Brittle Rock Fracture Propagation
In Rock Under Compression

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Abstract

The results of studies of the initiation and propagation of fracture from a single Griffith crack in a biaxial compressive stress field are reported. It is concluded that Griffith's theory of brittle fracture offers a reliable prediction of the fracture initiation stress but that the resulting fracture propagation from a single crack cannot account for the macroscopic fracture of a specimen. Some preliminary results of studies on crack arrays and on the effects of crack closure in compression are presented. The applicability of these results to the prediction of rock fracture in predominantly compressive stress fields is discussed.

Introduction

One of the most serious problems encountered in deep-level gold mining in South Africa is the sudden and violent fracture of rock, known in the mining industry as a rockburst. Seismic location of the foci of these rock-bursts (Cook, 1963) has established that they occur most frequently in the zones of high compressive stress which surround the working faces of the mining excavations. Since the mining industry is constantly striving to minimize the hazards created by these rockbursts, an understanding of the mechanism of rock fracture under compressive stress conditions is of vital interest.

Previous research (Brace, 1964; Hoek, 1964) has shown that Griffith's brittle fracture theory (Griffith, 1924) modified to account for the effects of crack closure in compression (McClintock and Walsh, 1962), is a useful basis for the study of the fracture of hard rock. Brace (Brace, 1964), in discussing the nature of the pre-existing cracks in rock, suggests that the grain boundaries act as or contain micro-cracks while joints and faults can be regarded as macro-cracks.

An analysis of the stress distribution around a crack (Erdogan and Sih, 1963) indicates the points of fracture initiation as well as the initial direction of crack propagation. As a result of the change in stress distribution associated with fracture propagation it is, however, impossible to predict the final path of the propagating crack. Consequently, a serious limitation of the Griffith theory lies in the fact that it can only be used to predict fracture initiation. In its usual form, it yields no information on the rate or direction of fracture propagation.

In studying the fracture of brittle materials subjected to tension, fracture is normally expected in a direction perpendicular to the applied tension, in other words, in the plane of the critically oriented crack. In the case of a brittle material subjected to compressive stress, one might therefore expect that fracture propagation will also follow the direction of the most critically oriented crack, i.e. the one which is inclined at 20-30° to the major principal field stress direction. It will be shown in this paper that this anticipated result is incorrect and that there is no simple relationship between the critical orientation of the original "Griffith crack" and the orientation of the macroscopic fracture surface of a specimen.

Theoretical conditions for fracture initiation

Griffith's original postulate on fracture initiation was based on energy considerations and his equations contained a surface energy term (Griffith, 1924). Because of the difficulty of evaluating experimentally the surface energy of a material, an alternative approach, which considers the stress concentration at the crack tip, has been adopted by most workers in rock mechanics.

The current interpretation (Orowan, 1949) of Griffith's theory is that fracture initiates when tensile stress induced at or near the tip of an inherent crack exceeds the molecular cohesive strength of the material. Since the molecular cohesive strength is difficult to determine by direct measurement, the fracture criterion is expressed in terms of the uniaxial tensile strength of the material (Hoek, 1964).

In order that the reader may readily follow the equations which are used in this paper, a brief derivation of these equations, based upon the work by Griffith and McClintock and Walsh, follows.

It is assumed that the crack from which the fracture of a brittle rock originates can be regarded as a flat elliptical opening in a two-dimensional body which is subjected to a stress system[1] as illustrated in Figure 1.

The stress field around an elliptical opening is related to the elliptical coordinates ε and η which are defined by the following equations of transformation of a rectangular system of coordinates x and z:

$$x = c \sinh \varepsilon \sin \eta$$
$$y = c \cosh \varepsilon \cos \eta$$

[1] Because of the predominance of compressive stress in rock mechanics problems, comprehensive stress is taken as positive.

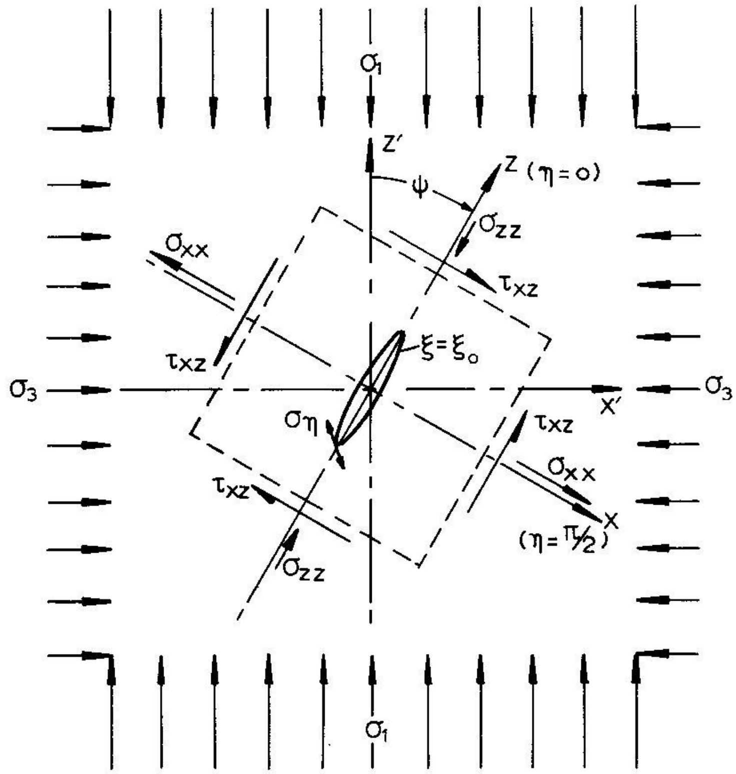

Figure 1. Stresses acting upon a crack which is inclined at an angle ψ to the direction of the major principal stress σ_1.

In Figure 1, the system of rectangular coordinates x, z is parallel to the axes of the elliptical opening: it is inclined at an angle ψ with respect to the system of rectangular coordinates x', z' which is parallel to the directions of the principal stresses σ_1 and σ_3. Of these, σ_1 is algebraically largest and σ_3 algebraically smallest of the three principal stresses[2].

The normal stress σ_{xx} and the shear stress τ_{xz} are related to the principal stresses σ_1 and σ_3 by the following equations:

$$2\,\sigma_{xx} = (\sigma_1 + \sigma_3)\ - (\sigma_1 - \sigma_3)\ \cos 2\psi \qquad (1)$$

$$2\tau_{xz} = (\sigma_1 - \sigma_3)\sin 2\psi \qquad (2)$$

The stress σ_{zz}, parallel to the major axis of the crack, has a negligible influence upon

[2] In this analysis, the intermediate principal stress σ_3 is assumed to have a negligible influence upon fracture.

the stresses induced near the crack tip and need not be considered in the following analysis.

The stresses σ_η and τ_η which act on the surface of the crack as shown in Figure 1, exist only when closure of the crack has occurred and their influence was considered by McClintock and Walsh (1963) in deriving their modification to Griffith theory.

