A modified Hoek-Brown failure criterion for jointed rock masses

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Synopsis

In order to eliminate some of the deficiencies which have been identified in ten years of practical experience with the application of the original Hoek-Brown failure criterion, a simplified rock mass classification and a modified rock mass failure criterion have been developed and are presented in this paper.

Introduction

The original Hoek- Brown failure criterion (Hoek and Brown, 1980) was developed in an attempt to provide a means of estimating the strength of jointed rock masses. The assumptions and limitations involved in the original criterion, derived empirically from the results of laboratory triaxial tests on intact rock samples, were discussed in a 1983 paper by Hoek (1983). Some aspects of the practical applications of the criterion were updated in a 1988 paper by Hoek and Brown (1988) but the criterion itself has remained essentially unchanged since it was first published in 1980.

On the basis of more than ten years of experience in using the Hoek-Brown criterion, a few deficiencies in the original system have become apparent. A modified criterion and an associated rock mass classification, both of which are presented for the first time in this paper, have been developed in an attempt to remedy these deficiencies. The most significant changes are:

a. A re-formulation of the criterion for jointed rock masses to eliminate the tensile strength predicted by the original criterion.
b. The introduction of a simplified qualitative rock mass classification for the estimation of the parameters in the modified criterion.
c. The presentation of a procedure for calculating the parameters defining the Mohr failure envelope or the modified criterion, and for determining the instantaneous friction angle and cohesive strength for a given normal stress value.

Applicability of failure criteria

For the analysis of laboratory test results on intact specimens or for rock engineering problems in which a feature such as a shear zone crosses the tunnel, the Hoek-Brown criterion is only applicable to the intact blocks of rock. This behaviour of the discontinuities should be considered in terms of a shear strength criterion such as the Mohr-Coulomb criterion used in soil mechanics or the criterion proposed by Barton (1971). The stability of the sparsely jointed system can be analyzed by utilizing solutions such as those proposed by Bray (1966) or Amadei (1988).

In cases where the rock mass can be considered to be ‘heavily jointed’ and where the behaviour is not dominated by one or two individual discontinuities, the modified Hoek-

Brown criterion presented later in this paper can be used. A typical application would be a 5 metre span tunnel in a rock mass with three or four similar discontinuity sets with an average spacing of approximately 100 mm. the overall stability of this tunnel would be controlled by the freedom of the rock pieces to translate and rotate and the rock mass would behave has an isotropic medium In some cases, a 'weak' rock mass such as this may contain a single dominant fault or shear zone. Here the modified Hoek-Brown criterion would be used to define the failure characteristics of the rock mass but the behaviour of the dominant discontinuity would be considered in terms of a shear strength criterion.

In deriving the classification scheme presented later in this paper, it has been assumed that the rock mass is undisturbed and that only its inherent properties are considered. External factors such as in situ or induced stresses, groundwater pressures and blasting damage are assumed to be included in the engineering analysis in which the failure criterion is used.

Failure criterion for intact rock

For intact rock the original Hoek-Brown failure criterion may be written in the following form:

$$\sigma_1' = \sigma_3' + \sigma_c \left(m_i \frac{\sigma_3'}{\sigma_c} + 1 \right)^{1/2} \tag{1}$$

Where

- σ_1' is the major principal effective stress at failure
- σ_3' is the minor principal effective stress at failure
- σ_c is the uniaxial compressive strength of the rock
- m_i is a constant for intact rock .

The derivation of the Mohr failure envelope corresponding to equation (1) is presented in the appendix.

The uniaxial compressive strength (σ_c) of intact rock is an important parameter in the Hoek-Brown failure criterion and should be determined by laboratory testing whenever possible. Where no laboratory test results are available, the value of the uniaxial compressive strength can be estimated from Table 1. When this is done, it is recommended that parametric studies be carried out to determine the sensitivity of the analysis to a range of uniaxial compressive strength values before a final selection is made.

