A brief history of the development of the Hoek-Brown failure criterion

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Abstract

The Hoek-Brown failure criterion was developed in the late 1970s to provide input for the design of underground excavations. Bieniawski's RMR was originally used to link the criterion to engineering geology input from the field but a more specific classification system called the Geological Strength Index (GSI) was introduced in 1995. Both the Hoek Brown criterion and the GSI classification have evolved and continue to evolve to meet new applications and to deal with unusual conditions encountered by users.

Introduction

The original Hoek-Brown failure criterion was developed during the preparation of the book *Underground Excavations in Rock* by E. Hoek and E.T. Brown, published in 1980. The criterion was required in order to provide input information for the design of underground excavations. Since no suitable methods for estimating rock mass strength appeared to be available at that time, the efforts were focussed on developing a dimensionless equation that could be scaled in relation to geological information. The original Hoek-Brown equation was neither new nor unique – an identical equation had been used for describing the failure of concrete as early as 1936.

The significant contribution that Hoek and Brown made was to link the equation to geological observations. It was recognised very early in the development of the criterion that it would have no practical value unless the parameters could be estimated from simple geological observations in the field. The idea of developing a 'classification' for this specific purpose was discussed but, since Bieniawski's RMR had been published in 1974 and had gained popularity with the rock mechanics community, it was decided to use this as the basic vehicle for geological input.

By 1995 it had become increasingly obvious that Bieniawski's RMR is difficult to apply to very poor quality rock masses and it was felt that a system based more heavily on fundamental geological observations and less on 'numbers' was needed. This resulted in the development of the Geological Strength Index, GSI, which continues to evolve as the principal vehicle for geological data input for the Hoek-Brown criterion.

Historical development

1980 Hoek E. and Brown E.T. 1980. *Underground Excavations in Rock*. London: Institution of Mining and Metallurgy

Hoek, E. and Brown, E.T. 1980. Empirical strength criterion for rock masses. *J. Geotech. Engng Div., ASCE* **106**(GT9), 1013-1035.

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The original criterion was conceived for use under the confined conditions surrounding underground excavations. The data upon which some of the original relationships had been based came from tests on rock mass samples from the Bougainville open pit copper mine in Papua New Guinea. The rock mass here is very strong andesite (uniaxial compressive strength about 270 MPa) with numerous clean, rough, unfilled joints. One of the most important sets of data was from a series of triaxial tests carried out by Professor John Jaeger at the Australian National University in Canberra. These tests were on 150 mm diameter samples of heavily jointed andesite recovered by triple-tube diamond drilling from one of the exploration adits at Bougainville.

The original criterion, with its bias towards hard rock, was based upon the assumption that rock mass failure is controlled by translation and rotation of individual rock pieces, separated by numerous joint surfaces. Failure of the intact rock was assumed to play no significant role in the overall failure process and it was assumed that the joint pattern was ‘chaotic’ so that there are no preferred failure directions and the rock mass can be treated as isotropic.

1983 Hoek, E. 1983. Strength of jointed rock masses, 23rd. Rankine Lecture. *Géotechnique* **33**(3), 187-223.

One of the issues that had been troublesome throughout the development of the criterion has been the relationship between Hoek-Brown criterion, with the non-linear parameters m and s, and the Mohr-Coulomb criterion, with the parameters c and ϕ. At that time, practically all software for soil and rock mechanics was written in terms of the Mohr-Coulomb criterion and it was necessary to define the relationship between m and s and c and ϕ in order to allow the criterion to be used for to provide input for this software.

An exact theoretical solution to this problem (for the original Hoek-Brown criterion) was developed by Dr John W. Bray at the Imperial College of Science and Technology and this solution was first published in the 1983 Rankine lecture. This publication also expanded on some of the concepts published by Hoek and Brown in 1980 and it represents the most comprehensive discussion on the original Hoek Brown criterion.

1988 Hoek E and Brown E.T. 1988. The Hoek-Brown failure criterion - a 1988 update. *Proc. 15th Canadian Rock Mech. Symp.* (ed. J.H. Curran), pp. 31-38. Toronto: Civil Engineering Dept., University of Toronto

By 1988 the criterion was being widely used for a variety of rock engineering problems, including slope stability analyses. As pointed out earlier, the criterion was originally developed for the confined conditions surrounding underground excavations and it was recognised that it gave optimistic results for shallow failures in slopes. Consequently, in 1998, the idea of *undisturbed* and *disturbed* masses was introduced to provide a method for downgrading the properties for near surface rock masses.

