Rock mass properties for surface mines

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Published in *Slope Stability in Surface Mining,* (Edited by W.A. Hustralid, M.K. McCarter and D.J.A. van Zyl), Littleton, Colorado: Society for Mining, Metallurgical and Exploration (SME), 2000, pages 59-70.

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1.1 INTRODUCTION

Reliable estimates of the strength and deformation characteristics of rock masses are required for almost any form of analysis used for the design of surface excavations. Hoek and Brown (1980a, 1980b) proposed a method for obtaining estimates of the strength of jointed rock masses, based upon an assessment of the interlocking of rock blocks and the condition of the surfaces between these blocks. This method was modified over the years in order to meet the needs of users who applied it to problems that were not considered when the original criterion was developed (Hoek 1983, Hoek and Brown 1988). The application of the method to very poor quality rock masses required further changes (Hoek, Wood and Shah 1992) and, eventually, the development of a new classification called the Geological Strength Index (Hoek 1994, Hoek, Kaiser and Bawden 1995, Hoek and Brown 1997, Hoek, Marinos and Benissi (1998)). A review of the development of the criterion and of the equations proposed at various stages in this development is given in Hoek and Brown (1997).

This chapter presents the Hoek-Brown criterion in a form that has been found practical in the field and that appears to provide the most reliable set of results for use as input for methods of analysis currently used in rock engineering.

For surface excavations, the rock mass properties are particularly sensitive to stress relief and blast damage and these two factors are discussed in his chapter.

1.2 GENERALISED HOEK-BROWN CRITERION

The Generalised Hoek-Brown failure criterion for jointed rock masses is defined by:

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b \frac{\sigma_3'}{\sigma_{ci}} + s \right)^a \qquad (1.1)$$

where σ_1' and σ_3' are the maximum and minimum effective stresses at failure,

m_b is the value of the Hoek-Brown constant m for the rock mass,

s and a are constants which depend upon the rock mass characteristics, and

σ_{ci} is the uniaxial compressive strength of the intact rock pieces.

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The Mohr envelope, relating normal and shear stresses, can be determined by the method proposed by Hoek and Brown (1980a). In this approach, equation 1.1 is used to generate a series of triaxial test values, simulating full scale field tests, and a statistical curve fitting process is used to derive an equivalent Mohr envelope defined by the equation:

$$\tau = A\sigma_{ci}\left(\frac{\sigma_n^{'} - \sigma_{tm}}{\sigma_{ci}}\right)^B \tag{1.2}$$

where A and B are material constants

$\sigma_n^{'}$ is the normal effective stress, and

σ_{tm} is the 'tensile' strength of the rock mass.

In order to use the Hoek-Brown criterion for estimating the strength and deformability of jointed rock masses, three 'properties' of the rock mass have to be estimated. These are

1. the uniaxial compressive strength σ_{ci} of the intact rock elements,
2. the value of the Hoek-Brown constant m_i for these intact rock elements, and
3. the value of the Geological Strength Index *GSI* for the rock mass.

1.3 INTACT ROCK PROPERTIES

For the intact rock pieces that make up the rock mass equation 1.1 simplifies to:

$$\sigma_1^{'} = \sigma_3^{'} + \sigma_{ci}\left(m_i \frac{\sigma_3^{'}}{\sigma_{ci}} + 1\right)^{0.5} \tag{1.3}$$

The relationship between the principal stresses at failure for a given rock is defined by two constants, the uniaxial compressive strength σ_{ci} and a constant m_i. Wherever possible the values of these constants should be determined by statistical analysis of the results of a set of triaxial tests on carefully prepared core samples. When laboratory tests are not possible, Table 1.1 and Table 1.2 can be used to obtain estimates of σ_{ci} and m_i.

In the case of mineralised rocks, the effects of alteration can have a significant impact on the properties of the intact rock components and this should be taken into account in estimating the values of σ_{ci} and m_i. For example, the influence of quartz-seritic alteration of andesite and porphyry is illustrated in the Figure 1.1. Similar trends have been observed for other forms of alteration and, where this type of effect is considered likely, the geotechnical engineer would be well advised to invest in a program of laboratory testing to establish the appropriate properties for the intact rock.

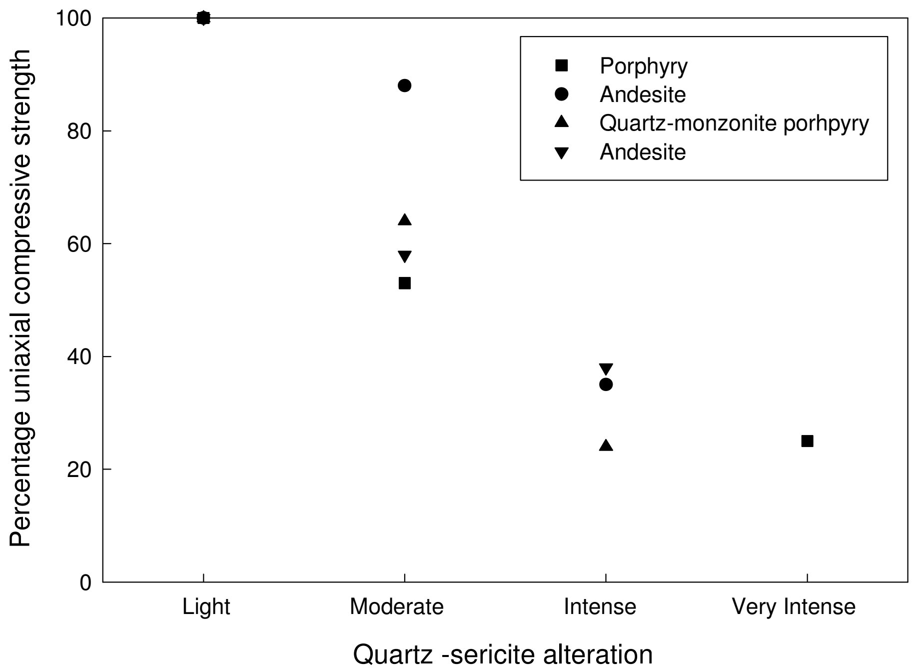


Figure 1.1: Influence of quartz-seritic alteration on the uniaxial compressive strength of "intact" specimens of andesite and porphyry.

The Hoek-Brown failure criterion, which assumes isotropic rock and rock mass behaviour, should only be applied to those rock masses in which there are a sufficient number of closely spaced discontinuities, with similar surface characteristics, that isotropic behaviour involving failure on multiple discontinuities can be assumed. When the structure being analysed is large and the block size small in comparison, the rock mass can be treated as a Hoek-Brown material.