Table 1. Estimates of uniaxial compressive strength σ_c for intact rock

Term	Uniaxial Comp. Strength σ_c MPa	Point Load Index I_s MPa	Field estimate of strength	Examples*
Extremely Strong	>250	>10	Rock material only chipped under repeated hammer blows	Basalt, chert, diabase, gneiss, granite, quartzite
Very Strong	100 - 250	4 - 10	Requires many blows of a geological hammer to break intact rock specimens	Amphibolite, andesite, basalt, dolomite, gabbro, gneiss, granite, granodiorite, limestone, marble, rhyolite, tuff
Strong	50 - 100	2 - 4	Hand held specimens broken by single blow of geological hammer	Limestone, marble, phyllite, sandstone, schist, slate
Medium Strong	25 - 50	1 - 2	Firm blow with geological pick indents rock to 5 mm, knife just scrapes surface	Claystone, coal, concrete, schist, shale, siltstone
Weak	5 - 25	**	Knife cuts material but too hard to shape into triaxial specimens	Chalk, rocksalt, potash
Very Weak	1 - 5	**	Material crumbles under firm blows of geological pick, can be shaped with knife	Highly weathered or altered rock
Extremely Weak	0.25 - 1	**	Indented by thumbnail	Clay gouge

* All rock types exhibit a broad range of uniaxial compressive strengths which reflect hetrogeneity in composition and anisotropy in structure. Strong rocks are characterized by well interlocked crystal fabric and few voids.
** Rocks with a uniaxial compressive strength below 25 MPa are likely to yield highly ambiguous results under point load testing.

The constant m_i has been found to depend upon the mineralogy, composition and grain size of the intact rock. Table 2 gives values of m_i for rocks described in a standard geological classification based on three major groupings of rock families. Values of m_i included in this table included in this table were obtained by a re-assessment of triaxial test data presented in Hoek and Brown (1980) together with more recently published triaxial data, using the Simplex Reflection technique described by Shah and Hoek (1992). This analysis was carried out by Doruk (1991), using a program called ROCKDATA developed by Shah (1992). This program has now been replaced by RocLab which can be downloaded (free) from www.rocscience.com.

Rock names given to test specimens have been taken directly from the literature; no attempt has been made to revise these names. The laboratory results indicate that high values of m_i are attributed to igneous and metamorphic rocks with well interlocked crystal structure, silicate mineralogy and coarse grain size. Lowest values are derived from tests on fine grained sedimentary rocks, and those with carbonate mineralogy.

Table 2. Values of constant m_i for intact rock, by rock group

Grain size	Sedimentary Carbonate	Sedimentary Detrital	Sedimentary Chemical	Metamorphic Carbonate	Metamorphic Silicate	Igneous Felsic	Igneous Mafic	Igneous Mafic
Coarse	Dolomite 10.1	Conglomerate (20)		Marble 9.3	Gneiss 29.2	Granite 32.7	Gabbro 25.8	Norite 21.7
Medium	Chalk 7.2	Sandstone 18.8	Chert 19.3		Amphibolite 31.2		Dolerite 15.2	
Fine	Limestone 8.4	Siltstone 9.6	Gypstone 15.5		Quartzite 23.7	Rhyolite (20)	Andesite 18.9	Basalt (17)
Very fine		Claystone 3.4	Anhydrite 13.2		Slate 11.4			

Values shown were derived from statistical analysis of triaxial test data for each rock type. Values in parenthesis have been estimated.

Failure criterion for jointed rock masses

In applying the original Hoek-Brown failure criterion to jointed rock masses, the predicted strengths are found to be acceptable where the rock mass is subjected to conditions in which the minor principal effective stress (σ_3') has a significant compressive value. For low values of σ_3', the criterion predicts too high an axial strength and also a finite tensile strength.

Most rock mechanics engineers consider that the type of jointed rock mass to which the Hoek-Brown failure criterion applies should have zero tensile strength. For the past 30 years, finite element numerical models for use in rock mechanics have included a 'no tension' option which allows tensile stresses developed in the model to be transferred onto adjacent elements.

In view of this deficiency in the original Hoek-Brown criterion when applied to jointed rock masses, it was decided that a modified criterion should be developed. This criterion should conform to the strength predictions given by the original criterion, for compressive stress conditions, and should predict a tensile strength of zero for the rock mass. A modified criterion, which satisfies these conditions, was developed by Shah (1992) and can be expressed in the following form:

$$\sigma_1' = \sigma_3' + \sigma_c \left(m_b \frac{\sigma_3'}{\sigma_c} \right)^a \qquad (2)$$

where m_b and a are constants for broken rock.

The derivation of the Mohr envelope corresponding to the modified criterion is discussed in the appendix.

For jointed rock masses, the strength characteristics are controlled by the block shape and size and the surface condition of the intersecting discontinuities and should be selected to represent the average condition of the rock mass. Specific features such as faults, dykes or shear zones, should be considered separately.