This paper also defined a method of using Bieniawski’s 1974 RMR classification for estimating the input parameters. In order to avoid double counting the effects of groundwater (an effective stress parameter in numerical analysis) and joint orientation (specific input for structural analysis), it was suggested that the rating for groundwater should always be set at 10 (completely dry) and the rating for joint orientation should always be set to zero (very favourable). Note that these ratings need to be adjusted in later versions of Bieniawski's RMR, for example, use 15 for ground water in the 1989 version.

1990 Hoek, E. 1990. Estimating Mohr-Coulomb friction and cohesion values from the Hoek-Brown failure criterion. *Intnl. J. Rock Mech. & Mining Sci. & Geomechanics Abstracts.* **12**(3), 227-229.

This technical note addressed the on-going debate on the relationship between the Hoek-Brown and the Mohr-Coulomb criterion. Three different practical situations were described and it was demonstrated how Bray's solution could be applied in each case.

1992 Hoek, E., Wood, D. and Shah, S. 1992. A modified Hoek-Brown criterion for jointed rock masses. *Proc. rock characterization, symp. Int. Soc. Rock Mech.: Eurock '92*, (J. Hudson ed.). 209-213.

The use of the Hoek Brown criterion had now become widespread and, because of the lack of suitable alternatives, it was now being used on very poor quality rock masses. These rock masses differ significantly from the tightly interlocked hard rock mass model used in the development of the original criterion. In particular it was felt that the finite tensile strength predicted by the original Hoek Brown criterion was too optimistic and that it needed to be revised. Based upon work carried out by Dr Sandip Shah for his Ph.D thesis at the University of Toronto, a modified criterion was proposed. This criterion contains a new parameter *a* that provides the means for changing the curvature of the failure envelope, particularly in the very low normal stress range. Basically, the modified Hoek Brown criterion forces the failure envelope to produce zero tensile strength.

1994 Hoek, E. 1994. Strength of rock and rock masses, *ISRM News Journal,* **2**(2), 4-16.

1995 Hoek, E., Kaiser, P.K. and Bawden. W.F. 1995. *Support of underground excavations in hard rock.* Rotterdam: Balkema

It soon became evident that the modified criterion was too conservative when used for better quality rock masses and a 'generalised' failure criterion was proposed in these two publications. This generalised criterion incorporated both the original and the modified criteria with a 'switch' at an RMR value of approximately 25. Hence, for excellent to fair quality rock masses, the original Hoek Brown criterion is used while, for poor and extremely poor rock masses, the modified criterion (published in 1992) with zero tensile strength is used.

These publications (which are practically identical) also introduced the concept of the Geological Strength Index (GSI) as a replacement for Bieniawski's RMR. It had become increasingly obvious that Bieniawski's RMR is difficult to apply to very poor quality rock masses and also that the relationship between RMR and *m* and *s* is no longer linear in these very low ranges. It was also felt that a system based more heavily on fundamental geological observations and less on 'numbers' was needed.

The idea of *undisturbed* and *disturbed* rock masses was dropped and it was left to the user to decide which GSI value best described the various rock types exposed on a site. The original *disturbed* parameters were derived by simply reducing the strength by one row in the classification table. It was felt that this was too arbitrary and it was decided that it would be preferable to allow the user to decide what sort of disturbance is involved and to allow users to make their own judgement on how much to reduce the GSI value to account for the strength loss.

1997 Hoek, E. and Brown, E.T. 1997. Practical estimates or rock mass strength. *Intnl. J. Rock Mech. & Mining Sci. & Geomechanics Abstracts.* **34**(8), 1165-1186.

This was the most comprehensive paper published to date and it incorporated all of the refinements described above. In addition, a new method for estimating the equivalent Mohr Coulomb cohesion and friction angle was introduced. In this method the Hoek Brown criterion is used to generate a series of values relating axial strength to confining pressure (or shear strength to normal stress) and these are treated as the results of a hypothetical large scale in situ triaxial or shear test. A linear regression method is used to find the average slope and intercept and these are then transformed into a cohesive strength c and a friction angle ϕ.

The most important aspect of this curve fitting process is to decide upon the stress range over which the hypothetical in situ 'tests' should be carried out. This was determined experimentally by carrying out a large number of comparative theoretical studies in which the results of both surface and underground excavation stability analyses, using both the Hoek Brown and Mohr Coulomb parameters, were compared.