Where the block size is of the same order as that of the structure being analysed or when one of the discontinuity sets is significantly weaker than the others, the Hoek-Brown criterion should not be used. In these cases, the stability of the structure should be analysed by considering failure mechanisms involving the sliding or rotation of blocks and wedges defined by intersecting structural features. Figure 1.2 summarises these statements in a graphical form.

1.4 GEOLOGICAL STRENGTH INDEX

The strength of a jointed rock mass depends on the properties of the intact rock pieces and also upon the freedom of these pieces to slide and rotate under different stress conditions. This freedom is controlled by the geometrical shape of the intact rock pieces as well as the condition of the surfaces separating the pieces. Angular rock pieces with clean, rough discontinuity surfaces will result in a much stronger rock mass than one which contains rounded particles surrounded by weathered and altered material.

Table 1.1: Field estimates of uniaxial compressive strength.

Grade*	Term	Uniaxial Comp. Strength (MPa)	Point Load Index (MPa)	Field estimate of strength	Examples
R6	Extremely Strong	> 250	>10	Specimen can only be chipped with a geological hammer	Fresh basalt, chert, diabase, gneiss, granite, quartzite
R5	Very strong	100 - 250	4 - 10	Specimen requires many blows of a geological hammer to fracture it	Amphibolite, sandstone, basalt, gabbro, gneiss, granodiorite, peridotite , rhyolite, tuff
R4	Strong	50 - 100	2 - 4	Specimen requires more than one blow of a geological hammer to fracture it	Limestone, marble, sandstone, schist
R3	Medium strong	25 - 50	1 - 2	Cannot be scraped or peeled with a pocket knife, specimen can be fractured with a single blow from a geological hammer	Concete, phyllite, schist, siltstone
R2	Weak	5 - 25	**	Can be peeled with a pocket knife with difficulty, shallow indentation made by firm blow with point of a geological hammer	Chalk, claystone, potash, marl, siltstone, shale, rocksalt,
R1	Very weak	1 - 5	**	Crumbles under firm blows with point of a geological hammer, can be peeled by a pocket knife	Highly weathered or altered rock, shale
R0	Extremely weak	0.25 - 1	**	Indented by thumbnail	Stiff fault gouge

* Grade according to Brown (1981).
** Point load tests on rocks with a uniaxial compressive strength below 25 MPa are likely to yield highly ambiguous results.

Table 1.2: Values of the constant m_i for intact rock, by rock group. Note that values in parenthesis are estimates.

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerates (21 ± 3) Breccias (19 ± 5)	Sandstones 17 ± 4	Siltstones 7 ± 2 Greywackes (18 ± 3)	Claystones 4 ± 2 Shales (6 ± 2) Marls (7 ± 2)
SEDIMENTARY	Non-Clastic	Carbonates	Crystalline Limestone (12 ± 3)	Sparitic Limestones (10 ± 2)	Micritic Limestones (9 ± 2)	Dolomites (9 ± 3)
SEDIMENTARY	Non-Clastic	Evaporites		Gypsum 8 ± 2	Anhydrite 12 ± 2	
SEDIMENTARY	Non-Clastic	Organic				Chalk 7 ± 2
METAMORPHIC	Non Foliated		Marble 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzites 20 ± 3	
METAMORPHIC	Slightly foliated		Migmatite (29 ± 3)	Amphibolites 26 ± 6		
METAMORPHIC	Foliated*		Gneiss 28 ± 5	Schists 12 ± 3	Phyllites (7 ± 3)	Slates 7 ± 4
IGNEOUS	Plutonic	Light	Granite 32 ± 3 Diorite 25 ± 5 Granodiorite (29 ± 3)			
IGNEOUS	Plutonic	Dark	Gabbro 27 ± 3 Norite 20 ± 5	Dolerite (16 ± 5)		
IGNEOUS	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
IGNEOUS	Volcanic	Lava		Rhyolite (25 ± 5) Andesite 25 ± 5	Dacite (25 ± 3) Basalt (25 ± 5)	Obsidian (19 ± 3)
IGNEOUS	Volcanic	Pyroclastic	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

* These values are for intact rock specimens tested normal to bedding or foliation. The value of m_i will be significantly different if failure occurs along a weakness plane.

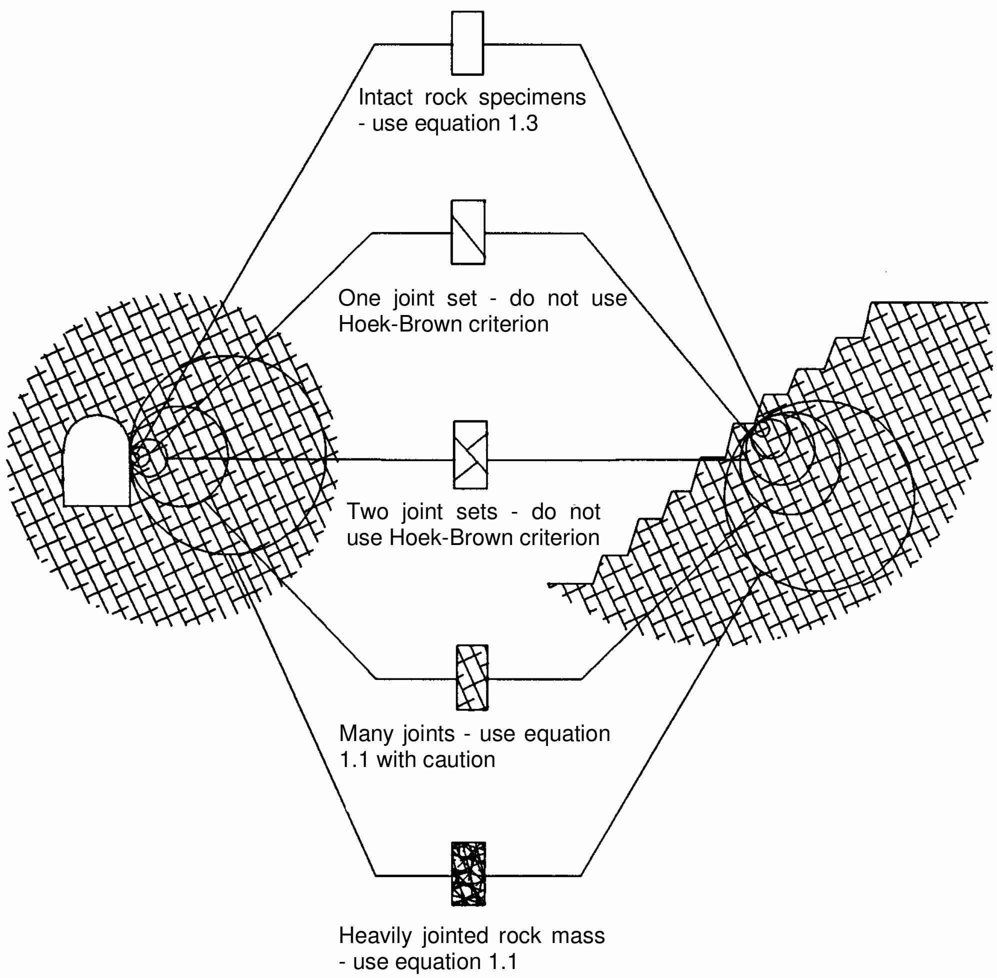


Figure 1.2: Idealised diagram showing the transition from intact to a heavily jointed rock mass with increasing sample size.

