A COMPARISON OF FINITE ELEMENT SLOPE STABILITY ANALYSIS WITH CONVENTIONAL LIMIT-EQUILIBRIUM INVESTIGATION

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ABSTRACT

In the assessment of slopes, factor of safety values still remain the primary indexes for determining how close or far slopes are from failure. Traditional limit-equilibrium techniques are the most commonly-used analysis methods. Recently, however, the significant computing and memory resources typically available to the geotechnical engineer, combined with low costs, have made the Finite Element Method (FEM) a powerful, viable alternative.

The Shear Strength Reduction (SSR) technique enables the FEM to calculate factors of safety for slopes. The method enjoys several advantages including the ability to predict stresses and deformations of support elements, such as piles, anchors and geotextiles, at failure. Despite the SSR finite element technique's many benefits, it has not received widespread adoption among geotechnical engineers for routine slope stability analysis. This could be probably due to the very limited experience engineers have had with the tool for slope stability analysis, and limited published information on the quality/accuracy of its results.

To help change this situation this paper will compare the method's performance to those of the most widely used limit-equilibrium methods on a broad range of slope cases. The SSR's performance will be tested on about 30 slope examples, which have been reported in literature and have been used by software developers to verify the results of traditional slope stability programs.

RÉSUMÉ

Dans l'évaluation des risques d'instabilité des pentes, on utilise encore aujourd'hui des indexes empiriques de sûreté pour déterminer la criticité et la rupture. Les techniques traditionnelles, fondées sur la notion de limite d'équilibre, sont les méthodes d'analyse les plus généralement utilisées pour ce faire. Cependant, les pouvoirs de calcul numériques en général disponibles à l'ingénieur géotechnique, combiné avec de bas coûts, ont récemment permis aux méthodes fondées sur les calculs d'éléments finis (FEM) de devenir une alternative puissante et viable.

La technique de la réduction de résistance au cisaillement (SSR) permet au FEM de calculer des facteurs de sûreté pour une pente. La méthode comporte plusieurs avantages, comme par exemple la capacité de prévoir les contraintes et les déformations des éléments de soutien, tels que les piliers, les ancres ou autre géotextiles au moment de la rupture. En dépit des nombreux avantages des calculs fondés sur les éléments finis et de la technique SSR, cette dernière n'a été adopté que par très peu d'ingénieurs géotechniques dans le cas de l'évaluation de la stabilité des pentes. Ceci peut probablement être mis sur le compte d'une expérience très limitée des ingénieurs avec les méthodes numériques d'analyse de stabilité de pente, et le peu d'information publiée sur la qualité ou l'exactitude de leurs résultats.

C'est dans le but de remédier à cette situation que cet article compare la méthode SSR à celles de limite-équilibre les plus largement répandues. Cette comparaison porte sur une large gamme de pente et les résultats de la méthode SSR sont examinés sur environ 30 exemples différents de pente, rapportées dans la littérature. Ces exemples ont permis aux informaticiens programmateurs de vérifier les résultats des programmes de stabilité de pente traditionnels.

1. INTRODUCTION

In the assessment of slopes, engineers primarily use factor of safety values to determine how close or far slopes are from failure. Conventional limit-equilibrium techniques are the most commonly-used analysis methods. Recently, however, the significant computing and memory resources available to the geotechnical engineer, combined with low costs, have made the Finite Element Method (FEM) a powerful, viable alternative.

The Shear Strength Reduction (SSR) technique [Dawson et al, 1999, Griffith and Lane, 1999, Hammah et al, 2004] enables the FEM to calculate factors of safety for slopes. The method enjoys several advantages including the ability to predict stresses and deformations of support elements, such as piles, anchors and geotextiles, at failure. As well the technique makes it possible to visualize the development of failure mechanisms. Advances in program interfaces and competitive computational times help account for the method's attractiveness.