The tangential stress σ_η around the boundary of an open elliptical crack, due to the stresses σ_{xx} and τ_{xz} can be calculated from the results presented by Inglis (1913) and is found to be:

$$\sigma_\eta = \frac{\sigma_{xx}\left(\sinh 2\varepsilon_o + e^{2\varepsilon_o}\cos 2\eta - 1\right) + 2\tau_{xz}e^{2\varepsilon_o}\sin 2\eta}{\cosh 2\varepsilon_o - \cos 2\eta} \tag{3}$$

where ε_o is the value of the elliptical coordinate ε on the crack boundary. The maximum boundary stresses, both tensile and compressive, occur near the ends of the crack (i.e., when the value of η is small). Since the value of ε_o is also small for a flat ellipse, (3) may be simplified by series expansion in which terms of the second order and higher which appear in the numerator are neglected. This simplification results in the following equation, valid only for the stresses near the crack tip:

$$\sigma_\eta = \frac{2\left(\sigma_{xx}\varepsilon_o + \tau_{xz}\eta\right)}{\cosh 2\varepsilon_o - \cos 2\eta} \tag{4}$$

Differentiation of (4) with respect to η and equating $\partial\sigma_\eta / \partial\eta$ to zero results in a quadratic equation in η from which the positions on the crack boundary at which the maximum and minimum stresses occur can be determined. Substituting these values of η into (4) gives the maximum and minimum stresses on the boundary of the crack as:

$$\sigma_{\mathrm{N}}\varepsilon_o = \sigma_{xx} \pm \left(\sigma_{xx}^2 + \tau_{xz}^2\right)^{1/2} \tag{5}$$

where σ_N is the maximum or minimum value of the tangential stress σ_η of the ellipse boundary.

Expressing equation (5) in terms of the principal stresses σ_1 and σ_3 from equations (1) and (2) gives

$$\sigma_{\mathrm{N}}\varepsilon_o = \frac{1}{2}\left[(\sigma_1 + \sigma_3) - (\sigma_1 - \sigma_3)\cos 2\psi\right] \pm \left[\frac{1}{2}\left\{\left(\sigma_1^2 + \sigma_3^2\right) - \left(\sigma_1^2 - \sigma_3^2\right)\cos 2\psi\right\}\right]^{\frac{1}{2}} \tag{6}$$

The critical crack orientation ψ_c at which the maximum and minimum stresses are induced near the crack tip is found by differentiating equation (6) with respect to ψ and letting $\partial\sigma_N / \partial\psi = 0$. This gives:

$$\cos 2\psi_c = \frac{\sigma_1 - \sigma_3}{2(\sigma_1 + \sigma_3)} \tag{7}$$

Note that this equation is only meaningful for values of $\sigma_3 / \sigma_1 \geq -0.33$ and the critical crack orientation for smaller values of σ_3 / σ_1 must be determined from other considerations.

The maximum and minimum stresses at the boundary of a crack oriented at the critical angle ψ_c under conditions where $\sigma_3 / \sigma_1 \geq -0.33$ are found by substituting ψ_c from equation (7) for ψ in equation (6). If it is accepted that fracture occurs as a result of tensile stress at or near the crack tip, only the minimum (negative) stress given by this substitution need be considered. Hence

$$\sigma_o \cdot \varepsilon_o = \frac{-(\sigma_1 - \sigma_3)^2}{4(\sigma_1 + \sigma_3)} \tag{8}$$

where σ_o denotes the minimum (algebraically smallest) value of the tangential stress on the boundary of the ellipse.

If it is postulated that the fracture of a brittle material initiates when the maximum tensile stress at the crack tip is equal to the molecular cohesive strength of the material (Orowan 1949), then equation (8) expresses a fracture criterion for a brittle material under conditions where $\sigma_3 / \sigma_1 \geq -0.33$, if σ_o is taken as the molecular cohesive strength of the material.

The molecular strength, σ_o, and the crack geometry, ε_o, cannot be determined by direct physical measurements. However, their product can be expressed in terms of the uniaxial tensile strength, σ_t, determined on a laboratory specimen. Since, for uniaxial tension ($\sigma_3 < 0$, $\sigma_1 = 0$), $\sigma_3 / \sigma_1 = -\infty$ equations (7) and (8), which are only valid for $\sigma_3 / \sigma_1 \geq -0.33$, cannot be used to find a relationship between σ_o, ε_o and σ_t and equation (6) must be resorted to for finding this relationship.

If the plane body containing the crack, illustrated in Figure 1, is subjected to a uniaxial tensile stress (i.e. $\sigma_3 < 0$, $\sigma_1 = 0$), the maximum stress at the crack tip (σ_N) is dependent upon the minor principal stress σ_3 only. Hence, equation (6) simplifies to

$$\sigma_N \cdot \varepsilon_o = \sigma_3 \left[\frac{1}{2}(1+\cos 2\psi) \pm \left[\frac{1}{2}(1+\cos 2\psi)\right]^{\frac{1}{2}} \right] \tag{9}$$

The maximum tensile stress at the crack tip occurs when the bracketed term on the right hand side of equation (9) is a maximum. This occurs when $\cos 2\psi = 1$ or when $\psi = 0$, giving

$$\sigma_o \cdot \varepsilon_o = 2\sigma_3 \qquad (\sigma_1 = 0, \quad \psi = 0) \tag{10}$$

If the minor principal stress σ_3 is tensile (negative), equation (10) defines the fracture criterion for uniaxial tensile stress conditions in terms of the molecular cohesive strength of the material (σ_o) and the crack geometry (ε_o). Denoting the uniaxial tensile strength of the material, measured on a laboratory specimen, as σ_t [(3)], equation (10) may, with $\sigma_3 = \sigma_t$, be re-written as:

$$\sigma_o \cdot \varepsilon_o = 2\sigma_1 \tag{11}$$

Equation (8), which is valid for $\sigma_3 / \sigma_1 \geq -0.33$, is of limited practical use because the term $\sigma_o . \ \varepsilon_o$ cannot be evaluated in the laboratory. If, however, this term is expressed in terms of the uniaxial tensile strength of the material (σ_t), according to equation (11), the fracture criterion becomes

$$\frac{(\sigma_1 - \sigma_3)^2}{(\sigma_1 + \sigma_3)} = -\ 8\sigma_1 \tag{12}$$

The authors have found that the most useful interpretation of this equation is in expressing the major principal stress (σ_1), at fracture, in terms of the principal stress ratio σ_3 / σ_1 and the uniaxial tensile strength (σ_1) or the uniaxial compressive strength (σ_c).

Thus:

$$\sigma_1 = \sigma_3 - 4\sigma_t \left[1 + \left(1 - {\sigma_3}/{\sigma_1} \right)^{\frac{1}{2}} \right] \tag{13}$$

or

[3] Note that the uniaxial tensile strength is negative by definition. Hence, in substituting a numerical value for σ_t, the negative sign must be shown; e.g. σ_t = - 100 lb/sq. in.

$$\sigma_1 = \sigma_3 + \sigma_c \left(2\, {}^{\sigma_3}\!/_{\sigma_c} + \frac{1}{4} \right)^{\frac{1}{2}} + \frac{1}{2}\sigma_c \tag{14}$$

Equation (14) is obtained by putting $\sigma_3 = 0$ and $\sigma_1 = \sigma_c$ in equation (12) and substituting thus obtained result ($\sigma_c = -8\,\sigma_t$) in equation (13).