Block shape and size give a measure of the overall geometry of the rock mass, as well as an indication of the proportion of the volume of rock which is occupied by discontinuities. The amount of geological deformation also has an influence; more highly deformed rock masses usually have a smaller block size.

The surface condition of the joints and other discontinuities also modified the rock mass strength. At best, an unweathered, massive rock with discontinuous, irregular, rough joints would be almost as strong as the intact rock material. At worst, a highly weathered, moderately folded, blocky and seamy rock mass with continuous, slickensided surfaces with soft infilling would be far weaker than the intact rock material pieces in the rock mass.

These consideration have been taken into account in constructing a classification system, presented in Table 3, which can be used to estimate values of the constants m_b and a. Approximate block sizes and discontinuity spacings in jointed rock masses are give in Table 4. These values are based upon recommendations published by the Engineering Group of the Geological Society (1997) and the International Society for Rock mechanics (1978).

The input descriptions of overall geological structure and surface conditions are used to select values of m_b/m_i and a in Table 3. Substitution of the value of m_i from Table 2 into m_b/m_i gives the value of m_b.

Worked example

Detailed engineering geological mapping of an area yields the following rock mass description: moderately weathered, bedded and jointed, pale grey, fine grained, medium strong SANDSTONE, with two sets of widely spaced joints orthogonal to bedding. Discontinuity surfaces are persistent, irregular and smooth with surface iron staining and minor calcite infilling.

In a pre-feasibility study it is unlikely that a comprehensive laboratory testing programme would be carried out so use would be made of Tables 1 and 2 to estimate values uniaxial compressive strength σ_c of 40 MPa, and intact m_i if 18.8. The rock mass has interlocked, medium sized blocks and would have an overall structural condition described as very blocky. The surface condition of the bedding planes and joints would be fair. Table 3 would give values of m_b/m_i of 0.1 and a of 0.5.

Table 3. Estimation of m_b/m_i and a based on rock structure and surface condition.

MODIFIED HOEK-BROWN FAILURE CRITERION

$$\sigma'_1 = \sigma'_3 + \sigma_c \left(m_b \frac{\sigma'_3}{\sigma_c} \right)^a$$

σ'_1 = major principal effective stress at failure
σ'_3 = minor principal effective stress at failure
σ_c = uniaxial compressive strength of *intact* pieces in the rock mass
m_b and a are constants which depend on the composition, structure and surface conditions of the rock mass

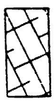
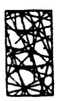
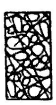
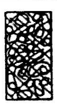
STRUCTURE	SURFACE CONDITION	VERY GOOD Unweathered, discontinuous, very tight aperture, very rough surface, no infilling	GOOD Slightly weathered, continuous, tight aperture, rough surface, iron staining to no infilling	FAIR Moderately weathered, continuous, extremely narrow, smooth surfaces, hard infilling	POOR Highly weathered, continuous, very narrow, polished/slickensided surfaces, hard infilling	VERY POOR Highly weathered, continuous, narrow, polished/slickensided surfaces, soft infilling
 BLOCKY - well interlocked, undisturbed rock mass; large to very block size	m_b/m_i a	0.7 0.3	0.5 0.35	0.3 0.4	0.1 0.45	
 VERY BLOCKY - interlocked, partially disturbed rock mass; medium block sizes	m_b/m_i a	0.3 0.4	0.2 0.45	0.1 0.5	0.04 0.5	
 BLOCKY/SEAMY - folded and faulted, many intersecting joints; small blocks	m_b/m_i a		0.08 0.5	0.04 0.5	0.01 0.55	0.004 0.6
 CRUSHED - poorly interlocked, highly broken rock mass; very small blocks	m_b/m_i a		0.03 0.5	0.015 0.55	0.003 0.6	0.001 0.65

Table 4. Approximate block sizes and discontinuity spacings for jointed rock masses

Term	Block size	Equivalent discontinuity spacings
Very large	$(>2\text{m})^3$	Extremelywide
Large	$(600\text{mm-}2\text{m})^3$	Very wide
Medium	$(200\text{-}600\text{mm})^3$	Wide
Small	$(60\text{-}200\text{mm})^3$	Moderately wide
Very small	$(<60\text{mm})^3$	<Moderately wide

Preliminary design analyses in this rock mass would be carried out using these strength parameters.