Despite the SSR finite element technique's many benefits, it has not received widespread adoption among geotechnical engineers for routine slope stability analysis. In the authors' opinion this is primarily due to the very limited experience engineers have had with the tool for slope stability analysis, and the limited published information on the quality/accuracy of its results.

To improve confidence in the SSR technique, this paper will compare the method's performance to those of well-established limit-equilibrium methods on a broad range of slope cases. These cases have all been reported in literature, and have been used to verify the results of some slope stability programs {Rocscience 2003].

Since FEM slope stability analysis is not very common in geotechnical engineering, the paper will discuss the impact of Young's modulus, Poisson's ratio, and dilation angle on computed factor of safety values. As well it will examine the choice of convergence criterion, number of iterations and tolerance that lead to accurate calculation of slope factor of safety. All the finite element models developed during the research for this paper employ six-noded triangular elements.

2. PARAMETERS OF SSR ANALYSIS AND THEIR INFLUENCE ON COMPUTED FACTOR OF SAFETY

The paper experiments with different values of Young's modulus, E, and Poisson's ratio, μ. As well it tests different values angle of dilatancy, and two models of post-peak material behaviour. It is expedient to establish the influence of these parameters, since they are absent from limit-equilibrium analysis. These experiments also help identify implied assumptions in limit-equilibrium analysis.

The study also looked into establishing a convergence criterion, tolerance level and number of iterations combination that allow factors of safety to be determined accurately and within reasonably computational time. Any such a combination had to work well over a wide range of slope problems.

To test the influence of the above-listed parameters two simple slopes were analysed (see Figures 1 and 2). The first slope consists of a single Mohr-Coulomb material, while the second comprises three horizontal layers of Mohr-Coulomb materials.

In all the experiments, the same basic Mohr-Coulomb strength parameters were used for all materials. Cohesion was assumed equal to 10.5 kPa, while friction angle was assigned a value of 35 degrees. For both slopes the Bishop's method factor of safety (computed with the program Slide [Rocscience, 2003]) was 1.21. This factor of safety was used as the benchmark for assessing deviations in SSR answers.

2.1 Post-Peak Material Behaviour

The authors tested the influence of two different assumptions regarding the post-peak strength of materials. The tests were performed on the homogeneous slope example with an E of 20,000 kPa and μ of 0.2.

The first assumption, the elastic-perfectly plastic strength model, supposed post-peak strength to remain the same as peak strength. This resulted in a factor of safety equal to 1.21 (Line 3 of Table 1). The second scenario assumed in post-peak regime both cohesion and friction angle experience a 50% reduction, i.e. post-peak cohesion and friction angle were 5.25 kPa and 17.5 degrees, respectively.

The assumption of a 50% reduction of post-peak cohesion and friction angle resulted in factor of safety of 0.67. This is an approximately 100% drop. This simple experiment indicates that elastic-perfectly plastic model best replicates limit-equilibrium slope stability analysis. As a result all subsequent SSR analysis in this paper are based on this assumption.

2.2 Young's Modulus, Poisson's Ratio and Angle of Dilation

The authors tested the impact of Young's modulus on factor of safety results first by assuming three different E values for the homogeneous slope. The E values used were 2,000 kPa, 20,000 kPa and 200,000 kPa. Similarly, for each of Poisson's ratio, μ, and angle of dilation, ψ, three different values were used in the tests: $\mu_1 = 0.2$, $\mu_2 = 0.3$, $\mu_3 = 0.48$, $\psi_1 = 0^\circ$, $\psi_2 = 17.5^\circ$, and $\psi_3 = 35^\circ$.

To keep the analysis simple, in the case of the three-material slope only different combinations of E and μ were considered. An additional E value of 2,000,000 kPa was also applied to this case to help highlight the consistency of factor of safety results even when high variations of Young's modulus ratios are considered for multiple material slopes. The results of the experiments are summarized in Tables 1 and 2 below.

