level of strain, and thus extents of squeezing problems expected, can be estimated from the curve on Figure 1.

Having established the anticipated degree of squeezing problems, the next step is to determine the support pressure required to limit the strain around the tunnel to a specified amount. This is accomplished through the use of Figure 2, a chart from Hoek, 1998. For a rock mass strength to in situ stress ratio and percent strain, the support pressure to *in situ* stress ratio, required to maintain the percent strain, can be estimated. Since the in situ stress is known the support pressure can then be readily calculated.

Using the diameter of the equivalent tunnel and the required support pressure in conjunction with the chart on Figure A1 (Hoek, 1998) the type and amount of support needed for the tunnel can be obtained.

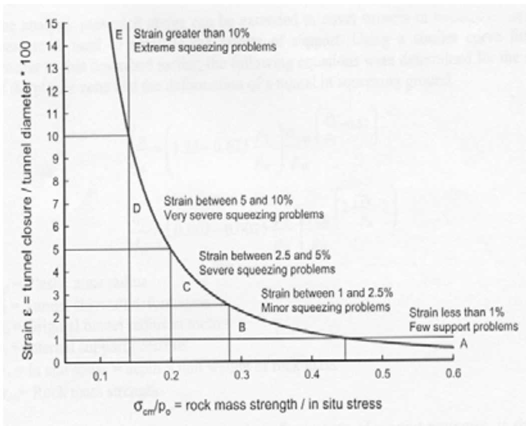

Figure 1. Approximate relationship between strain and the degree of difficulty associated with tunnelling through squeezing rock (Hoek, 2000, and Hoek and Marinos, 2000).

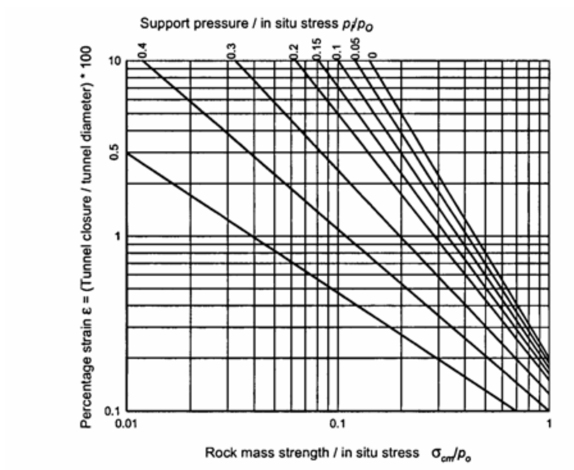

Figure 2. Approximate support pressure required for different strain values for a circular tunnel under hydrostatic in situ stresses (Hoek, 1998).

2.4 Step 4: Determine the Appropriate Modulus Softening Ratio

The last quantity to establish before embarking on the two-dimensional numerical modelling of the actual tunnel is the ratio by which the material modulus of elasticity in must be reduced the model. This step allows us to design realistic support by permitting the required tunnel displacement to occur in the model before support is installed.

The appropriate softening ratio can be determined by generating a plot of relative tunnel convergence to material softening ratio. This plot is created through running several tunnel models that have different reduced material moduli. Knowing the amount of displacement that occurs by the time support is installed (from Step 2), we can determine, from the plot, the ratio by which we must reduce the tunnel material modulus of elasticity.

2.5 Step 5: Model Tunnel and Support in Two Stages

The actual tunnel and support under design can now be analysed in two stages. In Stage I of modelling, the modulus of elasticity of the material to be excavated from the tunnel is reduced by the ratio established above. All other model parameters including in situ stresses, material strength and deformation properties, remain the same. Any suitable numerical analysis code can be used for this modelling.

This stage allows the expected level of deformation to occur around the tunnel before support is installed, realistically simulating true behaviour. Although the computed deformations are expected to be similar to those calculated for the equivalent tunnel, it is important to compare the two deformations to ensure that the assumption of equivalency is indeed true. If the two deformations differ significantly, then we suggest changing the material-softening ratio till the deformations match.

In the next stage of modelling, the selected type of support is installed and the tunnel material completely removed. The tunnel deformations and support loads calculated from this modelling stage are then checked to ensure that they meet design criteria.