1998 Hoek, E., Marinos, P. and Benissi, M. 1998. Applicability of the Geological Strength Index (GSI) classification for very weak and sheared rock masses. The case of the Athens Schist Formation. *Bull. Engg. Geol. Env.* **57**(2), 151-160.

This paper extends the range of the Geological Strength Index (GSI) down to 5 to include extremely poor quality schistose rock masses such as the 'schist' encountered in the excavations for the Athens Metro and the graphitic phyllites encountered in some of the tunnels in Venezuela. This extension to GSI is based largely on the work of Paul Marinos and Maria Benissi on the Athens Metro. Note that there were now 2 GSI charts. The first of these, for better quality rock masses published in 1994 and the new chart for very poor quality rock masses published in this paper.

2000 Hoek, E. and Marinos, P. 2000. Predicting Tunnel Squeezing. *Tunnels and Tunnelling International.* Part 1, **32/11**, 45-51 – November 2000, Part 2, **32/12**, 33-36 – December, 2000.

This paper introduced an important application of the Hoek-Brown criterion in the prediction of conditions for tunnel squeezing, utilising a critical strain concept proposed by Sakurai in 1983.

2000 Marinos, P. & Hoek, E. 2000. From The Geological to the Rock Mass Model: Driving the Egnatia Highway through difficult geological conditions, Northern

Greece, *Proc. 10th International Conference of Italian National Council of Geologists, Rome*, 325-334

This paper puts more geology into the Hoek-Brown failure criterion than that which has been available previously. In particular, the properties of very weak rocks are addressed in detail for the first time. There is no change in the mathematical interpretation of the criterion in these papers.

2000 Hoek, E. and Karzulovic, A. 2000. Rock-Mass properties for surface mines. In *Slope Stability in Surface Mining* (Edited by W. A. Hustralid, M.K. McCarter and D.J.A. van Zyl), Littleton, CO: Society for Mining, Metallurgical and Exploration (SME), pages 59-70.

This paper repeats most of the material contained in Hoek and Brown, 1997, but adds a discussion on blast damage.

2000 Marinos, P and Hoek, E. 2000. GSI: a geologically friendly tool for rock mass strength estimation. *Proc. International Conference on Geotechnical & Geological Engineering, GeoEng2000*, Technomic publ., 1422-1442, Melbourne.

2001 Marinos. P and Hoek, E. 2001. Estimating the geotechnical properties of heterogeneous rock masses such as flysch. *Bulletin of the Engineering Geology & the Environment (IAEG)*, **60**, 85-92

These papers do not add anything significant to the fundamental concepts of the Hoek-Brown criterion but they demonstrate how to choose appropriate ranges of GSI for different rock mass types. In particular, the 2001 paper on flysch discussed difficult weak and tectonically disturbed materials on the basis of the authors' experience in dealing with these rocks in major projects in northern Greece.

2002 Hoek, E., Carranza-Torres, C. and Corkum, B. 2002. Hoek-Brown criterion – 2002 edition. *Proc. NARMS-TAC Conference*, Toronto, 2002, **1**, 267-273.

This paper represents a major re-examination of the entire Hoek-Brown criterion and includes new derivations of the relationships between *m, s, a* and *GSI*. A new parameter D is introduced to deal with blast damage. The relationships between the Mohr Coulomb and the Hoek Brown criteria are examined for slopes and for underground excavations and a set of equations linking the two are presented. The final relationships were derived by comparing hundreds of tunnel and slope stability analyses in which both the Hoek-Brown and the Mohr Coulomb criteria were used and the best match was found by iteration. A Windows based program called *RocLab* was developed to include all of these new derivations and this program can be downloaded (free) from www.rocscience.com. A copy of the paper is included with the download.

2004 Chandler R. J., De Freitas M. H. and P. G. Marinos. 2004. Geotechnical Characterisation of Soils and Rocks: a Geological Perspective. Keynote paper in: *Advances in geotechnical engineering, The Skempton Conference,* **1**, 67-102, Thomas Telford, ICE, London

A brief contribution on the Geological Strength Index within a more general paper on engineering geology of soils and rock.

2005 V. Marinos, P. Marinos and E. Hoek 2005. The geological Strength index: applications and limitations, *Bull. Eng. Geol. Environ.*, **64**, 55-65

A discussion on the range of application and the limitations of GSI. General guidelines for the use of GSI are given.