First, we will discuss the impact of Young's modulus and Poisson's ratio. Although these parameters affect the magnitudes of computed deformations, they had minimal impact on factor of safety results.

In the case of the homogeneous slope, for the same μ changes in E did not lead to any changes in factor of safety. The two widely different Poisson's ratio values (0.2 and 0.48, respectively) yielded only a 2.5% change in answer (1.21 versus 1.18). Obvious as it may be, it is important to point out though that these parameters significantly affect the magnitudes of computed deformations.

The impact of dilation angle was tested on two cases of the homogeneous slope. In both cases the material had an E of 20,000 kPa, but different Poisson's ratios – 0.2 and 0.48, respectively.

For each of the cases three different dilation angles were considered 0°, 17.5°, and 5°. The factor of safety results are in lines 3 and 4, and lines 7 to 10 of Table 1.

Factors of safety ranged from 1.18 to 1.27. These minimum and maximum values are less than 5% away from the limit-equilibrium benchmark value of 1.21. This finding confirms that, as indicated by others [Griffith and Lane, 1999], the angle of dilation does not have significant impact in slope problems due to the generally low confinement environment.

The three-material slope example allowed exploration of different Young's moduli and Poisson's ratios for various slope materials. Table 2 below describes the different combinations of deformation properties tested, and the resulting factors of safety.

The factor of safety values ranged from 1.15 to 1.34, a -5% to +11% difference from the benchmark limit-equilibrium solution. Although in some cases the stiffest layer was 1,000 stiffer than the softest material (lines 6 and 7 of Table 2), computed factor of safety still remained within an acceptable range.

It is important to note, however, that material stiffness ratios impact deformation patterns, and in some instances produced failure mechanisms that differed significantly from the benchmark limit equilibrium solution. This hints that in multiple material cases involving reinforcement, factors of safety could be quite different for different stiffness ratios. For example, if deformations of support are required, and this is important for support types such as piles, then it is important to use properly estimated values of E and Poisson's ratio.

This simple, multi-material slope study establishes that to duplicate limit-equilibrium factor of safety results (at least for unreinforced slopes) with the SSR method, it is only necessary to:

(i) use the same E value for the materials in a multiple-material model
(ii) assume a single Poisson's ratio for materials
(iii) assume a dilation angle = 0, and
(iv) use the elastic-perfectly plastic assumption for post-peak behaviour.

These findings are consistent with assertions in Dawson et al, 1999.

3. THE ROLE OF CONVERGENCE PARAMETERS

Since in SSR analysis, instability of the numerical solution or non-convergence determines whether a slope has failed, the authors studied the impact of convergence parameters on results. The convergence of a finite element solution is characterized by three important attributes:

(i) the type of stopping criterion
(ii) the tolerance value of the stopping criterion, and
(iii) the number iterations allowed before a solutions is assessed to have not converged.

The paper examined three different stopping criteria – displacement, residual force, and energy [Cook et al, 1989, Crisfield, 1986, Owen and Hinton, 1980]. In finite element analysis the displacement convergence criterion is met for a current iteration when increments in displacement are negligible. Residual force convergence is satisfied when residual force change is negligible for a current configuration. The energy convergence stopping rule is a measure of energy balance in the system being solved, and is satisfied when the imbalance falls below a specified value.

It was important to establish a combination of convergence method, tolerance value, and number of iterations that provided 'accurate' and 'reliable' answers. An "ideal' stopping criterion has to be relatively insensitive to model attributes such as number of elements.

Once such a criterion was identified, it would then be necessary to establish a tolerance value and number of iterations combination that worked well over a wide range of problems. Such a combination would facilitate practical analysis especially if maintained reasonable solution times.

The convergence issues were studied using Phase2, an implicit finite element program [Rocscience, 2005]. The numerical experiments were performed using the Gaussian elimination matrix solver.