The Geological Strength Index (GSI), introduced by Hoek (1994) and Hoek, Kaiser and Bawden (1995) provides a system for estimating the reduction in rock mass strength for different geological conditions. This system is presented in Table 1.3, for blocky rock masses, and Table 1.4 for schistose metamorphic rocks.

Once the Geological Strength Index has been estimated, the parameters that describe the rock mass strength characteristics, are calculated as follows:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right) \quad (1.4)$$

For GSI > 25, i.e. rock masses of good to reasonable quality:

$$s = \exp\left(\frac{GSI - 100}{9}\right) \tag{1.5}$$

and

$$a = 0.5 \tag{1.6}$$

For GSI < 25, i.e. rock masses of very poor quality:

$$s = 0 \tag{1.7}$$

and

$$a = 0.65 - \frac{GSI}{200} \tag{1.8}$$

For better quality rock masses (GSI > 25), the value of GSI can be estimated directly from the 1976 version of Bieniawski's Rock Mass Rating, with the Groundwater rating set to 10 (dry) and the Adjustment for Joint Orientation set to 0 (very favourable) (Bieniawski 1976). For very poor quality rock masses the value of RMR is very difficult to estimate and the balance between the ratings no longer gives a reliable basis for estimating rock mass strength. Consequently, Bieniawski's RMR classification should not be used for estimating the GSI values for poor quality rock masses (RMR < 25) and the GSI charts should be used directly.

If the 1989 version of Bieniawski's RMR classification (Bieniawski 1989) is used, then GSI = RMR_{89}' - 5 where RMR_{89}' has the Groundwater rating set to 15 and the Adjustment for Joint Orientation set to zero.

1.5 MOHR-COULOMB PARAMETERS

Most geotechnical software is written in terms of the Mohr-Coulomb failure criterion in which the rock mass strength is defined by the cohesive strength c' and the angle of friction ϕ'. The linear relationship between the major and minor principal stresses, σ_1' and σ_3', for the Mohr-Coulomb criterion is

$$\sigma_1' = \sigma_{cm} + k\sigma_3' \tag{1.9}$$

where σ_{cm} is the uniaxial compressive strength of the rock mass and k is the slope of the line relating σ_1' and σ_3'. The values of ϕ' and c' can be calculated from

$$\sin\phi' = \frac{k - 1}{k + 1} \tag{1.10}$$

$$c' = \frac{\sigma_{cm}(1 - \sin\phi')}{2\cos\phi'} \tag{1.11}$$

Table 1.3: Characterisation of a blocky rock masses on the basis of particle interlocking and discontinuity condition. After Hoek, Marinos and Benissi (1998).

