Review of Post-Peak Parameters and Behaviour of Rock Masses: Current Trends and Research

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Introduction

Estimates of the strength and deformation characteristics of rock masses are required for the analysis of underground excavations. The Hoek-Brown failure criterion (Hoek et al. 2002) is widely accepted and has been applied to numerous projects and applications around the world. This failure criterion, however, only deals with the stress in the rock mass up to the point of failure. There has been comparatively little work in the literature that deals with the field scale behaviour of rock after significant damage or failure (i.e., the post-peak behaviour). Laboratory testing of hard rock often demonstrates strain-softening behaviour, resulting in relatively low confining stresses at high strains in the post-peak. The question remains as to what is the precise mode of stress-strain behaviour between these end-points.

In this article, the importance of post-peak parameters and behaviour are discussed, along with the need to model to failure conditions. The strength criterion that will be focussed upon is the Generalized Hoek-Brown (GHB) failure criteria, as it is currently the most widely used and accepted criterion. An email discussion about post-peak, or residual, rock parameters that recently occurred between Rocscience and several industry leaders in rock mechanics modelling is summarized. This is followed by a review of current publications and ongoing work at the University of Toronto into post-peak behaviour of rock masses.

Generalized Hoek-Brown Parameters

Support in underground mines provides a safe working environment, increases rock mass stability and controls dilution. Typical support methods such as a cable bolts are passive support and only provide reinforcement once the rock mass starts to dilate (the opening of fractures). Dilation and crack opening is a result of loss of strength of a rock mass after failure. Hence knowledge of the failed or post-peak rock mass parameters is quite important in the design of support for underground openings.

In the two-dimensional elasto-plastic finite element program, *Phase²*, users can currently define both elastic and plastic (i.e., peak and residual) rock mass properties. When an element of rock mass has exceeded its peak strength it fails in a simple brittle manner essentially switching directly from the peak parameters to post-peak, with no softening mechanism. Note that for this paper, all rock parameters or properties that define behaviour up to the point of failure, in the elastic behaviour range, will be referred to as the peak properties. As well, the properties that define behaviour of the rock mass after failure, in the plastic behaviour range, will be referred to as the post-peak properties.

For plastic numerical models, the most recent version of *Phase²* (version 5) allows the user to enter one value for unconfined compressive strength of intact rock samples σ_{ci} (also referred to as the UCS), as well as both peak and residual values for the Hoek-Brown m and s parameters (Hoek and Brown 1980a and 1980b). A dilation parameter is also defined as a measure of the increase in volume of the material when sheared. Using this version of the Hoek-Brown Failure Criterion (Equation 1), all three of the peak rock parameters are determined from laboratory triaxial tests.

[1] $$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m\frac{\sigma_3}{\sigma_{ci}} + s\right)^{0.5}$$

The next version of *Phase²*, Version 6, will include the most up to date version of the Hoek-Brown Failure Criterion, known as the Generalized Hoek-Brown (GHB) model (Hoek et al. 2002). It is expressed as

[2] $$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a$$

where m_b is a reduced value of the material constant m_i, given by

[3] $$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right)$$

and s and a are constants for the rock mass given by the following relationships:

[4] $$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right)$$

[5] $$a = \frac{1}{2} + \frac{1}{6}\left(e^{-GSI/15} - e^{-20/3}\right).$$

Peak rock mass properties used in the GHB criterion can be estimated by determining the unconfined compressive strength of intact rock samples (σ_{ci}), the Geological Strength Index (GSI), the rock parameter m_i (which is a Hoek-Brown material constant for intact samples), and the Disturbance factor (D). The Rocscience programs *RocLab* or *RocData* are capable of estimating σ_{ci} and m_i by either fitting laboratory data or by using built-in charts of typical parameter ranges. The Geological Strength Index (Hoek et al. 1995) is a parameter that is used to help link the m and s parameters to field observations of the rock mass. It has replaced the Rock Mass Rating (RMR) that was created by Bieniawski (1976). The Disturbance factor, which varies from zero (e.g., excellent quality controlled blasting or excavation) to one (e.g., significant disturbance due to heavy production blasting), is used to capture damage due to blasting or stress relaxation, not due to failure. All of these parameters are used in Equations 3, 4, and 5 to determine m_b, s, and a, respectively. Note that one must be careful to realize that GSI, m_i, and D are index parameters that are used to determine the actual peak strength parameters m_b, s, and a.