3. EXAMPLE

To illustrate the use of the suggested approach, support will be designed for the tunnel geometry shown on Figure A. The tunnel under a field stress regime where the horizontal stress is half the vertical stress (k=0.5). The material properties and in-situ stresses are given in Table 1. It will be assumed for this example that support is installed 2m from the tunnel face.

Using RocLab, a free program developed by Rocscience Inc., the global strength of the rock mass (Hoek *et al*, 2002), σ_{cm} was determined to be 2.49 MPa. The Young's modulus of the rock mass required for the numerical modelling of the tunnel can be estimated from the GSI, intact rock strength and disturbance factor (Hoek *et al*, 2002). Using Young's modulus calculator built into RocLab, we obtained a Young's modulus of 1732 MPa.

Next we calculated the area of the tunnel cross-section, using the coordinates of 54 vertices on the tunnel boundary using equation (1). We obtained an area of 37.17 m^2. From this area we computed the diameter of the equivalent circular tunnel to be 6.88 m.

Table 1. Material properties and in-situ stresses

Hoek-Brown Classification Parameters	
Uniaxial compressive strength of intact rock	30 MPa
Geological Strength Index (GSI)	35
Material constant m_i	10
Disturbance factor D	0.5
In situ Stresses	
Vertical stress	6 MPa
Horizontal stress	3 MPa

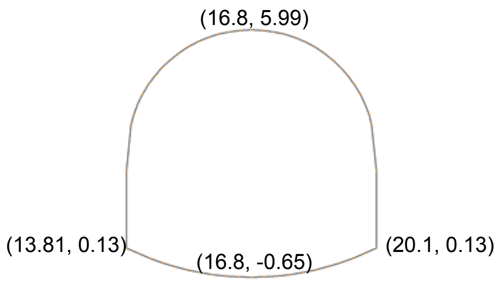

Figure 3. Tunnel cross-section

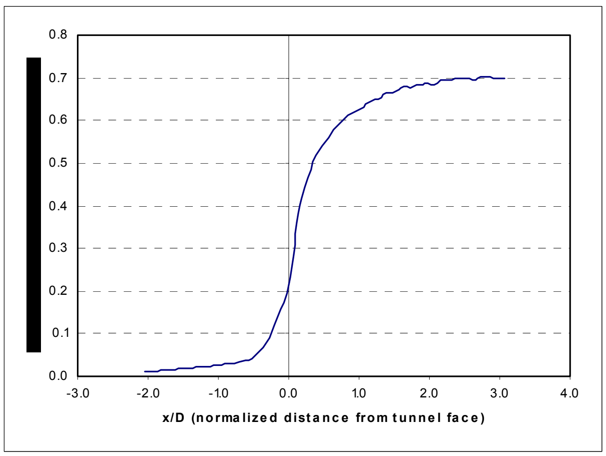

Figure 4. Plot of tunnel wall displacement as a function of distance from face developed from axisymmetric analysis in Phase[2].

Next, for the determined equivalent tunnel geometry and material and rock mass properties, we used the axisymmetric modelling option in the finite element program, Phase[2], to generate a curve of tunnel wall displacement as a function of distance, x, from the face (Figure 1). From the curve a tunnel strain (ratio of maximum displacement to tunnel diameter expressed as a percentage) of 0.48% (or a maximum displacement of 0.033 m) is expected when support is installed 2 m away from the face. The curve also indicates a 0.7% strain (maximum displacement of 0.048 m) for unsupported tunnel sections far from the face.

Now we look at selecting a suitable support system for the tunnel, given the stress and material strength conditions. Our rock mass compressive strength of 2.49 MPa and maximum in-situ principal stress of 6 MPa give a strength-to-stress ratio of 0.41. Examination of Figure 1 provides a corresponding percent strain that lies between 1 and 2.5%, signifying minor squeezing problems.

Next we use the guidelines described in Step 3 to select a support system for the tunnel. Since these guidelines have been implemented in RocSupport, a program by Rocscience, we used it to establish a support system comprising a 100 mm thick shotcrete layer.

We proceeded to generate the plot of convergence ratio against reduction ratios of Young's modulus for our rock mass. This plot is shown on Figure 5. Since we already established a percent strain of 0.48% in Step 3, we determine from Figure 5 that we need to soften the Young's modulus of the material to be excavated by a ratio of 0.005 to model the displacements that occur prior to installation of support.