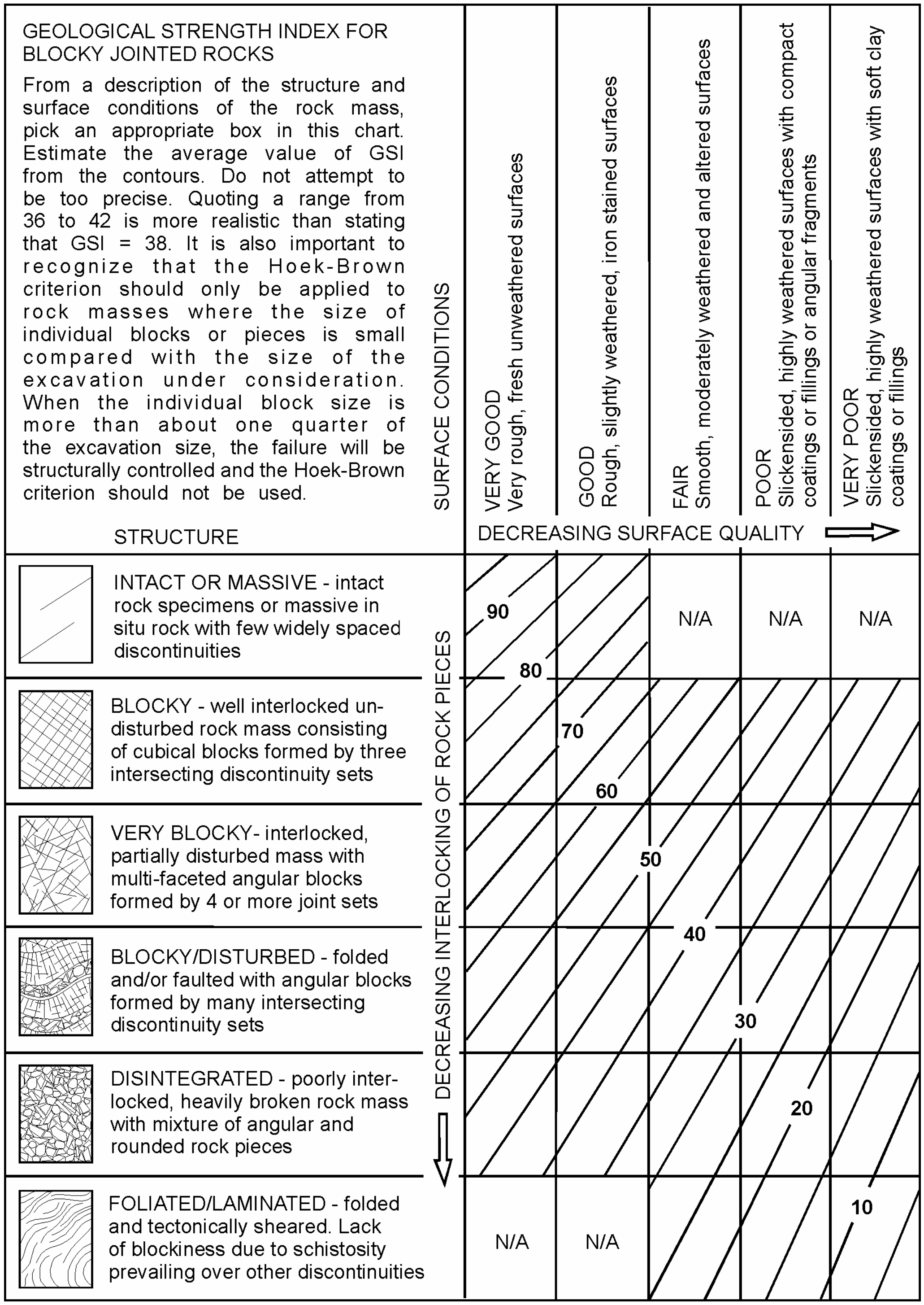
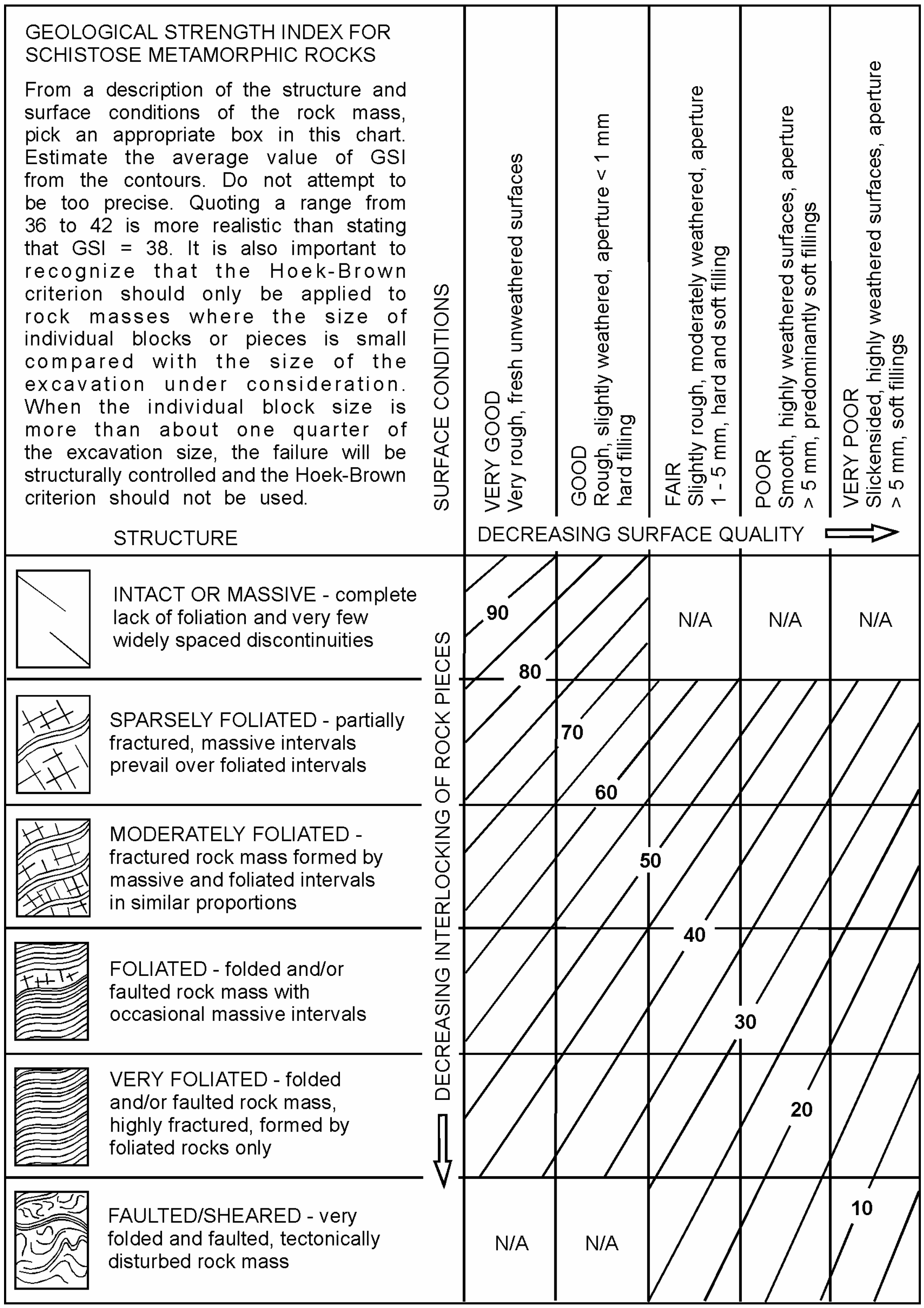


Table 1.4: Characterisation of a schistose metamorphic rock masses on the basis of foliation and discontinuity condition. (After M. Truzman, 1999)



There is no direct correlation between equation 1.9 and the non-linear Hoek-Brown criterion defined by equation 1.1. Consequently, determination of the values of c' and ϕ' for a rock mass that has been evaluated as a Hoek-Brown material is a difficult problem.

Having considered a number of possible approaches, it has been concluded that the most practical solution is to treat the problem as an analysis of a set of full-scale triaxial strength tests. The results of such tests are simulated by using the Hoek-Brown equation 1.1 to generate a series of triaxial test values. Equation 1.9 is then fitted to these test results by linear regression analysis and the values of c' and ϕ' are determined from equations 1.11 and 1.10. A full discussion on the steps required to carry out this analysis is presented in the Appendix, together with a spreadsheet for implementing this analysis.

The range of stresses used in the curve fitting process described above is very important. For the confined conditions surrounding tunnels at depths of more than about 30 m, the most reliable estimates are given by using a confining stress range from zero to $0.25\sigma_{ci}$, where σ_{ci} is the uniaxial compressive strength of the intact rock elements. For this stress range, the uniaxial compressive strength of the rock mass σ_{cm}, the cohesive strength c and the friction angle ϕ are given in Figures 1.3 and 1.4.

For slopes and shallow excavations the user is given the choice of the stress range for this curve fitting process. This is discussed in full in the Appendix.