After several tests involving numerous different models, the energy norm criterion proved to be the most robust stopping rule. It had the least sensitivity to model attributes. It was also established that a tolerance of 0.001 combined with 300 iterations produced consistently good results in acceptable time.

An experiment was conducted in which a few of the slope examples in this paper were re-analyzed with different numbers of six-noded triangular elements. The numbers of elements used were 200, 400, 800, 2,000, 4,000 and 10,000. In almost all of these unreinforced slope cases, the number of elements had minimal impact on computed factor of safety values. Whether or not these results can be generalized remains a subject for further research.

4. VERIFICATION EXAMPLES

After establishing the four simple rules above regarding the Young modulus, Poisson's ratio and dilation angles, post-peak strength assumption to use in order to simulate limit-equilibrium slope stability analysis, and establishing a robust convergence criterion alongside appropriate tolerance and number of iteration values, the authors proceeded to analyse a number of slope examples that have been reported in technical publications. The slopes analysed in this paper are all unreinforced, and are described in Rocscience 2003 and 2005.

Not all the verification examples involve Mohr-Coulomb materials. For a few of the slopes, material strength is represented with the power curve failure model. (An SSR technique for non-linear strength envelopes is described in [Hammah et al, 2005].) For some examples both circular and non-circular results are reported in Rocscience, 2003 and 2005. In such cases only the non-circular results are reported. Also wherever results for several different methods limit-equilibrium methods existed, primarily the Bishop and Spencer factor of safety values were reported in the paper.

The factors of safety computed by the SSR are compared against limit-equilibrium results in Table 3 below. In all cases the SSR gives answers that agree very well with the limit-equilibrium values.

5. SUMMARY OF RESULTS

In summary, this study helped establish that to duplicate limit-equilibrium factor of safety results (at least for unreinforced slopes) with the SSR method, it is only necessary to:

- (i) use the same E value for the materials in a multiple-material model
- (ii) assume a single valid Poisson's ratio for the materials
- (iii) assume a dilation angle = 0, and
- (iv) use the elastic-perfectly plastic assumption for post-peak behaviour.

Others have reported similar findings [Dawson et al, 1999].

In almost all of these unreinforced slope cases, the number of elements had little impact on SSR factor of safety answers. Further research must be conducted to determine how general this finding is.

Of the stopping rules tested, the energy norm proved to be the most robust stopping rule over the wide range of problems. The paper did not test combinations of different stopping rules, but it would certainly be valuable to do so. The authors established that for the energy norm, a tolerance of 0.001 combined with 300 iterations produced consistently good results.

As stated by Griffiths and Lane, 1999, opinions that the FE SSR may be complex overlook the fact that 'slip circle' analyses may produce misleading results. As such we encourage geotechnical engineers to adopt the SSR as an additional robust and powerful tool for designing and analysing slopes. It can help uncover important behaviour that may otherwise go unnoticed.

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FIGURES

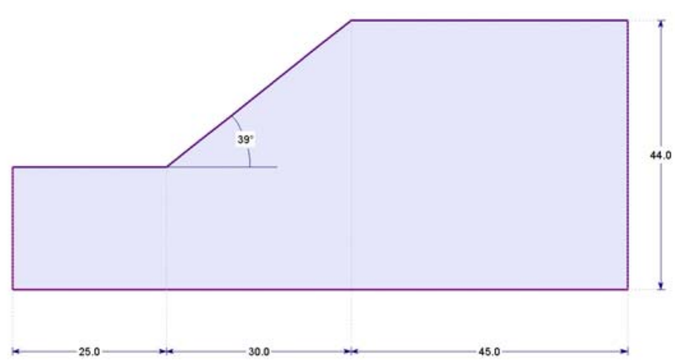

Figure 1. Geometry of homogeneous slope.