Equations (7), (13) and (14) are valid only when the principal stress ratio $\sigma_3 / \sigma_1 \geq -0.33$. For, $\sigma_3 / \sigma_1 < -0.33$ fracture occurs when the minor principal stress equals the uniaxial tensile strength of the material, i.e. when $\sigma_3 = \sigma_t$.

The Griffith fracture theory can also be represented by a parabolic Mohr envelope defined by the following equation:

$$\tau^2 = 4\sigma_t \left(\sigma_t - \sigma\right) \tag{15}$$

where τ is shear stress acting along the fracture surface; σ is normal stress perpendicular to the fracture surface.

Studies of rock fracture suggest that the inherent cracks from which fracture propagates are contained within grain boundaries (Brace, 1961) and can be simulated by very long elliptical openings. Under compressive stress conditions, closure of these cracks can occur before the tensile stress at the crack tip is high enough to initiate fracture. When crack closure has occurred, the shear resistance resulting from the contact pressure between the crack faces has to be overcome before propagation of the crack can occur.

McClintock and Walsh (1962) modified Griffith's original theory to account for the effects of crack closure in compression. If it is assumed that the inherent cracks are initially closed, the relationship between the principal stresses required to initiate fracture is

$$\sigma_1 \quad = \quad \sigma_3 \frac{\left(1+\mu^2\right)^{\frac{1}{2}} + \mu}{\left(1+\mu^2\right)^{\frac{1}{2}} - \mu} + \sigma_C \tag{16}$$

where μ is the coefficient of friction between the crack faces and σ_c is the uniaxial compressive strength of the material.

The critical orientation of a closed crack is given by

$$Tan\; 2\,\sigma_c = {}^{1}\!/_{\mu} \tag{17}$$

Equations (16) and (17) are valid when the normal stress σ_n acting across the crack is compressive, i.e. when

$$\sigma_n \quad = \quad \frac{1}{2}\left[(\sigma_1+\sigma_3)-(\sigma_1-\sigma_3)\cos 2\psi\right] > 0 \tag{18}$$

When σ_n is tensile, the original Griffith theory, defined by equations (7) and (12) is applicable.

The modified Griffith theory can be represented by a straight-line Mohr envelope having the following equation:

$$\tau = \mu\sigma - 2\sigma_t \tag{19}$$

In order to establish whether the original and modified Griffith theories are applicable to the prediction of rock fracture behaviour, a survey of published rock fracture data was undertaken. To facilitate comparison of the results they were reduced to a dimensionless form by dividing each strength value of a particular rock by its uniaxial compressive strength. These dimensionless strength values are plotted in Figure 2 together with the original and modified Griffith fracture loci, derived from equations (14) and (16).

It is evident from Figure 2 that, in spite of the wide variety of materials included (listed in Table 1), there is a remarkable agreement between the experimental results and the fracture initiation behaviour predicted by the modified Griffith theory. Detailed examination of the results reveals that the coefficient of internal friction, μ, given by the slope of the modified Griffith fracture locus, is closely related to the rock type tested. The igneous and metamorphosed sedimentary rocks (granites, dolerite, quartzites) are characterized by coefficients of friction of greater than 1.0; the sedimentary rocks (sandstones, limestones and shales) have coefficients of friction between 1.0 and 0.5.

A coefficient of internal friction of greater than unity implies that the shear resistance is greater than the normal stress acting across the crack. This apparent anomaly is probably due to interlocking of surface irregularities or, carrying the thought to the extreme, due to the fracture initiating from an elastic discontinuity rather than an actual crack.

It must be emphasized that both the original and modified fracture theories predict fracture initiation from a single crack and that, strictly, they cannot be applied to the fracture of a specimen as a whole. It has already been suggested by Brace and Bombolakis (1963) that a fracture propagation from a single crack follows a more complex path than is generally assumed and that it is the presence of favourable crack arrays which coalesce to form the macroscopic fracture surface, that make the Griffith theory applicable to predicting fracture of rock and rock specimens.