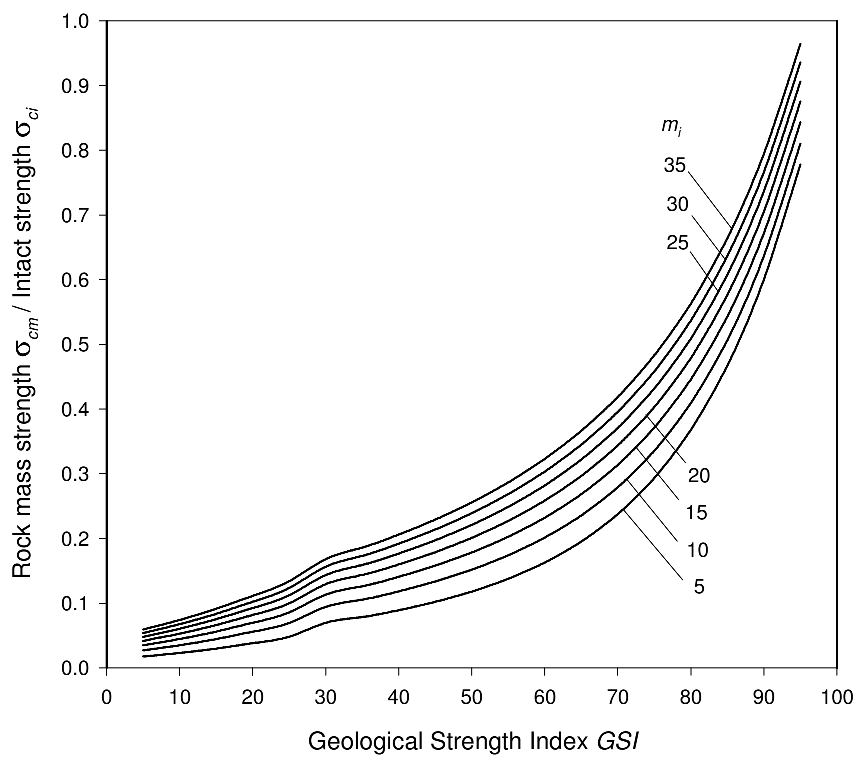
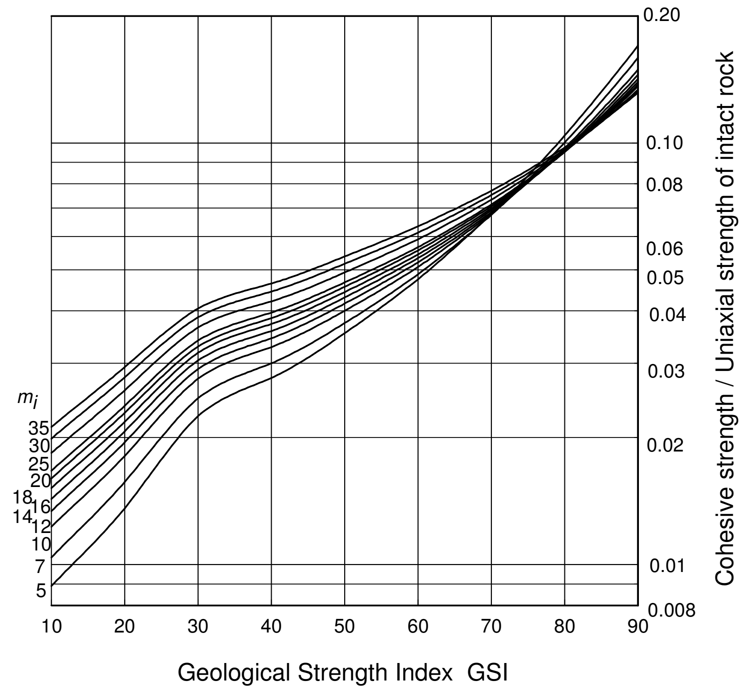
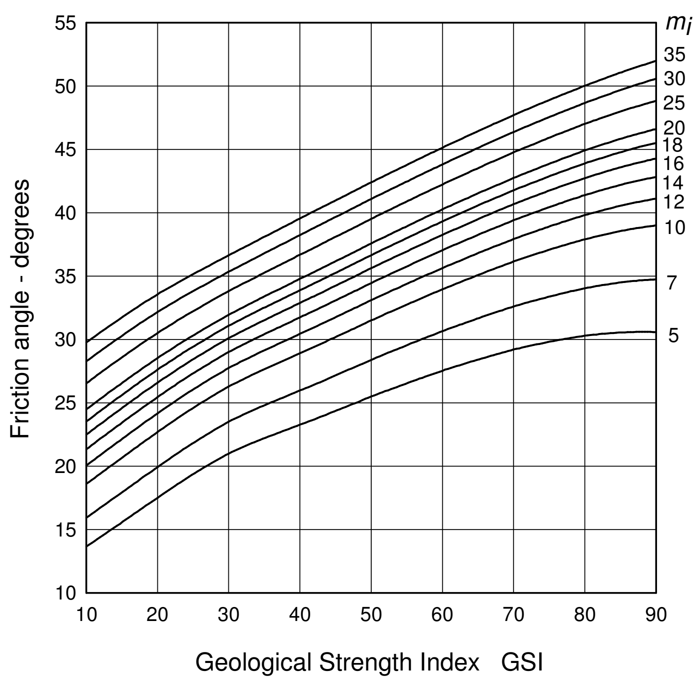


Figure 1.3: Ratio of uniaxial compressive strength of rock mass to intact rock versus Geological Strength Index GSI for depths of more than 30 m.



a. Plot of ratio of cohesive strength c' to uniaxial compressive strength σ_{ci} for depths of more than 30 m.



b. Plot of friction angle ϕ'

Figure 1.4: Cohesive strengths and friction angles for different GSI and m_i values for depths of more than 30 m.

1.6 DEFORMATION MODULUS

Serafim and Pereira (1983) proposed a relationship between the in situ modulus of deformation and Bieniawski's RMR classification. This relationship is based upon back analysis of dam foundation deformations and it has been found to work well for better quality rocks. However, for many of the poor quality rocks it appears to predict deformation modulus values that are too high. Based upon practical observations and back analysis of excavation behaviour in poor quality rock masses, the following modification to Serafim and Pereira's equation is proposed for $\sigma_{ci} < 100$:

$$E_m = \sqrt{\frac{\sigma_{ci}}{100}} 10^{\left(\frac{GSI-10}{40}\right)} \tag{1.12}$$

Note that GSI has been substituted for RMR in this equation and that the modulus E_m is reduced progressively as the value of σ_{ci} falls below 100. This reduction is based upon the reasoning that the deformation of better quality rock masses is controlled by the discontinuities while, for poorer quality rock masses, the deformation of the intact rock pieces contributes to the overall deformation process.

Based upon measured deformations, equation 1.12 appears to work reasonably well in those cases where it has been applied. However, as more field evidence is gathered it may be necessary to modify this relationship.

