# One is not Enough

> *"Knowing is not enough; we must apply. Being willing is not enough; we must do."*
>
> *Leonardo da Vinci*

## Introduction

The rock mechanics community recognized long ago that the geometric arrangement of joint networks significantly contributes to rock mass behaviour. Joint networks lower the strength and deformational properties of rocks, cause directional (anisotropic) behaviour, and influence the manner in which excavations fail. Because the sizes and locations of joints are not well known, fracture networks are also cloaked in uncertainty. As a result of network geometry uncertainties, any prediction based on a single network realization has practically zero likelihood of ever being manifest. This is why the article is titled *"One is not Enough."*

To help rock engineers better assess the influence of ensembles of fractures on excavations, a feature for automatically generating joint networks was included in the recently released Phase$^2$ version 7. It generates ensembles of randomly located joints that satisfy specified statistical attributes such as length, spacing and orientation.

The automatic joint network generator combines very well with the Shear Strength Reduction (SSR) technique (for calculating factor of safety) to make the Finite Element program a powerful and worthy alternative to Discrete Element Methods, traditionally used to analyze excavation stability in blocky rock masses. *Phase$^2$* 7 is versatile. It can model a broad range of continuous and discontinuous rock mass problems and failure mechanisms. Its capabilities are helping soften the boundaries between the classification of rock mass stability problems into categories such as wedge-type failures controlled by discontinuities and rotational-type failures in which rock masses essentially behave as continua.

In this article, we demonstrate an additional advantage of the new tools in *Phase$^2$* 7 – they allow engineers to understand the degree to which the geometric uncertainties of joint networks impact behaviour and design.

We will show that although it may not yet be feasible to perform full blown Monte Carlo analysis comprising hundreds of individual simulations with *Phase$^2$*, this numerical tool can still be used to obtain an adequate idea of how variable results can be. This will be accomplished through the stability analysis of a slope in highly broken rock.

First we will briefly examine the goal of Monte Carlo analysis and current limitations of applying the technique to Finite Element SSR modeling. We will look at how to use rough estimates of factor of safety mean and variance obtained from a few Monte Carlo simulations to estimate probability of failure. The ease of use of *Phase$^2$* facilitates such analyses.

## Probabilistic Stability Analysis

The ultimate goal of probabilistic stability analysis is to obtain the complete distribution of factors of safety, given a set of random (uncertain) input variables with specified statistical properties. From this distribution, probability of failure can be determined.

It is difficult to directly obtain an output probability distribution from Finite Element SSR analysis because the approach is implicit and computationally expensive. With the computing resources generally available to users today, it may be more appropriate to only determine the statistical moments of the output distribution, and not the distribution itself.

Statistical moments are quantities that capture both overall and in-depth information on the geometry (location and appearance) of a probability distribution function. The first statistical moment is the mean. It provides information on the location and central tendency of a distribution. The other moments, which are of higher order, are commonly taken about the mean. The second moment is variance. It describes the spread or dispersion of the distribution about the mean. The third and fourth moments, skewness and kurtosis, provide further information on distribution shape.

The powerful and flexible Monte Carlo method can be used to estimate distribution moments. It is very simple to use and can be applied to existing deterministic programs without modifications. This is why we use it in this example. In the Monte Carlo method samples of probabilistic input variables are generated and then used to perform a number of deterministic simulations. Information on the distribution and moments of the response variable is then obtained from the simulation results. The calculated moments can be used to determine a probability of failure by assuming either a normal or log-normal distribution for factor of safety.

## Example of a Simple Slope in Highly Broken Rock

We will analyze an example of a rock slope in highly broken rock in which there are no preferred jointing directions. The rock mass is modeled with a Voronoi joint network model. The test model is shown on Figure 1 on the next page. Due to the highly broken nature of the rock mass, one would not expect much variation in the failure mechanism, and thus factor of safety. We shall examine whether this is the case.

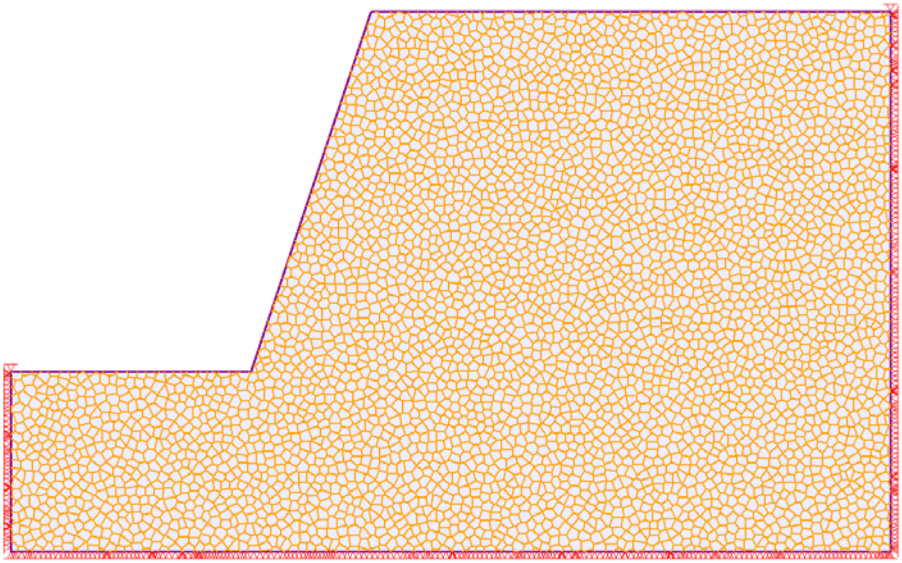

*Figure 1: Example of slope in highly broken rock mass*

We generated 10 random realizations of the Voronoi network and performed *Phase*$^2$ SSR analysis on each of them. The factor of safety results are given in the table below. They range from a low of 0.5 to a maximum of 1.26, with mean equal to 1.037 and variance of 0.042. If we assume that the factors of safety are distributed according to a normal distribution, we can calculate a probability of failure of 43% for the slope. We used the following relationships to calculate probability of failure:

$$reliability\ index = \frac{mean - 1}{\sqrt{variance}}$$

$probability\ of\ failure = 1 - \Phi\left[reliability\ index\right]$, where $\Phi$ is the standard normal cumulative distribution function.