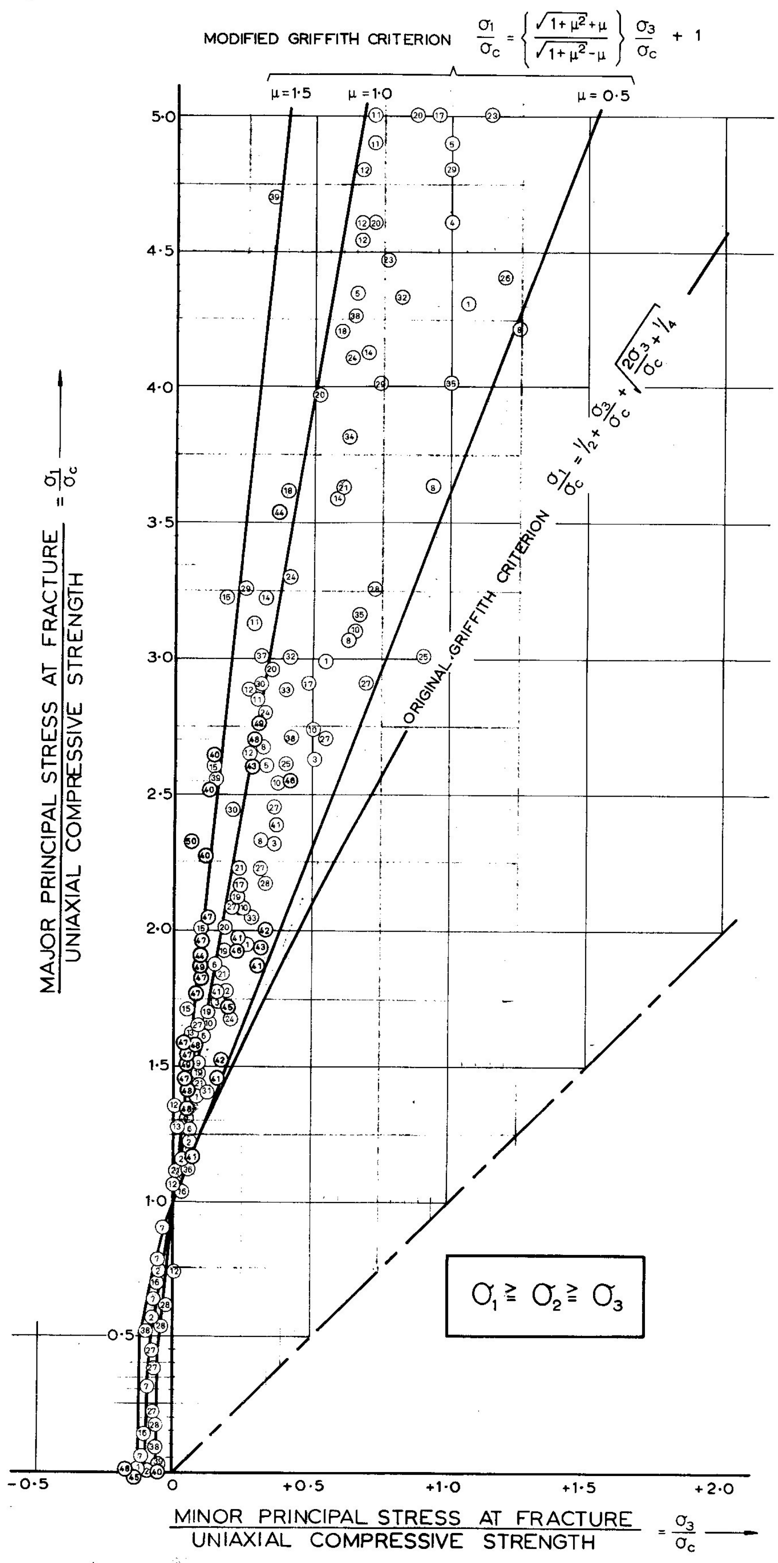

Figure 2. Triaxial fracture data for rock materials.

Table 1. Summary of triaxial test results on rock and concrete.

Graph Point	Material	σ_c lb/sq. in	Tested by
1	Marble	13 700	Ros. and Eichinger
2	Marble	18 000	Ros and Eichinger
3	Marble	20 000	Von Karman
4	Carthage Marble	10 000	Bredthauer
5	Carthage Marble	7 500	Bredthauer
6	Wombeyan Marble	10 000	Jaeger
1	Concrete	2 380	McHenry and Kami
8	Concrete	3 200	Akroyd
9	Concrete	6 000	Jaeger
10	Concrete	5 700	Fumagalli
11	Concrete (28 day)	3 510	Balmer
12	Concrete (90 day)	4 000	Balmer
13	Granite Gneiss	25 500	Jaeger
14	Barre Granite	24 200	Robertson
15	Granite (slightly alt)	10 000	Wreuker
16	Westerly Granite	33 800	Brace
17	Iwaki Sandstone	1 780	Horibe & Kobayashi
18	Rush Springs Sandstone	26 000	Bredthauer
19	Pennant Sandstone	22 500	Price
20	Darley Dale Sandstone	5 780	Price
21	Sandstone	9 000	Jaeger
22	Oil Creek Sandstone	**	Handin
23	Dolomite	24 000	Bredthauer
24	White Dolomite	12 000	Bredthauer
25	Clear Fork Dolomite	*	Handin
26	Blair Dolomite	**	Handin
27	Blair Dolomite	75 000	Brace
28	Webtuck Dolomite	22 000	Brace
29	Chico Limestone	10 000	Bredthauer
30	Virginia Limestone	48 000	Bredthauer
31	Limestone	20 000	Jaeger
32	Anhydrite	6 000	Bredthauer
33	Knippa Basalt	38 000	Bredthauer
34	Sandy Shale	8 000	Bredthauer
35	Shale	15 000	Bredthauer
36	Porphry	40 000	Jaeger
3-7	Sioux Quartzite	**	Handin
38	Frederick Diabase	71 000	Brace
39	Cheshire Quartzite	68 000	Brace
40	Chert dyke material	83 000	Hoek
41	Quartzitic Shale (Dry)	30 900	Colback and Wiid
42	Quartzitic Shale (Wet)	17 100	Colback and Wiid
43	Quartzitic Sandstone (Dry)	9 070	Colback and Wiid
44	Quartzitic Sandstone (Wet)	4 970	Colback and Wiid
45	Slate (primary cracks)	4 300	Hoek
46	Slate (secondary cracks)	15 900	Hoek
47	Dolerite	37 000	CSIR
48	Quartzite (ERPM Footwall)	31 000	CSIR
49	Quartzite (ERPM Hanging wall)	43 200	CSIR
50	Glass	91 000	CSIR

* Uniaxial compressive strength

** Presented in dimensionless form by McClintock and Walsh

Fracture propagation from a single crack

In order to study the propagation of fracture from a single crack, 6 inch square by ¼ inch thick plates of annealed grass were carefully prepared. Open "Griffith cracks" were ultrasonically machined into these plates. The length of the crack was kept constant at ½ inch and its axis ratio at 25:1. The cracks were oriented at their critical angles as determined by Equation (7).

The plates were subjected to uniformly distributed edge loading in the tension and compression loading devices described in the Appendix to this paper. The specimens were studied photoelastically while under load and the stress at which fracture initiated as well as the direction of crack propagation was noted. A typical isochromatic pattern obtained in a plate subjected to the uniaxial compression is reproduced in Figure 3.

The stresses at which fracture is initiated are plotted in terms of the major and minor principal stresses, in Figure 4 and as Mohr circles in Figure 5. The theoretical fracture loci according to the original Griffith fracture initiation criterion, defined by equations (13) and (15), are given by the curves in Figures 4 and 5. The agreement between these and the experimental plots is considered satisfactory.

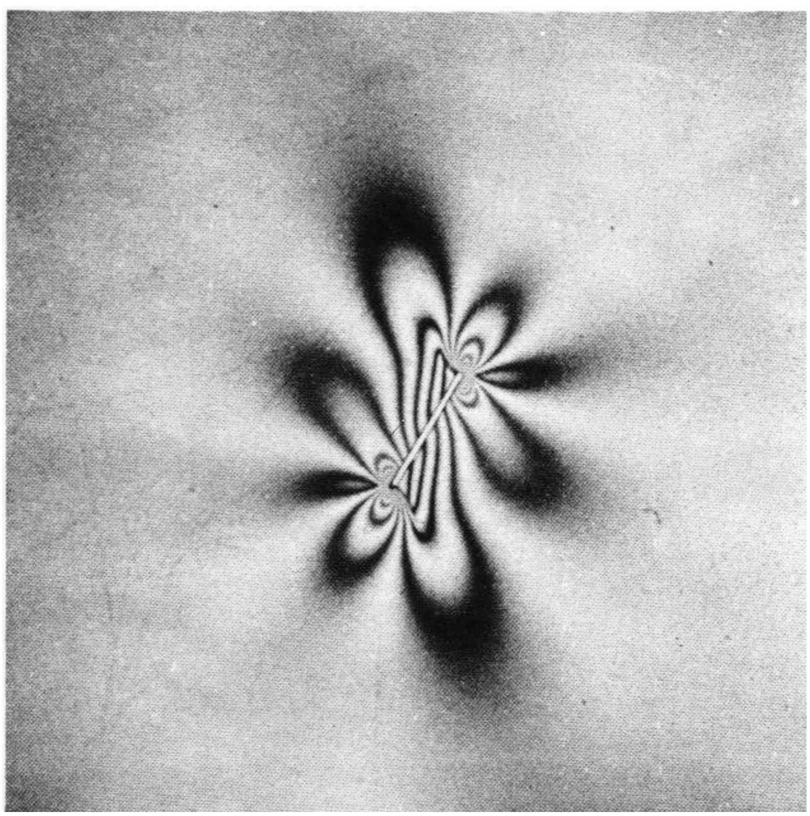

Figure 3. Photoelastic pattern in a glass plate containing an elliptical crack from which fracture has propagated.