2005 E. Hoek, P. Marinos and V. Marinos. 2005. Characterization and engineering properties of tectonically undisturbed but lithologically varied sedimentary rock masses, *International Journal of Rock Mechanics and Mining Sciences*, **42/2**, 277-285

A significant paper in which a new GSI chart for molassic rock masses is introduced. Molasse consists of a series of tectonically undisturbed sediments of sandstones, conglomerates, siltstones and marls, produced by the erosion of mountain ranges after the final phase of an orogeny. They behave as continuous rock masses when they are confined at depth and, even if lithologically heterogeneous, the bedding planes do not appear as clearly defined discontinuity surfaces. The paper discusses the difference between these rock masses and the flysch type rocks which have been severely disturbed by orogenic processes.

2006 Marinos, P., Hoek, E., Marinos, V. 2006. Variability of the engineering properties of rock masses quantified by the geological strength index: the case of ophiolites with special emphasis on tunnelling. *Bull. Eng. Geol. Env.*, **65**(2), 129-142.

The paper presents the geological model in which the ophiolitic complexes develop, their various petrographic types and their tectonic deformation, mainly due to overthrusts. The structure of the various rock masses include all types from massive strong to sheared weak, while the conditions of discontinuities are in most cases fair to poor or very poor due to the fact that they are affected by serpentinisation and shearing. Serpentinisation also reduces the initial intact rock strength. Associated pillow lavas, and tectonic mélanges are also characterised. A GSI chart for ophiolitic rock masses is presented.

2006 Hoek, E and Diederichs, M.S. 2006. Empirical estimation of rock mass modulus. *International Journal of Rock Mechanics and Mining Sciences*, **43**, 203–215.

While not directly related to the Hoek-Brown failure criterion, the deformation modulus of a rock mass is an important input parameter in any analysis of rock mass behaviour that includes deformations. Field tests to determine this parameter directly are time consuming, expensive and the reliability of the results of these tests is sometimes questionable. Consequently, several authors have proposed empirical relationships for estimating the value of rock mass deformation modulus on the basis of classification schemes. These relationships are reviewed and their limitations are discussed. Based on data from a large number of in situ measurements from China and Taiwan, a new relationship between the deformation modulus and GSI is proposed. The properties of the intact rock as well as the effects of disturbance due to blast damage and/or stress relaxation are also included in this new relationship. The program RocLab has been updated (January 2007) to incorporate the method proposed by Hoek and Diederichs for estimating the rock mass deformation modulus.

Conclusions and recommendations

The historical development of the Hoek Brown failure criterion and the associated Geological Strength Index (GSI) has been presented. Evolution of both will continue in order to accommodate processes such as brittle spalling and anisotropy and to include a wider range of rock types. Great care is taken to retain the fundamental components of the system and to avoid changing "ratings" so that users need not go back to question or redo previous applications.

A fundamental assumption of the Hoek-Brown criterion is that the rock mass to which it is being applied is *homogeneous* and *isotropic*. It should *not be applied* to the analysis of structurally controlled failures in cases such as hard rock masses where the discontinuity spacing is similar to the size of the tunnel or slope being analysed and where the failure processes are clearly *anisotropic*.

The criterion also assumes that there is contact between intact rock pieces within the rock masses and it is these contacts that give rise to the highly non-linear characteristics of the criterion at low confining stresses. Where no such contact exists, for example when the components of the rock mass are predominantly soil or clay as in the case of fault gouges, the use of the Mohr-Coulomb criterion, with cohesion and friction parameters determined from laboratory tests, is more appropriate.

One of the greatest sources of error in applying the Hoek-Brown criterion is a misunderstanding of the contribution of the intact rock strength σ_{ci}, the role of which is almost equivalent to GSI in the evaluation of the rock mass properties. It is very common to see geologists confusing the intact strength with the rock mass strength and this results in significant under-estimates of the final rock mass strength. The authors encourage users to pay particular attention to the intact strength of the rock pieces that make up the rock mass. Measurement of the intact strength, using direct compression tests or point load tests where appropriate, should be considered.

Many engineers have requested that the GSI classification should be made more numerical so that in input parameters can be "measured" from core or rock exposures rather than estimated from geological observations. The authors and their colleagues have taken note of these request and work on providing quantitative methods for estimating GSI is ongoing, without however neglecting the basic geologic logic expressed by the GSI chart.

Many geotechnical software packages can now accommodate the Hoek-Brown criterion directly and, where this is the case, the exclusive use of the criterion is recommended. All of the necessary parameters can be calculated by means of the free program *RocLab* (www.rocscience.com) and this avoids the approximations and uncertainty associated with trying to determine equivalent Mohr Coulomb parameters.