Discussion of Post-Peak Parameters

During development of the newest version of *Phase²*, Rocscience asked some industry leaders, who are also users of *Phase²*, to help define what properties constitute residual strength in the Generalized Hoek-Brown model. The participants in the email discussion were: C. Carranza-Torres (Itasca Consulting Group Inc.), J. Carvalho (Golder Associates Ltd.), B. Corkum (Rocscience Inc.), M. Diederichs (Queen's University), E. Hoek (Evert Hoek Consulting Engineer Inc.), and D. Martin (University of Alberta). A summary of various communications amongst these experts follows. The comments have been sorted with respect to the various GHB parameters.

Unconfined Compressive Strength, σ_{ci}

This is essentially a "fixed" index parameter that is determined from intact rock specimens, used for normalization purposes. The idea of a residual value of this parameter does not make physical sense. However, this parameter has been altered to a residual parameter in one proposed numerical model that will be discussed later in this paper.

Hoek-Brown Parameter, m_b

This parameter (which may be considered similar to the friction angle in the Mohr-Coulomb criterion) should be allowed to change after the rock mass reaches failure by decreasing as the rock mass is subjected to internal shear. The amount of change is quite dependent on the rock mass and type of failure. For example, in a massive rock mass that fails in a brittle manner, the value of m_b should experience a large reduction, whereas very weak rock that behaves in a very plastic manner should experience very low or no reduction of m_b (essentially the rock mass is already at a residual state).

Hoek-Brown Parameter, s

This parameter is the "cohesive" component of the GHB criterion and is already a small number. This parameter should be allowed to decrease upon rock mass failure essentially to zero in order to decrease the value of the intact rock strength.

Hoek-Brown Parameter, *a*

This parameter essentially controls the curvature of the GHB failure envelope, especially at low confining stresses. Allowing for post-peak a-values would give users of *Phase*^{2} full parametric flexibility. Having a fixed value of $a = 0.5$ does not allow the strength to increase quickly enough with confinement for highly fractured rock masses.

Disturbance Factor, *D*

The disturbance factor should not be altered to a residual value as it used to calculate the peak rock mass parameters m_b and s. The value of this parameter is based on existing damage due to blasting or disturbance, not due to failure. The disturbance factor was originally introduced to capture damage due to blasting and / or stress relaxation particularly in very large open pit mine slopes, and hence experience using this parameter in underground mining applications is limited at this time.

Geological Strength Index, *GSI*

Similar to the disturbance factor, this parameter is used to establish peak rock mass parameters m_b, s, and a. It is a basic classification parameter resulting from field observations and should not be changed to reflect residual strength.

Hoek's Post-Peak Strength Guidelines

Dr. E. Hoek, in one of his email responses, presented the general guidelines, which he follows in practice. Note that these are his choices for residual parameters and are not suggested for use by other engineers given that a great deal of judgement is required on a job-specific basis. The guidelines are based on the rock type from massive brittle rocks of high GSI value through to very weak rock of low GSI.

1. Massive Brittle Rocks ($70 < GSI < 90$)
   - High stress resulting in intact rock failure
   - All strength lost at failure
   - $s_r = 0$, $m_r = 1$, and dilation = 0

2. Jointed Strong Rocks (50 < GSI < 65)
   - Moderate stress levels resulting in failure of joint systems
   - Rock fails to a 'gravel'
   - $s_r = 0$, $m_r = 15$, and dilation = 0.3(m_r)

3. Jointed Intermediate Rocks (40 < GSI < 50)
   - Weathered granite, schist, sandstone
   - Assume strain softening, loss of tensile strength, retains shear strength
   - $s_r = 0$, $m_r = 0.5(m_b)$, and dilation is small