In uniaxial tension, fracture initiation and fracture of the specimen occurred in the period of a few milliseconds. As can be expected, the fracture propagated in a direction normal to the direction of applied tension.

In biaxial compression, the fracture propagation followed a consistent pattern. Fracture initiated at a point on the crack boundary near but not at the crack tip (Ode, 1963) and followed a curved path (e.g. Figure 6, or upon careful observation, Figure 3). Generally, fracture propagation ceased when the crack path had become parallel to the major principal stress direction. In all cases, the applied stress was increased to at least three times the fracture initiation stress and, if the cracks showed no tendency to propagate further, the test was discontinued on the assumption that fracture of the specimen would not occur except at much higher stress levels.

The lengths of the stable cracks were found to be related to the ration of the applied principal stresses as illustrated in Figure 6. This finding is similar to that previously reported by Hoek (1965) for the propagation of cracks from a circular hole in a biaxial compressive stress field.

Under uniaxial compressive stress conditions, fracture propagation commenced with the sudden appearance of a small cracks of approximately 0.2 times the original crack length. Normally a crack would appear at only one end of the initial crack but would be followed, within a period of a few seconds[4] and at the same applied stress level, by a mirror image crack at the other end of the initial crack. Further propagation of these cracks required increased applied stress and this is plotted against crack length in Figure 7.

In the case of biaxial compression, insufficient information is available to permit plotting graphs similar to that shown in figure 7. From examination of available records, however, it appears that the length of the initial and final cracks did not differ by more than a few percent. This implies that, if the crack can be propagated, the stress required to do so would be as much as about ten times greater than the initiation stress and, as such, ceases to be of practical interest.

From these results, it can be concluded[5] that a single Griffith crack cannot account for the failure of a specimen in a compressive field unless the ration of applied principal stress is less than or equal to zero i.e. in uniaxial compression or when one principal stress in tensile. It is also suggested that, where failure of the specimen originates from a single crack, the direction of macroscopic fracture is normal to the minor (algebraically smallest) principal stress direction, i.e. in uniaxial compression parallel to the compressive stress direction.

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[4] In many of these tests fracture initiation was observed to be significantly time-dependent but no conclusions can be drawn from the present results because of a lack of adequate records in this respect.
[5] Somewhat similar conclusions have been reached by Brace and Bombolakis (1963).

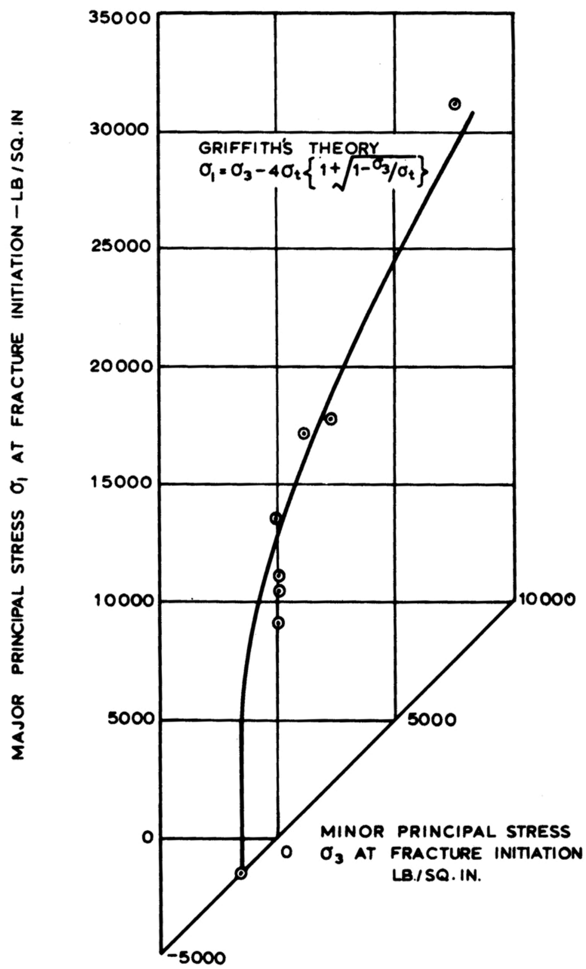


Figure 4. Relationship between principal stresses at fracture initiation at the boundary of an open crack.

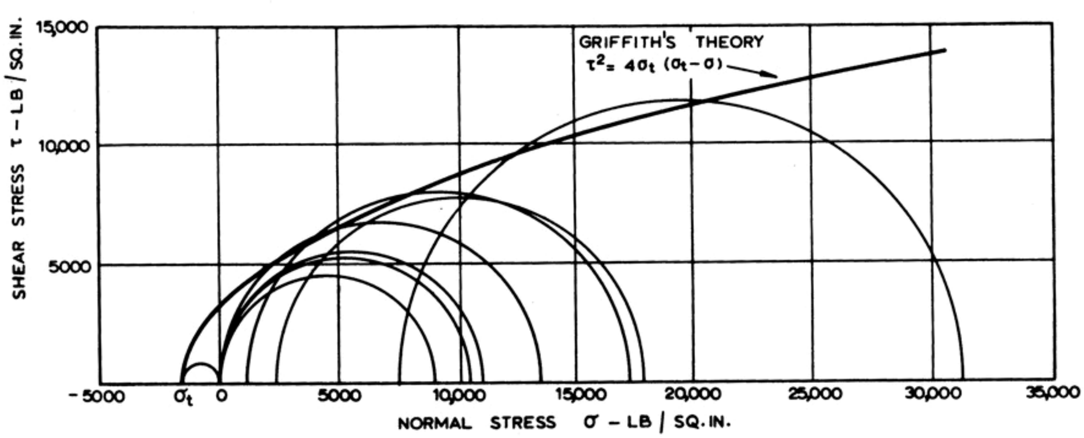


Figure 5. Mohr circles for fracture initiation at the boundary of an open crack.

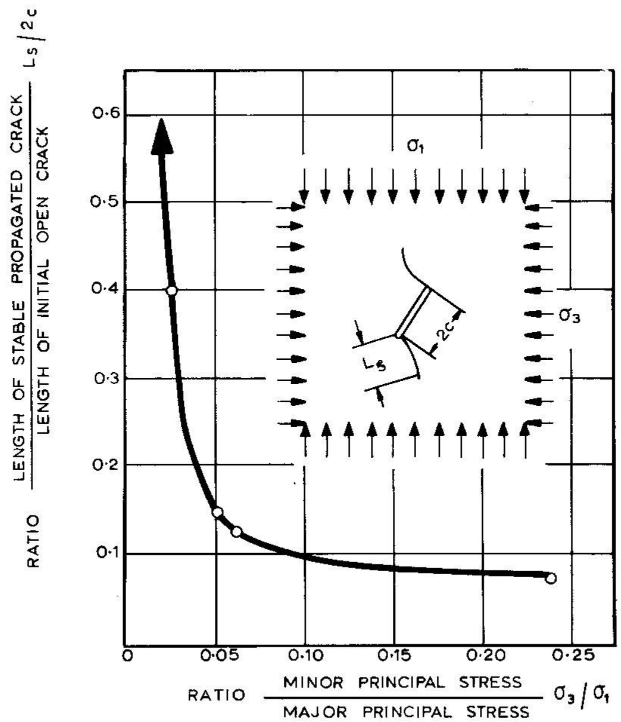


Figure 6. Relationship between stable crack length and ratio of applied principal stresses.