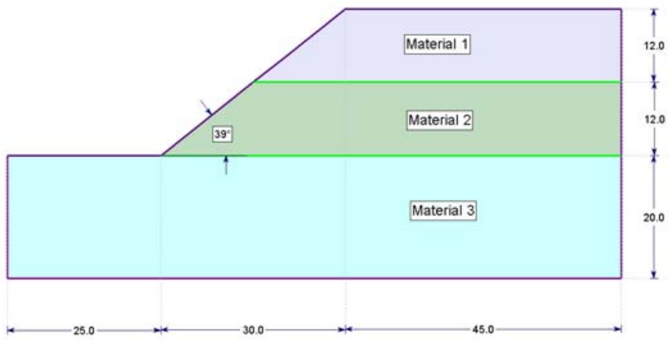

Figure 2. Geometry of three-material slope.

TABLES

Table 1. Factor of safety results for the homogeneous slope example.

No.	Homogeneoeus Slope Example			
	E (kPa)	μ	ψ **(degrees)**	**Factor of Safety**
1	2,000	0.2	0	1.21
2	2,000	0.48	0	1.18
3	20,000	0.2	0	1.21
4	20,000	0.48	0	1.18
5	200,000	0.2	0	1.21
6	200,000	0.48	0	1.18
7	20,000	0.2	17.5	1.26
8	20,000	0.2	35	1.25
9	20,000	0.48	17.5	1.22
10	20,000	0.48	35	1.27

Table 2. Factor of safety results for the three-material slope example.

No.	Three-Material Slope Example						
	Material 1		**Material 2**		**Material 3**		**Factor of Safety**
	E (kPa)	μ	E (kPa)	μ	E (kPa)	μ	
1	20,000	0.2	20,000	0.2	20,000	0.2	1.19*
2	2,000	0.2	20,000	0.2	200,000	0.2	1.15
3	200,000	0.2	20,000	0.2	2,000	0.2	1.34
4	20,000	0.2	2,000	0.2	200,000	0.2	1.16
5	200,000	0.2	2,000	0.2	20,000	0.2	1.23
6	200,000	0.2	2,000	0.2	2,000,000	0.2	1.23
7	2,000,000	0.2	2,000	0.2	200,000	0.2	1.29
8	20,000	0.48	20,000	0.3	20,000	0.2	1.19
9	20,000	0.2	20,000	0.48	20,000	0.3	1.16
10	20,000	0.2	2,000	0.3	200,000	0.48	1.17

*The difference this factor and safety and that in Line 3 of Table 1 is a result of different meshing.

Table 3. Performance of SSR method on verification examples reported in literature.

No.	Original Reference (See List of References below)	Slide Verification Number[1]	Phase2 Verification Number[2]	Limit-Equilibrium Analysis Method	Reported Factors of Safety[3]	SSR Results
1	[1]		1	-- Spencer GLE	1.000 [1] 0.986 [S] 0.986 [S]	1.00
2	[1]		2	-- Spencer GLE	1.390 [1] 1.375 [S] 1.374 [S]	1.36
3	[1]		3	-- Spencer GLE	1.000 [S] 0.991 [S] 0.989 [S]	0.97
4	[1]		4	-- Spencer GLE	1.950 [1] 1.948 [S] 1.948 [S]	1.93
5	[1]		5	-- Spencer GLE	1.24-1.27 [1] 1.258 [S] 1.246 [S]	1.28
6	[1]		6	-- Spencer GLE	0.780 [1] 0.707 [S] 0.683 [S]	0.79
7	[1]		7	 Spencer GLE	1.530 [1] 1.501 [S] 1.500 [S]	1.48
8	[2]		8	-- Spencer GLE	1.040 [2] 1.065 [S] 1.059 [S]	0.96
9	[2]		9	-- Spencer GLE	1.240 [2] 1.334 [S] 1.336 [S]	1.33
10	[3]		10	Bishop Janbu Corrected Bishop Spencer	1.451 [3] 1.346 [3] 1.409 [S] 1.406 [S]	1.40
11	[3]		11	Bishop Janbu Corrected Bishop Spencer	0.417 [3] 0.430 [3] 0.421 [S] 0.424 [S]	0.39
12	[3]		12	Janbu Simplified Janbu Corrected Janbu Corrected Spencer	0.995 [3] 1.071 [3] 1.050 [S] 1.094 [S]	1.09
13	[4], [5]		13	Spencer Spencer Spencer	1.339 [4] 1.330 [5] 1.324 [S]	1.34
14	[6], [7]		14	Spencer Spencer Spencer	1.080 [6] 1.020 [7] 1.010 [S]	1.01
15	[6], [5]		15	Spencer Spencer Spencer	1.40-1.42 [5] 1.40-1.42 [6] 1.398 [S]	1.39
16	[8], [5]		16	Spencer Spencer Spencer	1.080 [5] 1.01-1.03 [8] 1.007 [S]	1.01
17	[5], [7], [9], [10]		17-1 Dry 17-2 Ru 17-3 WT	Spencer Spencer Spencer Spencer Spencer Spencer	2.073 [9] 2.075 [S] 1.761 [9] 1.760 [S] 1.830 [9] 1.831 [S]	2.00 1.68 1.79