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Hoek E., Marinos P., Benissi M. (1998) "Applicability of the Geological Strength Index (GSI) classification for very weak and sheared rock masses. The case of the Athens Schist Formation". *Bull. Eng. Geol. Env.* **57**(2), 151-160.

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Marinos V., Marinos P., Hoek E. (2005) “The geological Strength index: applications and limitations”. *Bull. Eng. Geol. Environ.*, **64**, 55-65

Hoek E., Marinos P., Marinos V. (2005) “Characterization and engineering properties of tectonically undisturbed but lithologically varied sedimentary rock masses”. *International Journal of Rock Mechanics and Mining Sciences*, **42/2**, 277-285

Marinos P., Hoek E., Marinos V. (2006) “Variability of the engineering properties of rock masses quantified by the geological strength index: the case of ophiolites with special emphasis on tunnelling”. *Bull. Eng. Geol. Env.*, **65**(2), 129-142.

Hoek E, Diederichs M.S. (2006) “Empirical estimation of rock mass modulus”. *International Journal of Rock Mechanics and Mining Sciences*, **43**, 203–215.

Appendix - Summary of equations

Publication	*Coverage*	*Equations*
Hoek & Brown 1980	Original criterion for jointed hard rock masses tightly interbedded with no fines. Mohr envelope was obtained by statistical curve fitting to a number of (σ'_n, τ) pairs calculated by the method published by Balmer σ'_1, σ'_3 are major and minor effective principal stresses at failure, respectively σ_{ci} is the uniaxial compressive strength of the intact rock σ_t is the tensile strength of the rock mass m and s are material constants (s=1 for intact rock) σ'_n, τ are effective normal and shear stresses, respectively.	$\sigma'_1 = \sigma'_3 + \sigma_{ci}\sqrt{m\sigma'_3/\sigma_{ci} + s}$ $\sigma_t = \frac{\sigma_{ci}}{2}\left(m - \sqrt{m^2 + 4s}\right)$ $\tau = A\sigma_{ci}\left((\sigma'_n - \sigma_t)/\sigma_{ci}\right)^B$ $\sigma'_n = \sigma'_3 + \left((\sigma'_1 - \sigma'_3)/(1 + \partial\sigma'_1/\partial\sigma'_3)\right)$ $\tau = (\sigma'_n - \sigma'_3)\sqrt{\partial\sigma'_1/\partial\sigma'_3}$ $\partial\sigma'_1/\partial\sigma'_3 = m\sigma_{ci}/2(\sigma'_1 - \sigma'_3)$
Hoek 1983	Original criterion for jointed hard rock masses tightly interlocked with no fines with a discussion on anisotropic failure and an exact solution for the Mohr envelope by Dr J.W. Bray.	$\sigma'_1 = \sigma'_3 + \sigma_{ci}\sqrt{m\sigma'_3/\sigma_{ci} + s}$ $\tau = \left(Cot\,\phi'_i - Cos\phi'_i\right)m\sigma_{ci}/8$ $\phi'_i = \arctan\left(1/\sqrt{4h\cos^2\theta - 1}\right)$ $\theta = \left(90 + \arctan(1/\sqrt{h^3 - 1})\right)/3$ $h = 1 + \left(16(m\sigma'_n + s\sigma_{ci})/(3m^2\sigma_{ci})\right)$
Hoek & Brown 1988	As for Hoek 1983 but with the addition of relationships between constants *m* and *s* and a modified form of *RMR* in which the Groundwater rating was assigned a fixed value of 10 and the Adjustment for Joint Orientation was set at 0. Also a distinction between *disturbed* and *undisturbed* rock masses was introduced together with means of estimating deformation modulus *E* (after Serafim and Pereira). Note that the ground water rating assigned a final value of 15 in the RMR 1989 version.	*Disturbed rock masses:* $m_b/m_i = \exp\left((RMR - 100)/14\right)$ $s = \exp\left((RMR - 100)/6\right)$ *Undisturbed or interlocking rock masses* $m_b/m_i = \exp\left((RMR - 100)/28\right)$ $s = \exp\left((RMR - 100)/9\right)$ $E = 10^{\left((RMR - 10)/40\right)}$ m_b, m_i are petrographic constants for broken and intact rock, respectively.
Hoek, Wood & Shah 1992	Modified criterion to account for the fact the heavily jointed rock masses have zero tensile strength. Balmer's technique for calculating shear and normal stress pairs was utilised. Material parameter *a* is introduced.	$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m_b\sigma'_3/\sigma_{ci}\right)^a$ $\sigma'_n = \sigma'_3 + \left((\sigma'_1 - \sigma'_3)/(1 + \partial\sigma'_1/\partial\sigma'_3)\right)$ $\tau = (\sigma'_n - \sigma'_3)\sqrt{\partial\sigma'_1/\partial\sigma'_3}$ $\partial\sigma'_1/\partial\sigma'_3 = 1 + \alpha m_b^a\left(\sigma'_3/\sigma_{ci}\right)^{(a-1)}$
Hoek 1994 Hoek, Kaiser & Bawden 1995	Introduction of the Generalised Hoek-Brown criterion, incorporating both the original criterion for excellent to fair quality rock masses and the modified criterion for poor to very poor quality rock masses	$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m\sigma'_3/\sigma_{ci} + s\right)^a$ for *GSI* >25 $m_b/m_i = \exp\left((GSI - 100)/28\right)$