We then developed a two-stage Phase2 model of the actual tunnel (Figure 6) under design. The maximum displacements (strain value of 0.47%) around the tunnel computed in Stage 1 were very similar to the displacements determined for the equivalent circular tunnel at a 2 m distance behind the face. This demonstrates the ability of the proposed approach in obtaining a good starting point for design. If the displacements were very different, we would have had to appropriately increase or decrease the modulus-softening ratio.

The stresses computed in the liner were all below the tensile and compressive strengths of the liner concrete. We are therefore able to accept this 100 mm shotcrete layer support system for the tunnel.

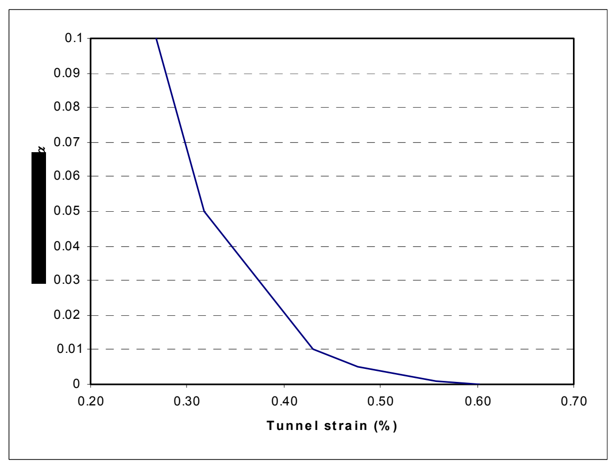

Figure 5. Plot of convergence ratio against Young's modulus reduction ratio.

4. CONCLUDING REMARKS

In reality, tunnel support systems are often installed in areas that are under three-dimensional stress regimes. However, tunnel designers often limited to the use of two-dimensional analysis tools. This is because the two-dimensional tools offer speed and ease-of-use advantages, especially in studying alternative support solutions and trade-offs.

The approach advocated in this paper is designed to help engineers to use available two-dimensional analysis tools to design realistic tunnel support, which takes into account the three-dimensional stress environment in the region of support installation. The approach suggested, although simple, allows tunnel designers to at least obtain a good start off point for designing support.

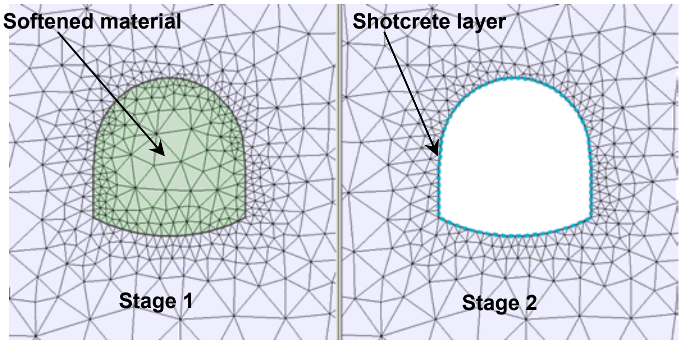

Figure 6. Two-stage Phase2 model of actual tunnel.

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Hoek, E. 2000. Big tunnels in bad rock. 2000 Terzaghi Lecture, ASCE National Convention in Seattle, WA.

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Phase^2 v5.0. 2002. Two-dimensional finite element program. Rocscience Inc.

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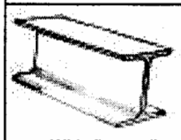
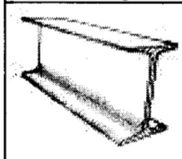
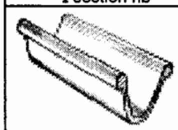
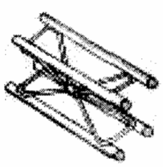
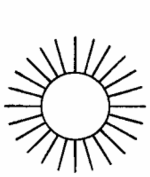
RocSupport v2.0. 2002. Program for estimating tunnel support using ground reaction curves.