Since it appears reasonable to assume that the inherent cracks form which the fracture of hard rock initiates are initially closed (Brace and Bombolakis, 1963), an attempt was made to produce initially closed cracks in annealed glass plates.

The method used to produce theses cracks involves inducing a hairline crack of the required length on the surface of the glass plate. A hardened roller type glass tool which induces this crack as a result of the stress distribution under the contact point has been found preferable to a diamond tool which scores the glass surface. The shallow hairline crack is propagated throughout the thickness of the plate by reflected tensile stress waves generated by impacting the glass plate on the face opposite to that containing the crack.

At the time of writing it has not been possible to produce closed cracks of the same length in sufficient quantity to permit similar tests to those described in the previous section to be carried out. However, the authors feel that precise control of the main parameters involved in the process of closed crack formation, namely the quality of the initial hairline crack and the magnitude of the impact required to propagate it though the plate, will ultimately enable them to reproduce these cracks as required.

A few of the better quality cracks which have been obtained were tested in uniaxial compression and a typical result obtained is illustrated in Figure 8.

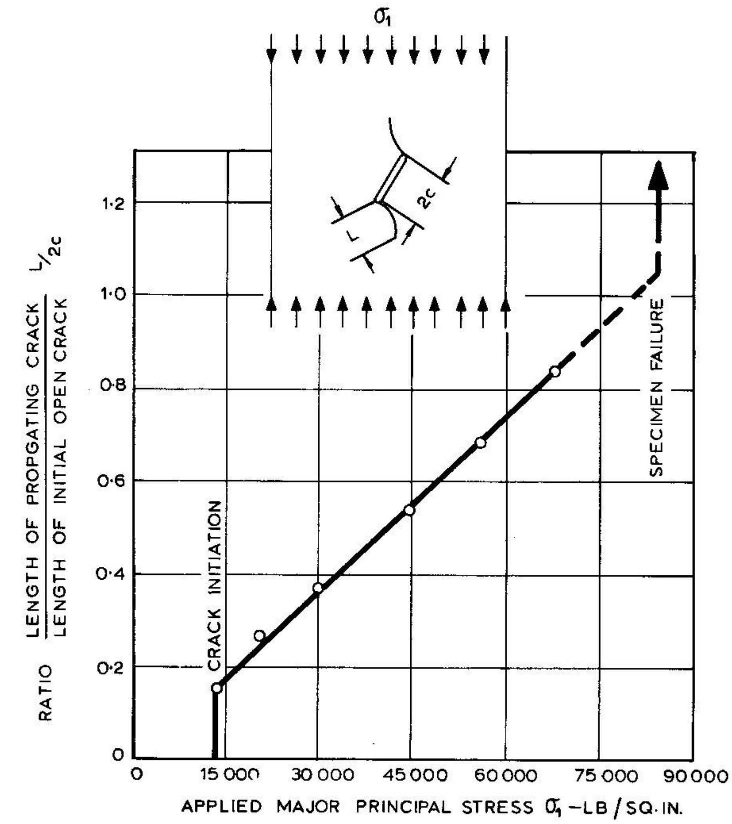


Figure 7. Relationship between propagating crack length and applied uniaxial compressive stress

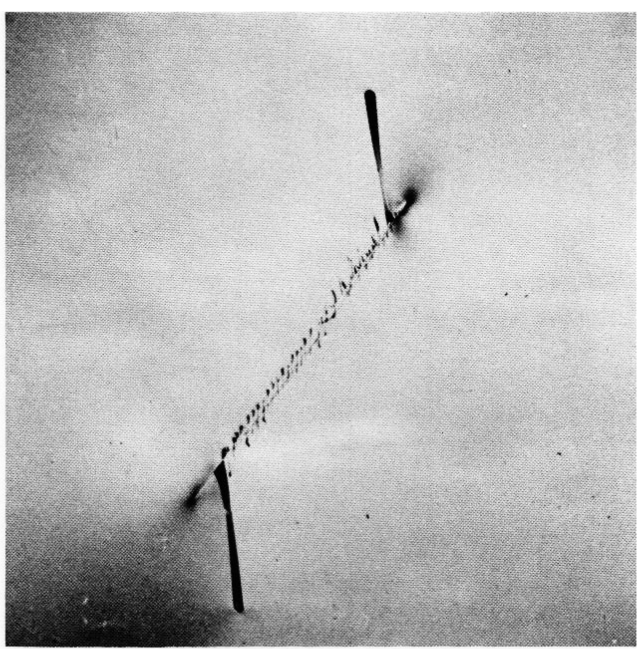


Figure 8. Fracture propagation from a closed crack in glass.

This photograph suggests some modification of the mechanism of fracture propagation assumed by McClintock and Walsh (1962). In their study of the friction effects on closed cracks, they postulated that the shear stress acting parallel to the crack surfaces exceeds the shear resistance due to friction, relative movement of the crack faces will occur and fracture will propagate from the crack tip.

The authors' observation observations of fracture propagation from closed cracks in glass leads to the conclusion that the crack tip itself plays a very minor role in the fracture process. The primary factor responsible for fracture initiation is the relative movement of the crack faces. Slight irregularities in the crack surface result in a n uneven stress distribution along the crack surfaces and tensile fracture initiates in the tensile stress zones which occur at points where the crack surface is relatively free to move. The formation of these tensile cracks is clearly illustrated in Figure 8.

In all the tests carried out by the authors it was found that these short tensile cracks formed at regular intervals over a fairly wide stress range. Fracture propagation occurred which the cracks closest to the tips of the initial crack propagated as illustrated in Figure 8. Once this state of fracture propagation had commenced, initiation and propagation of the other short tensile cracks ceased.

While the actual fracture initiation process may differ form that postulated by McClintock and Walsh, the authors feel that it may eventually be possible to express the fracture initiation criterion by means of an equation very similar in form to equation (16).
Once sufficient experimental evidence is available, the theoretical conditions for fracture initiation will be re-examined and, if necessary, modified.