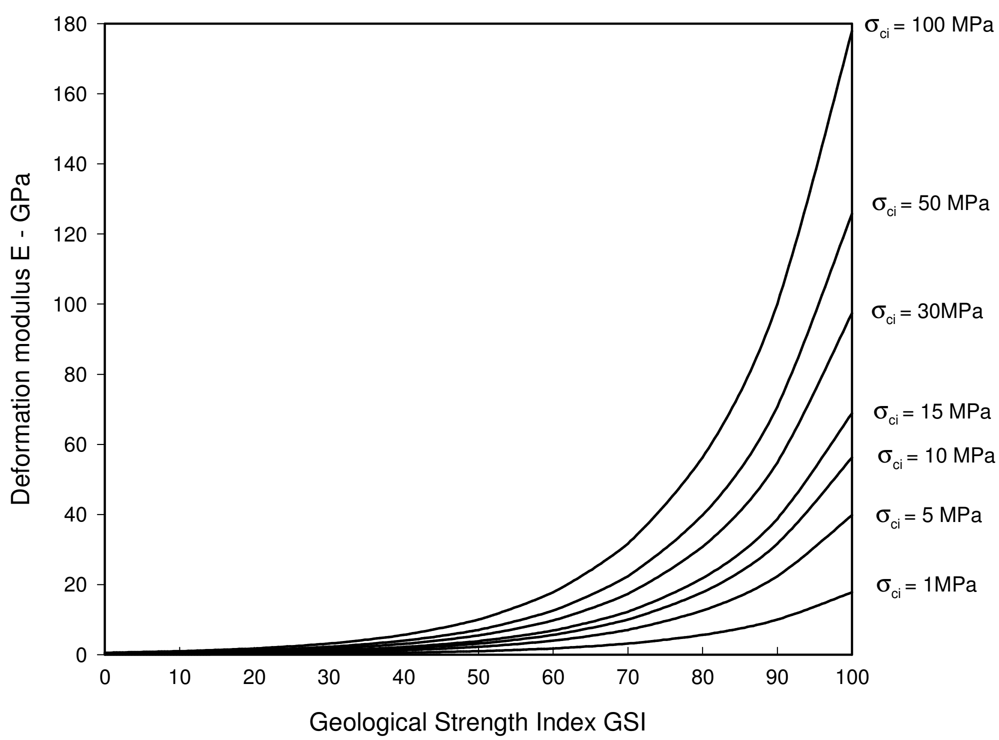


Figure 1.5: Deformation modulus versus Geological Strength Index GSI.

1.7 STRESS RELAXATION

When the rock mass adjacent to a tunnel wall or a slope is excavated, a relaxation of the confining stresses occurs and the remaining material is allowed to expand in volume or to dilate. This has a profound influence on the strength of the rock mass since, in jointed rocks, this strength is strongly dependent upon the interlocking between the intact rock particles that make up the rock mass.

As far as the authors are aware, there is very little research evidence relating the amount of dilation to the strength of a rock mass. One set of observations that gives an indication of the loss of strength associated with dilation is derived from the support required to stabilise tunnels. Sakurai (1983) suggested that tunnels in which the 'strain', defined as the ratio of tunnel closure to tunnel diameter, exceeds 1% are likely to suffer significant instability unless adequately supported. This suggestion was confirmed in observations by Chern et al (1998) who recorded the behaviour of a number of tunnels excavated in Taiwan. They found that all of those tunnels that exhibited strains of greater than 1 to 2% required significant support. Tunnels exhibiting strains as high as 10% were successfully stabilised but the amount of effort required to achieve this stability increased in proportion to the amount of strain.

While it is not possible to derive a direct relationship between rock mass strength and dilation from these observations, it is possible to conclude that the strength loss is significant. An unconfined surface that has deformed more than 1 or 2% (based upon Sakurai's definition of strain) has probably reached residual strength in which all of the effective 'cohesive' strength of the rock mass has been lost. While there are no similar observations for rock slopes, it is reasonable to assume that a similar loss of strength occurs as a result of dilation. Hence, a 100 m high slope which has suffered a total crest displacement of more than 1 m (i.e. more than 1% strain) may start to exhibit significant signs of instability as a result of loss of strength of the rock mass.

1.8 BLAST DAMAGE

Blast damage results in a loss of rock mass strength due to the creation of new fractures and the wedging open of existing fractures by the penetration of explosive gasses. In the case of very large open pit mine blasts, this damage can extend as much as 100 m behind the final row of blast holes.

In contrast to the strength loss due to stress relaxation or dilation, discussed in the previous section, it is possible to arrive at an approximate quantification of the strength loss due to blast damage. This is because the blast is designed to achieve a specific purpose which is generally to produce a fractured rock mass that can be excavated by means of a given piece of equipment.

Figure 1.6 presents a plot of 23 case histories of excavation by digging, ripping and blasting published by Abdullatif and Cruden (1983). These case histories are summarised in Table 1.5. The values of GSI are estimated from the data contained in the paper by Abdullatif and Cruden while the rock mass strength values were calculated by means of the spreadsheet given in the appendix, assuming an average slope height of 15 m.

These examples shows that rock masses can be dug, obviously with increasing difficulty, up to GSI values of about 40 and rock mass strength values of about 1 MPa. Ripping can be used up to GSI values of about 60 and rock mass strength values of about 10 MPa, with two exceptions where heavy equipment was used to rip strong rock masses. Blasting was used for GSI values of more than 60 and rock mass strengths of more than about 15 MPa.

Consider the case of an open pit slope excavated in granodiorite. The uniaxial compressive strength of the intact rock is σ_{ci} = 60 MPa and the Geological Strength Index is GSI = 55. For granodiorite, Table 2 gives the value of m_i = 30. Substitution of these values into the spreadsheet given in the appendix, for a single 18 m high bench, gives a rock mass strength σ_{cm}= 5.7 MPa. In order to create conditions for easy digging, the blast is designed to reduce the GSI value to below 40 and/or the rock mass strength to less than 1 MPa. In this case the controlling parameter is the rock mass strength and the spreadsheet given in the appendix shows that the GSI value has to be reduced to about 22 on order to achieve this rock mass strength.

In another example of a 15 m high slope in weak sandstone, the compressive strength of the intact rock is σ_{ci} = 10 MPa, m_i = 17 and GSI = 60. These values give a rock mass strength σ_{cm}= 1.4 MPa and this is reduced to 0.7 by reducing the GSI to 40. Hence, in this case, both the conditions for efficient digging in this soft rock are satisfied by designing the blast to give a GSI value of 40.