APPENDIX

Support type	Flange width - mm	Section depth - mm	Weight – kg/m	Curve number	Maximum support pressure p_{imax} (MPa) for a tunnel of diameter D (metres) and a set spacing of s (metres)
 Wide flange rib	305	305	97	1	$p_{i\max} = 19.9D^{-1.23}/s$
	203	203	67	2	$p_{i\max} = 13.2D^{-1.3}/s$
	150	150	32	3	$p_{i\max} = 7.0D^{-1.4}/s$
 I section rib	203	254	82	4	$p_{i\max} = 17.6D^{-1.29}/s$
	152	203	52	5	$p_{i\max} = 11.1D^{-1.33}/s$
 TH section rib	171	138	38	6	$p_{i\max} = 15.5D^{-1.24}/s$
	124	108	21	7	$p_{i\max} = 8.8D^{-1.27}/s$
 3 bar lattice girder	220	190	19	8	$p_{i\max} = 8.6D^{-1.03}/s$
	140	130	18		
 4 bar lattice girder	220	280	29	9	$p_{i\max} = 18.3D^{-1.02}/s$
	140	200	26		
 Rockbolts or cables spaced on a grid of s x s metres	34 mm rockbolt			10	$p_{i\max} = 0.354/s^2$
	25 mm rockbolt			11	$p_{i\max} = 0.267/s^2$
	19 mm rockbolt			12	$p_{i\max} = 0.184/s^2$
	17 mm rockbolt			13	$p_{i\max} = 0.10/s^2$
	SS39 Split set			14	$p_{i\max} = 0.05/s^2$
	EXX Swellex			15	$p_{i\max} = 0.11/s^2$
	20mm rebar			16	$p_{i\max} = 0.17/s^2$
	22mm fibreglass			17	$p_{i\max} = 0.26/s^2$
	Plain cable			18	$p_{i\max} = 0.15/s^2$
	Birdcage cable			19	$p_{i\max} = 0.30/s^2$

Support type	Thickness - mm	Age - days	UCS - MPa	Curve number	Maximum support pressure p_{imax} (MPa) for a tunnel of diameter D (metres)
 Concrete or shotcrete lining	1m	28	35	20	$p_{i\max} = 57.8D^{-0.92}$
	300	28	35	21	$p_{i\max} = 19.1D^{-0.92}$
	150	28	35	22	$p_{i\max} = 10.6D^{-0.97}$
	100	28	35	23	$p_{i\max} = 7.3D^{-0.98}$
	50	28	35	24	$p_{i\max} = 3.8D^{-0.99}$
	50	3	11	25	$p_{i\max} = 1.1D^{-0.97}$
	50	0.5	6	26	$p_{i\max} = 0.6D^{-1.0}$

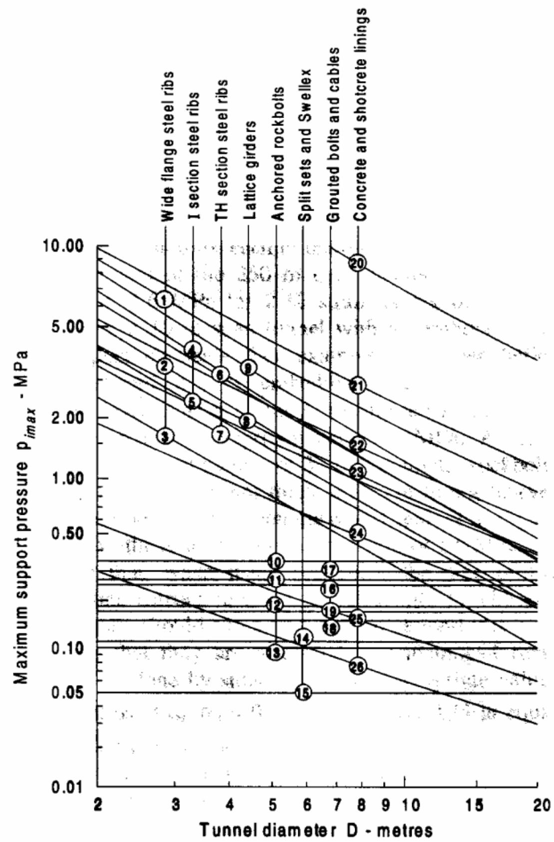


Figure A1: Approximate maximum capacities for different support systems installed in circular tunnels (Hoek, 1998).