Uniaxial material strength	σ_c	40 MPa
Broken material constant	m_b	1.88
Broken material constant	a	0.5

Figure 1 gives a plot of axial strength $\sigma_1^{'}$ versus minor principal stress $\sigma_3^{'}$ and also the Mohr failure envelope for these parameters. These curves were generated using the program ROCKDATA.

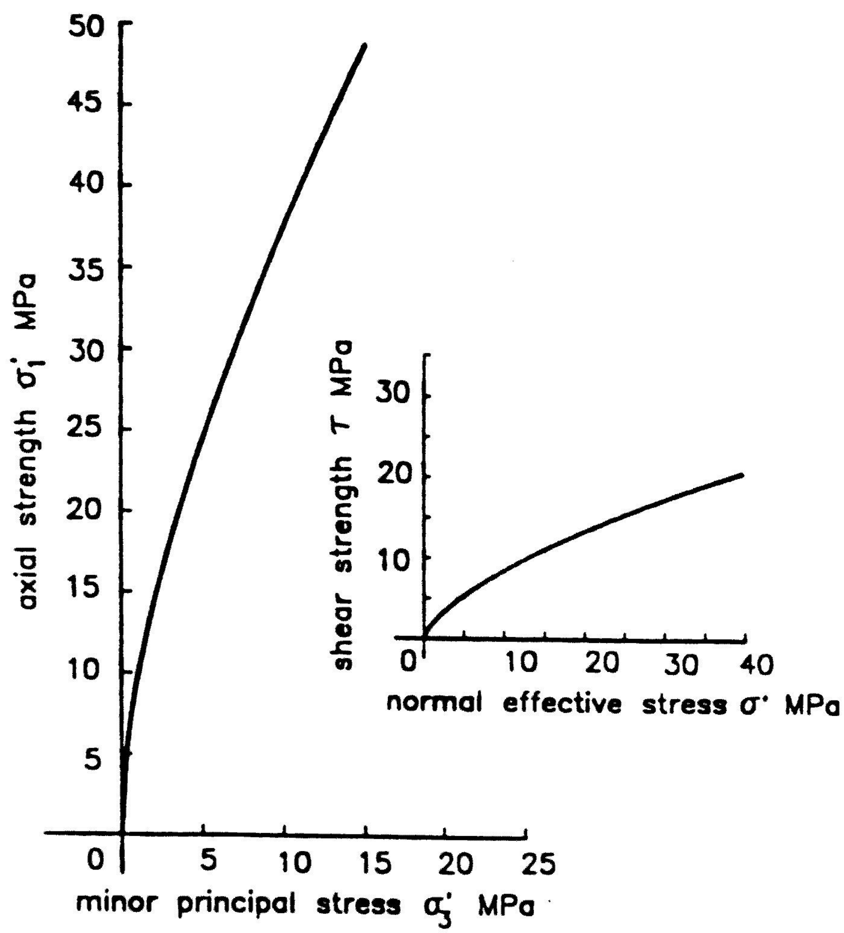


Figure 1. Plot of principal stresses at failure and the Mohr envelope for the example considered

Conclusion

A modified failure criterion for jointed rock masses and a simplified classification scheme, for estimating the parameters for this criterion, have been presented. These tools are intended to provide engineers and geologists with a means of estimating the strength of jointed rock masses during preliminary feasibility studies. It is strongly recommended that more detailed studies, including laboratory or in situ tests, should be carried out for detailed engineering design and that parametric studies should be carried out to check the influence of the rock mass strength before final decisions are made

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APPENDIX - MOHR ENVELOPES

The Mohr envelope corresponding to the original Hoek-Brown failure criterion is defined by the following relationship, (ref. 2):

$$\tau = \sigma_c \frac{m_i}{8}(\cot \phi'_i - \cos \phi'_i) \tag{3}$$

where

τ is the shear stress at failure

ϕ'_i is the instantaneous friction angle for a given value of the effective normal stress σ'

The value of the instantaneous friction angle ϕ'_i is given by:

$$\phi'_i = \arctan\left(4h\cos^2\left(30 + \frac{1}{3}\arcsin\frac{1}{\sqrt{h^3}}\right) - 1\right)^{-1/2} \tag{4}$$

where

$$h = 1 + \frac{16(m_i\sigma' + \sigma_c)}{3{m_i}^2\sigma_c} \tag{5}$$

The instantaneous cohesive strength c'_i is given by:

$$c'_i = \tau - \sigma' \tan \phi'_i \tag{6}$$

For the modified Hoek-Brown criterion it is not possible to derive a closed form solution for the Mohr envelope as for the original criterion. Consequently a numerical technique is used to determine the constants α and β in the following empirical equation :

$$\tau = \sigma_c \alpha \left(\frac{\sigma'}{\sigma_c}\right)^\beta \tag{7}$$

A general analytical solution for Mohr envelope published by Balmer (ref. 12) is used in the following calculation of the constants α and β :