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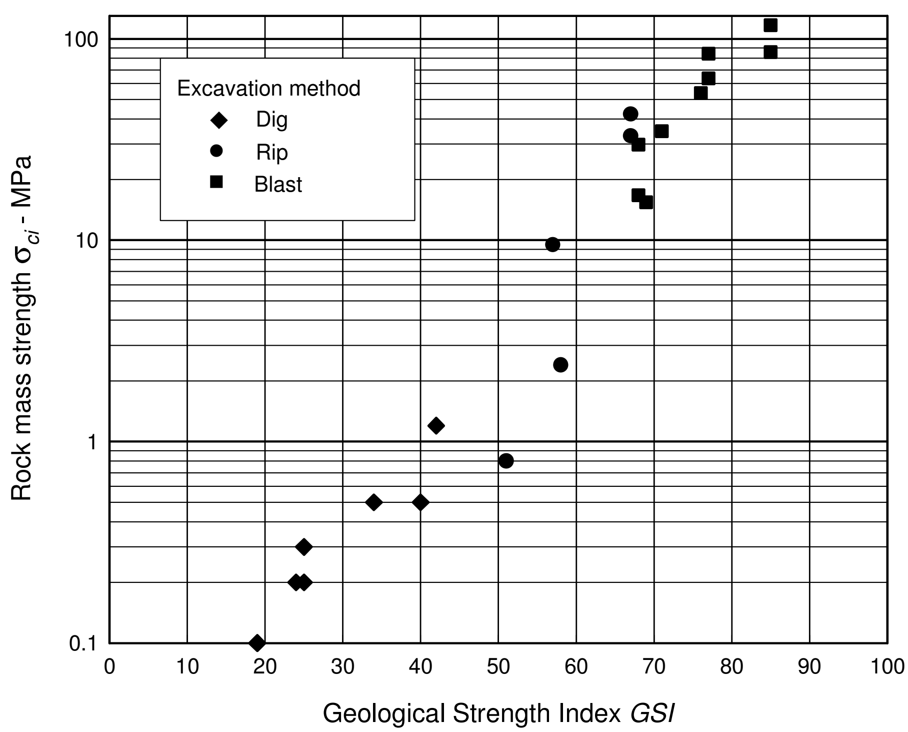

Figure 1.6: Plot of rock mass strength versus GSI for different excavation methods, after Abdullatif and Cruden (1983).

Table 1.5: Summary of methods used to excavate rock masses with a range of uniaxial compressive strength values, based on data published by Abdullatif and Cruden (1983).

GSI	Rock mass strength σ_{cm} - MPa	Excavation method
85	86	Blasting
85	117	Blasting
77	64	Blasting
77	135	Blasting
77	84	Blasting
76	54	Blasting
71	35	Blasting
69	15	Blasting
68	17	Blasting
68	30	Blasting
67	42	Ripping by D9L bulldozer
67	33	Ripping by D9L bulldozer
58	2.4	Ripping by track loader
57	9.5	Ripping by 977L track loader
51	0.8	Ripping by track loader
42	1.2	Digging by 977L track loader
40	0.5	Digging by wheel loader
34	0.5	Digging by hydraulic face shovel
25	0.3	Digging by 977L track loader
24	0.2	Digging by wheel loader
25	0.2	Digging by hydraulic backhoe
19	0.1	Digging by D9 bulldozer
19	0.1	Digging by 977L track loader

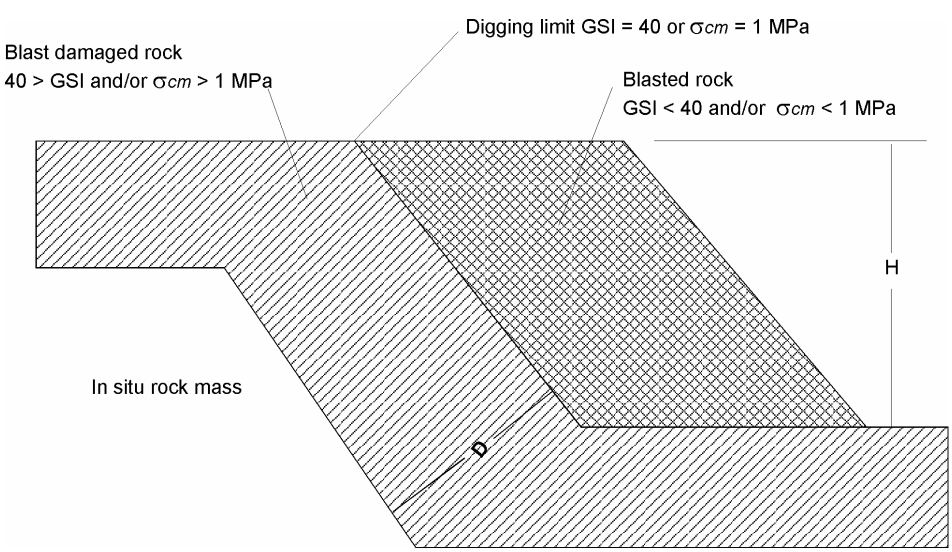

Figure 1.7: Diagrammatic representation of the transition between the in situ rock mass and blasted rock that is suitable for digging.

Figure 1.7 summarises the conditions for a muckpile that can be dug efficiently and the blast damaged rock mass that lies between the digging limit and the in situ rock mass. The properties of this blast damaged rock mass will control the stability of the slope that remains after digging of the muckpile has been completed.

The thickness D of the blast damaged zone will depend upon the design of the blast. Based upon experience, the authors suggest that the following approximate relationships can be used as a starting point in judging the extent of the blast damaged zone resulting from open pit mine production blasting:

- Large production blast, confined and with little or no control — D = 2 to 2.5 H
- Production blast with no control but blasting to a free face — D = 1 to 1.5 H
- Production blast, confined but with some control, e.g. one or more buffer rows — D = 1 to 1.2 H
- Production blast with some control, e.g. one or more buffer rows, and blasting to a free face — D = 0.5 to 1 H
- Carefully controlled production blast with a free face — D = 0.3 to 0.5 H

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1.10 APPENDIX – DETERMINATION OF MOHR COULOMB CONSTANTS

The steps required to determine the parameters A, B, c' and ϕ' are given below. A spreadsheet for carrying out this analysis, with a listing of all the cell formulae, is given in Figure 1.8.

The relationship between the normal and shear stresses can be expressed in terms of the corresponding principal effective stresses as suggested by Balmer (1952):

$$\sigma_n' = \sigma_3' + \frac{\sigma_1' - \sigma_3'}{\partial\sigma_1'/\partial\sigma_3' + 1} \tag{1.13}$$

$$\tau = (\sigma_1' - \sigma_3')\sqrt{\partial\sigma_1'/\partial\sigma_3'} \tag{1.14}$$

For the $GSI > 25$, when $a = 0.5$:

$$\frac{\partial\sigma_1'}{\partial\sigma_3'} = 1 + \frac{m_b\sigma_{ci}}{2(\sigma_1' - \sigma_3')} \tag{1.15}$$

For $GSI < 25$, when $s = 0$:

$$\frac{\partial\sigma_1'}{\partial\sigma_3'} = 1 + a m_b^a \left(\frac{\sigma_3'}{\sigma_{ci}}\right)^{a-1} \tag{1.16}$$

The tensile strength of the rock mass is calculated from:

$$\sigma_{tm} = \frac{\sigma_{ci}}{2}\left(m_b - \sqrt{m_b^2 + 4s}\right) \tag{1.17}$$

The equivalent Mohr envelope, defined by equation 1.2, may be written in the form

$$Y = \log A + BX \tag{1.18}$$

where

$$Y = \log\left(\frac{\tau}{\sigma_{ci}}\right),\ X = \log\left(\frac{\sigma_n' - \sigma_{tm}}{\sigma_{ci}}\right) \tag{1.19}$$

Using the value of σ_{tm} calculated from equation 1.17 and a range of values of τ and σ_n' calculated from equations 1.13 and 1.14 the values of A and B are determined by linear regression where :

$$B = \frac{\sum XY - (\sum X \sum Y)/T}{\sum X^2 - (\sum X)^2/T} \tag{1.20}$$

$$A = 10\text{^}\left(\sum Y/T - B(\sum X/T)\right) \tag{1.21}$$

and T is the total number of data pairs included in the regression analysis.