Table 3 continued. Performance of SSR method on verification examples reported in literature.

18	[7], [9], [11], [12]		18-1 Dry 18-2 Ru 18-3 WT	Spencer Spencer Spencer Spencer Spencer Spencer	1.373 [9] 1.382 [S] 1.118 [9] 1.124 [S] 1.245 [9] 1.244 [S]	1.39 1.04 1.18
19	[13]		19	Bishop Bishop	1.440 [13] 1.439 [S]	1.43
20	[8], [14]		20	Spencer Spencer	1.050 [8] 1.051 [S]	1.01
21	[14]		21	Theory Spencer	1.000 0.941 [S]	1.01
22	[10], [15]		22	Spencer Spencer	1.513 [15] 1.510 [S]	1.52
23	[16]		23	-- Spencer	1.170 [16] 1.150 [S]	1.12
24	[17]		25	-- Bishop	1.310 [17] 1.305 [S]	1.27
25	[18]		26	-- Spencer	2.360 [18] 2.383 [S]	2.35
26	[19], [20]		27	-- Bishop	1.334 [20] 1.339 [S]	1.31
27	[21]		28-1 (61 m) 28-2 (62 m) 28-3 (63 m)	-- Bishop -- Bishop -- Bishop	1.636 [21] 1.616 [S] 1.527 [21] 1.535 [S] 1.436 [21] 1.399 [S]	1.65 1.56 1.42
28	[22]		29-Sand 29-Clay	Spencer Spencer Spencer Spencer	1.219 [22] 1.189 [S] 0.941 [22] 0.975 [S]	1.25 0.99
29	[23]		30	-- Janbu Simplified	0.980 [23] 0.944 [S]	0.91
30	[24]		31-PC 31-MC	-- Spencer -- Spencer	0.970 [24] 0.960 [S] 1.500 [24] 1.540 [S]	1.14 1.54
31	[24]		32-PC 32-MC	-- Spencer -- Spencer	2.640 [24] 2.660 [S] 2.660 [24] 2.760 [S]	2.74 2.83
32	[25]		33	Spencer Spencer	1.290 [25] 1.300 [S]	1.28
33	[24]		34-PC 34-MC	-- Spencer -- Spencer	1.480 [24] 1.470 [S] 1.350 [24] 1.370 [S]	1.47 1.38

1. This number refers to the Verification Number for this problem as described in the Slide Verification Manual.
2. This number refers to the Verification Number for this problem as described in the $Phase^2$ Verification Manual.
3. The values in the parentheses besides each reported factor of safety refer to the reference number of the technical paper that reports that factor of safety. An [S] beside a factor of safety indicates a value computed by Slide.

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