The normal and shear stresses for a given value of σ'_3/σ_c are calculated as follows:

$$\sigma' = \sigma'_3 + \frac{\sigma'_1 - \sigma'_3}{1 + \frac{\partial \sigma'_1}{\partial \sigma'_3}} \tag{8}$$

$$\tau = (\sigma' - \sigma'_3)\sqrt{\frac{\partial \sigma'_1}{\partial \sigma'_3}} \tag{9}$$

where, for the modified Hoek-Brown criterion defined by equation (2),

$$\frac{\partial \sigma'_1}{\partial \sigma'_3} = 1 + a m_b^a \left(\frac{\sigma'_3}{\sigma_c}\right)^{a-1} \tag{10}$$

Equation (7) can be re-written in the form

$$\log \frac{\tau}{\sigma_c} = \log \alpha + \beta \log \frac{\sigma'}{\sigma_c} \tag{11}$$

Let $x = \log \sigma'/\sigma_c$ and $y = \log \tau/\sigma_c$, then

$$\beta = \frac{\Sigma x_i y_i - \frac{\Sigma x_i \Sigma y_i}{n}}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}} \tag{12}$$

$$\log \alpha = \frac{\Sigma y_i}{n} - \beta . \frac{\Sigma x_i}{n} \tag{13}$$

Where x_i and y_i are the ith values of x and y and n is the total number of each of the quantities.

The instantaneous friction angle ϕ'_i for a specified value of the normal stress σ'/σ_c is given by:

$$\phi'_i = \arctan \alpha\beta \left(\frac{\sigma'}{\sigma_c}\right)^{\beta - 1} \tag{14}$$

The corresponding cohesive strength is given by equation (6).

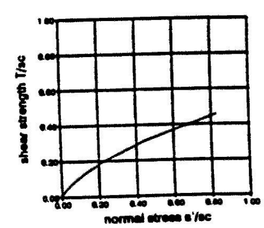
A convenient method for determining α and β from equations (12) and (13) is to set up this calculation in a spreadsheet as shown in Figure 2. In order to capture the pronounced curvature at low normal stresses it has been found that the values of σ'_3/σ_c which are substituted into equation (10) should follow a geometric progression defined by :

$$\frac{\sigma'_3}{\sigma_c} = \frac{1}{2^k} \tag{15}$$

where k varies from 9 to 1 in increments of -1.

This gives a maximum value of $\sigma'_3/\sigma_c = 0.5$ which has been found to cover the range of stresses encountered in most practical applications of the Hoek-Brown failure criterion.

Input data:	
Uniax strength sc	40
Constant mb	1.88
Constant a	0.500
Maximun s3	20
Output data :	
Calculated alpha	0.547
Calculated beta	0.695



s3/sc	s1/sc	ds1/ds3	s'/sc	T/sc	x	y	xy	x^2
0.0020	0.0625	16.5126	0.0054	0.0141	-2.2665	-1.8520	4.1976	5.1372
0.0039	0.0896	11.9690	0.0105	0.0229	-1.9782	-1.6409	3.2461	3.9134
0.0078	0.1290	8.7563	0.0202	0.0368	-1.6939	-1.4347	2.4302	2.8693
0.0156	0.1870	6.4845	0.0385	0.0583	-1.4143	-1.2342	1.7455	2.0001
0.0313	0.2736	4.8781	0.0725	0.0911	-1.1398	-1.0406	1.1860	1.2990
0.0625	0.4053	3.7423	0.1348	0.1398	-0.8704	-0.8544	0.7436	0.7575
0.1250	0.6098	2.9391	0.2481	0.2110	-0.6054	-0.6758	0.4091	0.3665
0.2500	0.9356	2.3711	0.4534	0.3131	-0.3436	-0.5042	0.1732	0.1180
0.5000	1.4695	1.9695	0.8265	0.4582	-0.0828	-0.3389	0.0281	0.0068
				Sums	-10.3948	-9.5758	14.1595	16.4881

Figure 2. Example of spreadsheet calculation of α and β for the modified Hoek-Brown criterion.