Plots of total displacement contours at failure for each of the simulations yielded rich insights into the range of possible slope failure modes. Three of the failures are shown on Figures 2a – 2c. They range from shallow, sloughing-type failure to deep-seated rotational mechanisms. Given that the interplay between excavation geometry, joint network geometry, and joint and rock mass strength and stiffness properties is complex and almost impossible to anticipate before analysis, the ability of Finite Element SSR analysis to capture the correct failure mode without special treatment or assumptions is useful.

Table: Factor of Safety Results

| **Realization** | **Factor of Safety** |
|---|---|
| 1 | 0.93 |
| 2 | 1.09 |
| 3 | 1.07 |
| 4 | 1.07 |
| 5 | 1.22 |
| 6 | 1.18 |
| 7 | 0.96 |
| 8 | 1.26 |
| 9 | 1.09 |
| 10 | 0.5 |
| Mean | 1.037 |
| Variance | 0.042 |

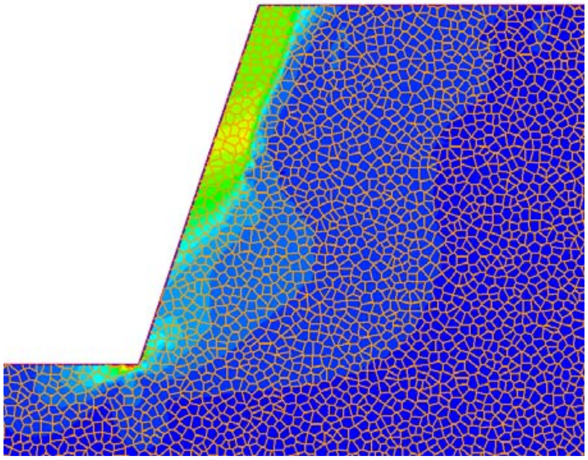

*Figure 2a: Contours of total displacement reveal a shallow – sloughing-type – failure with factor of safety equal to 0.5.*

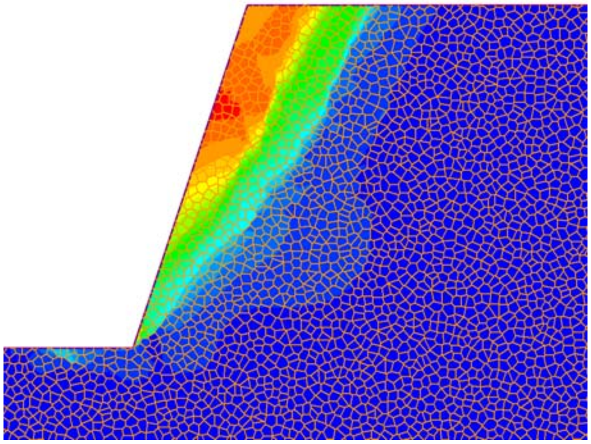

*Figure 2b: Contours of total displacement show a failure mass of intermediate depth with factor of safety equal to 0.93. The failure mechanism looks rotational and passes through the slope toe, just as is the case for homogeneous soils.*

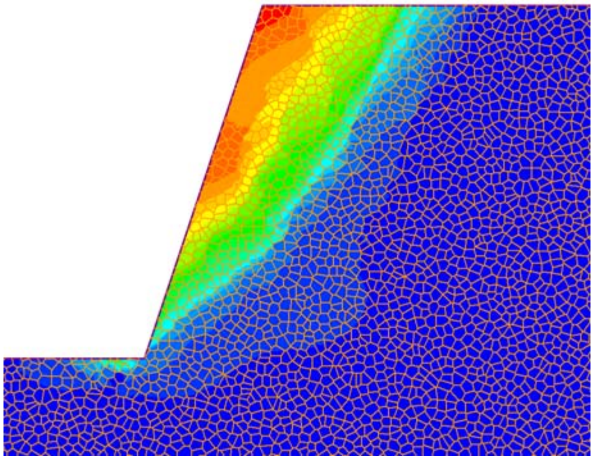

*Figure 2c: Contours of total displacement show a failure mass of greater depth with factor of safety equal to 1.26. Again the failure mechanism seems rotational and passes through the slope toe.*

## Concluding Remarks

As a result of Finite Element SSR analysis being computationally expensive, it may not be feasible today to *routinely* perform hundreds of Monte Carlo simulations. However, because such simulations are highly amenable to parallel computation, we believe that the time for detailed Monte Carlo studies is not far away. It is achievable on networks of multi-processor desktops. In a recent test, we performed about 50 Monte Carlo simulations (each model with about 30,000 degrees of freedom) in parallel on a cluster of four desktops in only about 8 hours.

Our simple example, which yields different failures and factors of safety for the same stochastic joint networks, underlines the need to perform probabilistic analysis on excavations in jointed rock. It promotes greater understanding of problems and minimizes the likelihood of unpleasant surprises in performance. At the same time, it increases our chances of developing more robust designs, stabilization measures and improved monitoring. One is certainly not enough.