4. Very Weak Rock (GSI < 30)
   - Severe tectonic shearing/folding (flysch, phyllite)
   - Elastic-perfectly plastic behaviour, no dilation – i.e. already at residual
   - $s_r = s$, $m_r = m_b$, and dilation = 0

Review of Current Research on Post-Peak Behaviour of Rock Masses

Implicit finite element stress analysis programs such as *Phase*^{2} apply peak rock mass properties according to the GHB criterion until failure is reached. As soon as the peak rock mass strength is exceeded, the properties of the rock instantly change to residual parameters in a brittle manner. It is more likely, however that the rock actually starts to "soften", and hence gradually change, rather than abruptly changing from peak to residual parameters. It is generally understood that softening describes the failure process of rock masses, which starts with micro-cracks, followed by crack nucleation, crack propagation and finally failure (Brace et al. 1966, Lockner et al. 1991). Figure 1 demonstrates that the true nature of a strain-softening response is not known for large field-scale rock masses. This is the thrust of ongoing research at the University of Toronto and of several other researchers at this time.

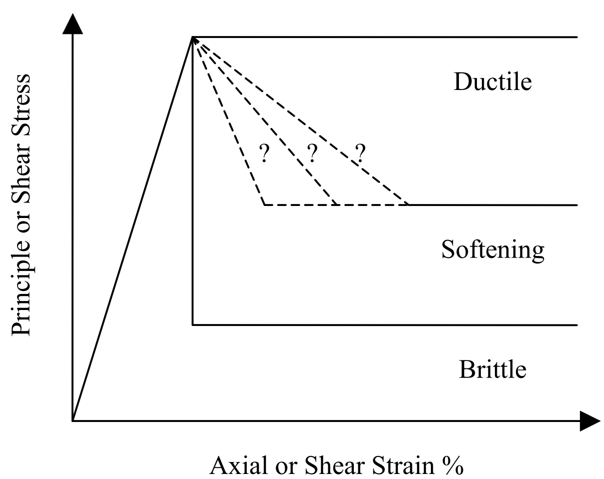

Figure 1. Possible failure mechanisms of rock masses.

The question is then, if peak rock mass parameters and criteria are well established and accepted, and in the field it is known that rock masses have failed and exist at high strains and low stresses, what is the transition between these two end-points? Cundall et al. (2003) states that material softening can be simulated by specifying that Hoek-Brown mechanical properties change and hence reduce the overall material strength. The following discussion introduces two technical papers and research at the University of Toronto that address the issue of post-peak behaviour.

Carranza-Torres et al. (2002) presents a solution to quantify the strength of intact and broken rock in terms of the Mohr-Coulomb yield criterion. The approach uses the "strength loss parameter, β" to quantify the jump of strength from the peak to the residual condition, as shown in Equation 6:

$$[6] \qquad \sigma_1 = [1 + (1-\beta)(K_\phi - 1)]\sigma_3 + (1-\beta)\sigma_{ci}$$

where the slope of the Mohr-Coulomb failure envelope in principle stress space (σ_1 vs. σ_3) is

$$[7] \qquad K_\phi = \frac{1+\sin\phi}{1-\sin\phi}$$

and ϕ is the internal angle of friction of the rock mass, expressed in radians. The strength loss parameter varies as $0 \le \beta \le 1$, such that

$$[8] \qquad \sigma_1 = K_\phi \sigma_3 + \sigma_{ci}, \qquad \text{for } \beta = 0 \text{ (no strength loss)},$$

$$[9] \qquad \sigma_1 = \sigma_3, \qquad \text{for } \beta = 1 \text{ (residual strength condition)},$$

The similar post-peak form of the Hoek-Brown failure criterion (Carranza-Torres 2004, Cundall et al. 2003) is:

$$[10] \qquad \sigma_1 = \sigma_3 + \sigma_{ci}^R \left(m_b^R \frac{\sigma_3}{\sigma_{ci}^R} + s \right)^a$$

where the residual, or post-peak, rock mass properties, $\sigma_{ci}{}^R$ and $m_b{}^R$, are defined as

$$[11] \qquad \sigma_{ci}^R = (1-\beta)\sigma_{ci}$$

$$[12] \qquad m_b^R = (1-\beta)m_b.$$

Substitute Equations 11 and 12 into 10,

$$[13] \qquad \sigma_1 = \sigma_3 + (1-\beta)\sigma_{ci} \left(m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a.$$