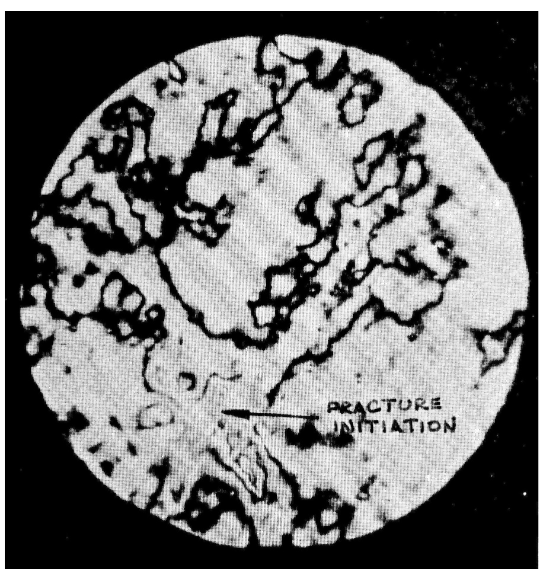


Figure 9. Reflected light photoelastic pattern showing strain distribution in a large grained granite plate subjected to uniaxial compression.

Preliminary studies of crack arrays

It is evident, from the results presenting in this paper, that fracture of a specimen is unlikely to occur unless a large number of cracks are present. Obviously, the spatial distribution of the cracks will influence the mechanism of fracture initiation and propagation and it is considered essential that this aspect of the problem be investigated if the fracture mechanism of rock is to be fully understood.

The results of a preliminary study of arrays of open cracks are included in Table 2. It is evident from these results that the interaction of cracks within the array influences both the initiation stress and the mechanism of crack propagation. The future research problem calls for a detailed study of the various parameters which influence the behaviour of crack arrays.

In addition to the tests on glass plates described above, the authors are also studying the mechanism of crack initiation and propagation in plates of rack. The plates, measuring 6 inches square by ¼ inch thick, have a dish-shaped central portion ground out of each face, giving a three inch diameter section of 1/8 inch thickness. One side of this reduced section is covered with a birefringent layer and the strain pattern associated with fracture initiation is studied by means of reflected polarised light (Hoek and Bieniawski, 1963). A typical photoelastic pattern obtained in these studies is illustrated in Figure 9 which shows stress concentrations around individual grains and the point of fracture initiation.

A study of the reverse side of the plates used for these tests permits detailed examination of the crack path. A low magnification micrograph of a typical crack in quartzite is reproduced in Figure 10. The stepped path followed by the propagation crack is evident in this photograph and the authors hope that, by studying the fracture paths in such specimens, a rational picture of rack fracture can be built up.

Conclusions

The results presented in this paper have shown that the Griffith theory offers a reliable basis for the prediction of fracture initiation from a single open crack. It is concluded, however, that a single crack cannot account for the failure of a specimen unless one of the applied principal stresses is zero or tensile.

The effects of crack closure in compression have been shown to differ form those assumed by McClintock and Walsh but it is anticipated that this difference will not significantly influence the final fracture criterion.

Examples have been given of studies of crack arrays and of fracture propagation in rock which indicate the direction of future research.

Table 2. Preliminary study of arrays of open cracks.

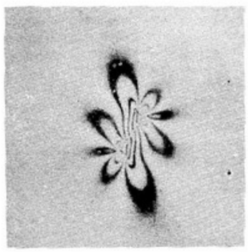
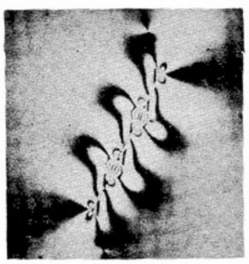
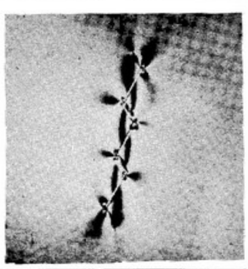
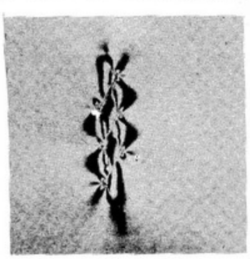
Array	Fracture Initiation	Remarks
	11 180 lb/sq. in	Fracture propagation occurred with increasing applied stress as illustrated in Figure 7
	8 750 lb/sq. in	Fracture initiation at lower stress than single crack due to greater "crack length". Individual cracks do not join.
	11 590 lb/sq. in	Individual cracks do not join. Array does not appear to influence initiation or propagation.
	9 900 lb/sq. in	Crack array influences propagation so that centre cracks tend to move together.
	13 420 lb/sq. in	Cracks join although initiation stress is higher.

Figure 10. Crack path in a quartzite specimen subjected to uniaxial compression.

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Appendix: Apparatus for the application of uniformly distributed edge loads to plate models

Tensile apparatus

The apparatus for applying uniformly distributed uniaxial tension to 6 inch square by ¼ inch thick plate models is illustrated in Figure 11. Load distribution is achieved by means of a "whipple tree" arrangement of pinjointed segments. The eight small segments which transmit the load to the model itself are bonded onto the model edge with epoxy resin.

Loading of the model and the photoelastic study of the stress distribution around the crack is carried out on the 12 inch diameter lens polariscope which has been described elsewhere by Hoek (1965).

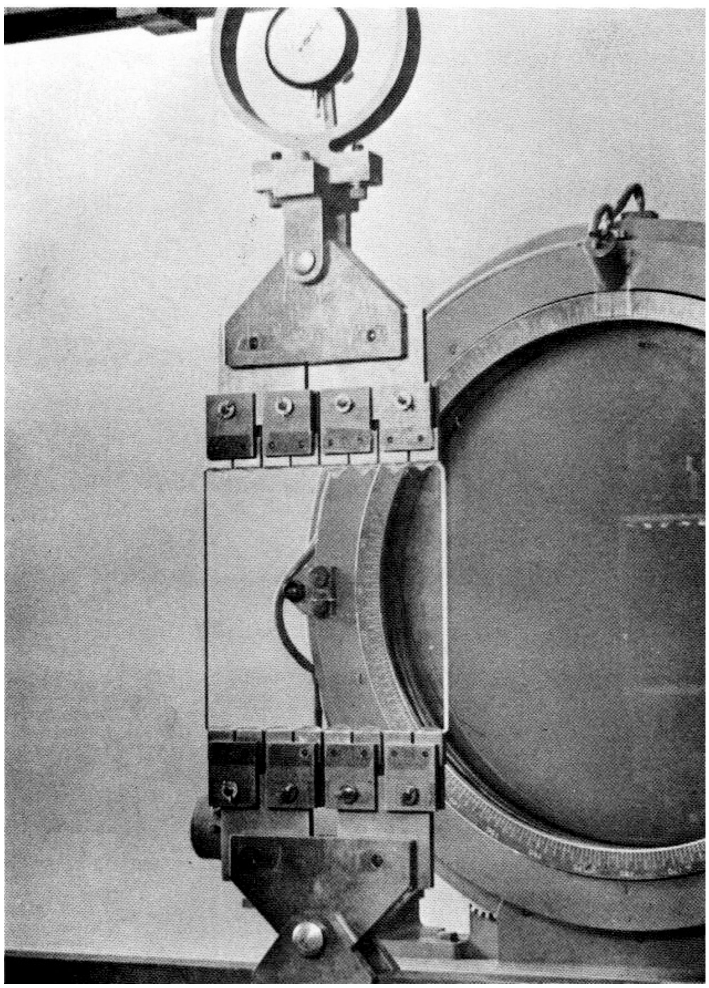

Figure 11. Apparatus for subjecting plate models to uniaxial tension.