The most critical step in this process is the selection of the range of σ_3' values. As far as the authors are aware, there are no theoretically correct methods for choosing this range and a trial and error method, based upon practical compromise, has been used for selecting the range included in the spreadsheet presented in Figure 1.9.

For a Mohr envelope defined by equation 1.2, the friction angle ϕ_i' for a specified normal stress σ_{ni}' is given by:

$$\phi_i' = \arctan\left(AB\left(\frac{\sigma_{ni}' - \sigma_{tm}}{\sigma_{ci}}\right)^{B-1}\right) \tag{1.22}$$

The corresponding cohesive strength c_i' is given by:

$$c_i' = \tau - \sigma_{ni}' \tan \phi_i' \tag{1.23}$$

and the corresponding uniaxial compressive strength of the rock mass is :

$$\sigma_{cmi} = \frac{2c_i' \cos\phi_i'}{1 - \sin\phi_i'} \tag{1.24}$$

The values of c' and ϕ' obtained from this analysis are very sensitive to the range of values of the minor principal stress σ_3' used to generate the simulated full-scale triaxial test results. On the basis of trial and error, it has been found that the most consistent results for deep excavations (depth > 30 m below surface) are obtained when 8 equally spaced values of σ_3' are used in the range $0 < \sigma_3' < 0.25\sigma_{ci}$. For shallow excavations and slopes, the user should input the depth below surface of the anticipated failure surface and the unit weight of the rock mass. For typical slopes, the depth of the failure surface can be assumed to be equal to the slope height.

Figure 1.8: Spreadsheet for calculation of Hoek-Brown and equivalent Mohr-Coulomb parameters for shallow excavations and slopes.

Input:

sigci =	30	MPa	mi =	15		GSI =	55	
Depth of failure surface or tunnel below slope =				25	m	Unit wt. =	0.027	MN/n3

Output:

stress =	0.68	MPa	mb =	3.01		s =	0.0067	
a =	0.5		sigtm =	-0.0672	MPa	A =	0.7086	
B =	0.7263		k =	9.19		phi =	53.48	degrees
coh =	0.494	MPa	sigcm =	3.00	MPa	E =	7304.0	MPa

Calculation:

									Sums
sig3	1E-10	0.10	0.19	0.29	0.39	0.48	0.58	0.68	2.70
sig1	2.46	3.94	5.04	5.96	6.78	7.52	8.21	8.86	48.77
ds1ds3	19.32	12.74	10.31	8.95	8.06	7.41	6.91	6.51	80.20
sign	0.12	0.38	0.62	0.86	1.09	1.32	1.54	1.76	7.70
tau	0.53	1.00	1.38	1.70	2.00	2.28	2.54	2.78	14.21
x	-2.20	-1.83	-1.64	-1.51	-1.41	-1.34	-1.27	-1.21	-12.42
y	-1.75	-1.48	-1.34	-1.25	-1.18	-1.12	-1.07	-1.03	-10.21
xy	3.85	2.71	2.19	1.88	1.66	1.49	1.36	1.25	16.41
xsq	4.85	3.35	2.69	2.28	2.00	1.78	1.61	1.47	20.04
sig3sig1	0.00	0.38	0.97	1.72	2.61	3.63	4.75	5.98	20
sig3sq	0.00	0.01	0.04	0.08	0.15	0.23	0.33	0.46	1
taucalc	0.53	1.00	1.37	1.70	2.00	2.28	2.54	2.79	
sig1sig3fit	3.00	3.88	4.77	5.65	6.54	7.42	8.31	9.20	
signtaufit	0.66	1.00	1.33	1.65	1.97	2.28	2.58	2.88	

Cell formulae:

```
   stress = if(depth>30, sigci*0.25,depth*unitwt*0.25)
       mb = mi*EXP((GSI-100)/28)
        s = IF(GSI>25,EXP((GSI-100)/9),0)
        a = IF(GSI>25,0.5,0.65-GSI/200)
    sigtm = 0.5*sigci*(mb-SQRT(mb^2+4*s))
     sig3 = Start at 1E-10 (to avoid zero errors) and increment in 7 steps of  stress/28 to stress/4
     sig1 = sig3+sigci*(((mb*sig3)/sigci)+s)^a
   ds1ds3 = IF(GSI>25,(1+(mb*sigci)/(2*(sig1-sig3))),1+(a*mb^a)*(sig3/sigci)^(a-1))
     sign = sig3+(sig1-sig3)/(1+ds1ds3)
      tau = (sign-sig3)*SQRT(ds1ds3)
        x = LOG((sign-sigtm)/sigci)
        y = LOG(tau/sigci)
       xy = x*y                x sq = x^2
       A =    acalc =    10^(sumy/8 - bcalc*sumx/8)
       B =    bcalc =    (sumxy - (sumx*sumy)/8)/(sumxsq - (sumx^2)/8)
        k = (sumsig3sig1 - (sumsig3*sumsig1)/8)/(sumsig3sq-(sumsig3^2)/8)
      phi = ASIN((k-1)/(k+1))*180/PI()
      coh = sigcm/(2*SQRT(k))
    sigcm = sumsig1/8 - k*sumsig3/8
        E = IF(sigci>100,1000*10^((GSI-10)/40),SQRT(sigci/100)*1000*10^((GSI-10)/40))
     phit = (ATAN(acalc*bcalc*((signt-sigtm)/sigci)^(bcalc-1)))*180/PI()
     coht = acalc*sigci*((signt-sigtm)/sigci)^bcalc-signt*TAN(phit*PI()/180)
 sig3sig1= sig3*sig1        sig3sq = sig3^2
  taucalc = acalc*sigci*((sign-sigtm)/sigci)^bcalc
   s3sifit = sigcm+k*sig3
sntaufit =   coh+sign*TAN(phi*PI()/180)
```