	with increasing fines content. The Geological Strength Index *GSI* was introduced to overcome the deficiencies in Bieniawski's *RMR* for very poor quality rock masses. The distinction between disturbed and undisturbed rock masses was dropped on the basis that disturbance is generally induced by engineering activities and should be allowed for by downgrading the value of *GSI*.	$s = \exp\left((GSI - 100)/9\right)$ $a = 0.5$ for $GSI < 25$ $s = 0$ $a = 0.65 - GSI/200$
Hoek, Carranza-Torres and Corkum, 2002	A new set of relationships between GSI, m_b, s and a is introduced to give a smoother transition between very poor quality rock masses (GSI < 25) and stronger rocks. A disturbance factor D to account for stress relaxation and blast damage is also introduced. Equations for the calculation of Mohr Coulomb parameters *c* and ϕ are introduced for specific ranges of the confining stress σ'_{3max} for tunnels and slopes. All of these equations are incorporated into the Windows program RocLab that can be downloaded from the Internet site www.rocscience.com. A copy of the full paper is included with the download.	$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m_b \sigma'_3/\sigma_{ci} + s\right)^a$ $m_b = m_i \exp\left(GSI - 100/28 - 14D\right)$ $s = \exp\left(GSI - 100/9 - 3D\right)$ $a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right)$ $E_m(GPa) = \left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_{ci}}{100}} \cdot 10^{((GSI-10)/40)}$ $\phi' = \sin^{-1}\left[\frac{6am_b(s + m_b\sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma'_{3n})^{a-1}}\right]$ $c' = \frac{\sigma_{ci}\left[(1+2a)s + (1-a)m_b\sigma'_{3n}\right](s + m_b\sigma'_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1 + \left(6am_b(s + m_b\sigma'_{3n})^{a-1}\right)/\left((1+a)(2+a)\right)}}$ where, for tunnels $\frac{\sigma'_{3max}}{\sigma'_{cm}} = 0.47\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0.94}$ - H is the depth below surface for slopes $\frac{\sigma'_{3max}}{\sigma'_{cm}} = 0.72\left(\frac{\sigma'_{cm}}{\gamma H}\right)^{-0.91}$ - H is the slope height γ is the unit weight of the rock mass
Hoek and Diederichs, 2006	Based on an analysis of a data set from China and Taiwan, a new relationship between the rock mass deformation modulus E_{rm} and GSI is proposed. This is based on a sigmoid function and two forms of the relationship are presented. The simplified equation depends on GSI and D only and it should be used with caution, only when no information in the intact rock properties are available. The more comprehensive equation includes the intact rock modulus. When laboratory data for the modulus are not available a means of estimating this modulus from the intact rock strength σ_{ci} is given, based on a modulus reduction factor MR.	Sigmoid function: $y = c + \frac{a}{1 + e^{-((x - x_0)/b)}}$ Simplified Hoek and Diederichs equation: $E_{rm}(MPa) = 100\,000\left(\frac{1 - D/2}{1 + e^{((75 + 25D - GSI)/11)}}\right)$ Hoek and Diederichs equation: $E_{rm} = E_i\left(0.02 + \frac{1 - D/2}{1 + e^{((60 + 15D - GSI)/11)}}\right)$ Estimated intact rock modulus: $E_i = MR \cdot \sigma_{ci}$

Geological Strength Index Chart