Biaxial compression loading apparatus

The biaxial compression loading apparatus which was used in the tests described in this paper was designed for general rock mechanics model studies and extreme care was taken to ensure uniformity of the load distribution.

The load is applied to the accurately ground edges of the model though stacks of semi-circular segments. These segments are held in alignment by means of a set of copperberryllium leaf springs. The photograph of the partially dismantled load distribution frame reproduced in Figure 12 shows the arrangement of these segments and springs.

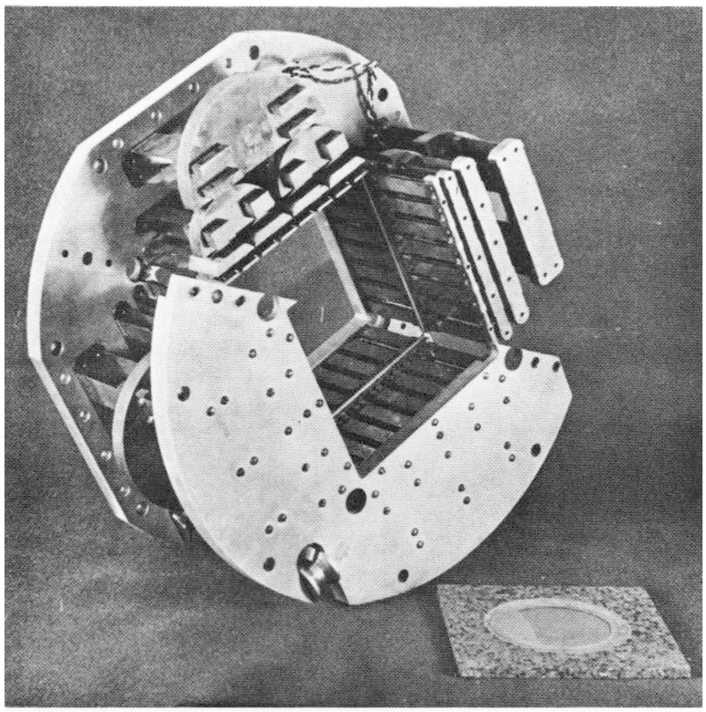

Figure 12. Partially dismantled biaxial loading fram showing details of the load distribution mechanism.

The load is applied onto the large segments by means of 4 hydraulic jacks, designed to exert a thrust of 100 tons each. These jacks are located in a circular frame illustrated in Figure 13. All the jacks are interconnected and fed by a single variable volume high pressure hydraulic pump.

Load control is achieved by means of a needle bleed-off valve in the hydraulic circuit. The horizontal jacks can be isolated to allow the application of uniaxial compression in the vertical direction. Alternatively, different diameter pistons can be fitted into the horizontal jacks to give a constant ratio of vertical to horizontal load.

The applied load is measured by means of strain gauges bonded onto the four large segments onto which the jacks act. The signals from these gauges are displayed on a digital voltmeter which permits direct load read-out in tens of pounds.

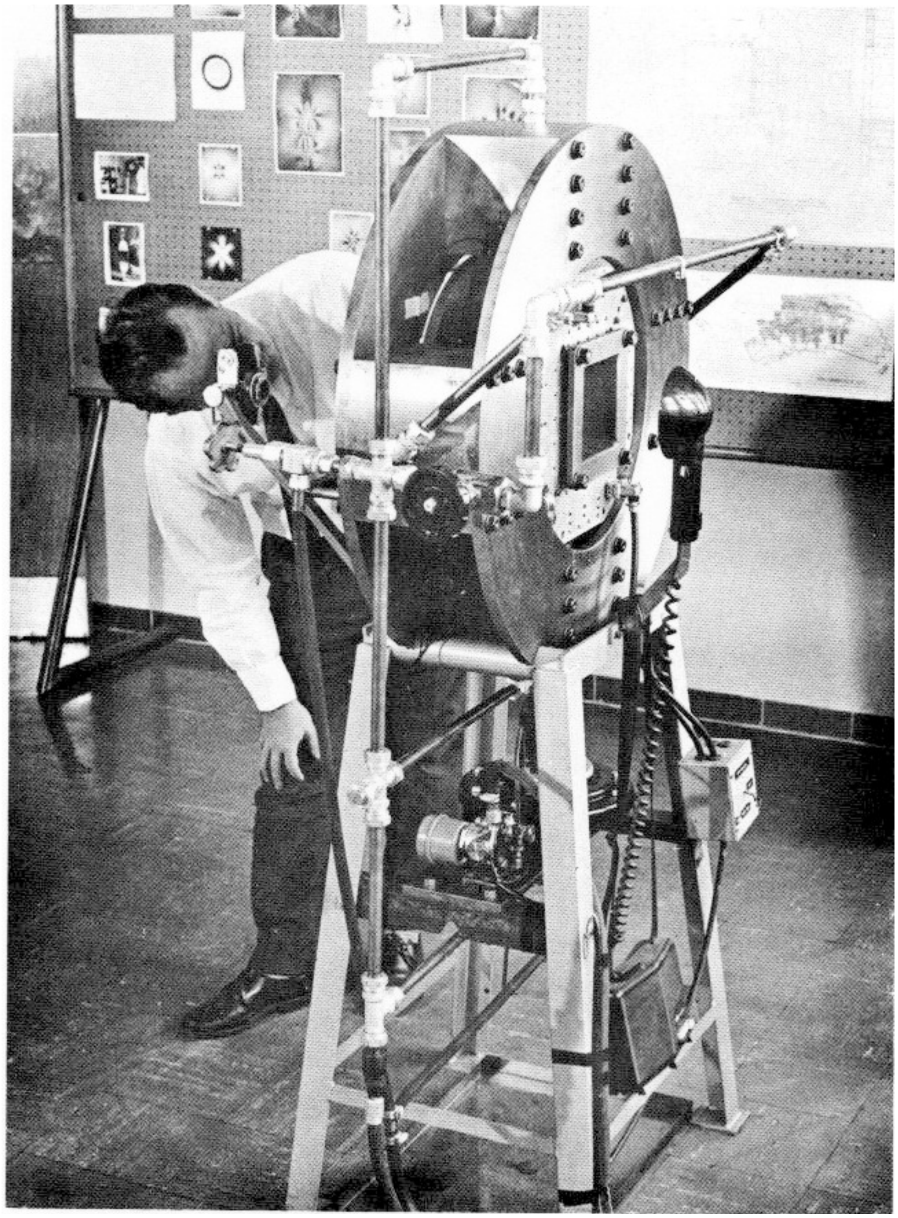

Figure 13. Apparatus for subjecting plate models to biaxial compression.

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* Title missing from original paper.