Again, the strength loss parameter varies as $0 \leq \beta \leq 1$. An example of how the β parameter affects the Hoek-Brown envelope is shown in Figure 2. The post-peak parameters defined in this model include a residual value for both σ_{ci} and m_b, which are obtained by multiplying the peak values by the factor (1-β). Both of the Hoek-Brown parameters s and a do not change when the rock mass fails. In this present formulation a post-peak value for the unconfined strength of intact rock specimens is inconsistent with the majority of comments from the industry leaders, as discussed earlier in this paper. As well, the industry leaders also suggested that post-peak values of the Hoek-Brown parameters *s* and *a* are desired. This is not the case in this model. Perhaps the method of how the strength loss factor, β, is applied should be rethought.

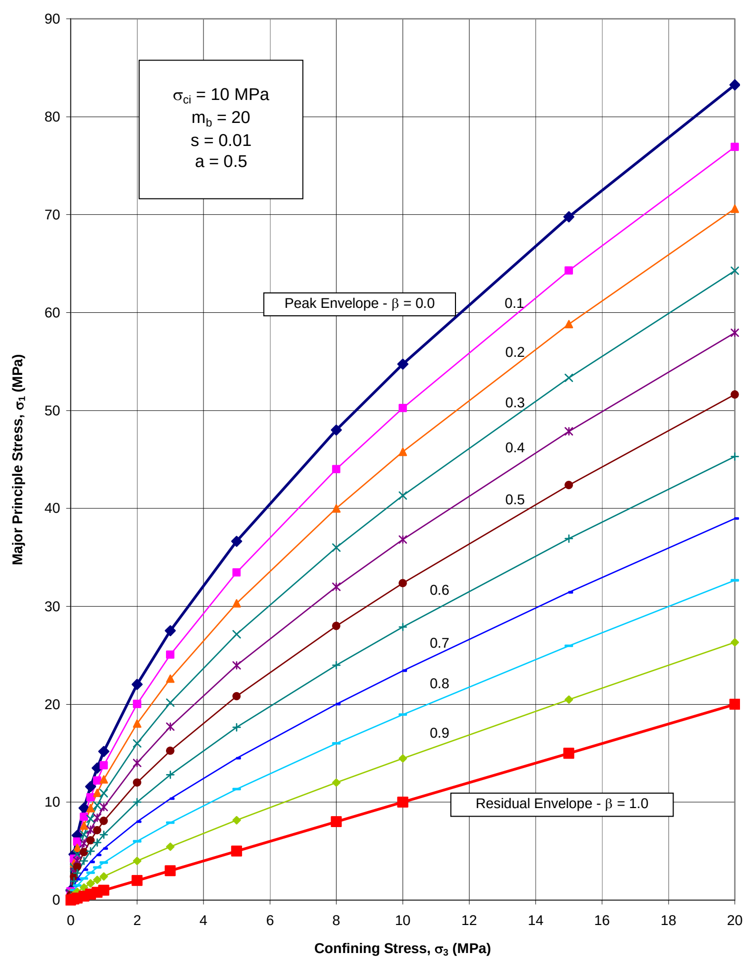

Figure 2. The effect of the β parameter on a given Hoek-Brown envelope. The original parameters of the rock mass are: σ_{ci} = 10 MPa, m_b = 20, s = 0.01, a = 0.5.

Carranza-Torres et al. (2002) also present a potential solution for the stress-strain softening behaviour of a rock mass after failure using a drop modulus parameter, μ. This parameter controls the slope of the softening branch of the stress strain curve, as shown in Figure 1. If $\mu = 0$ then the stress strain curve is brittle, and as μ approaches very high values, the material is ductile.

Cundall et al. (2003) presents another model based on the Hoek-Brown failure criterion. The authors essentially incorporate a three-part plasticity flow rule (a relation between the components of strain rate at failure) that varies as a function of confining stress. The model was developed for implementation into the explicit 2D and 3D finite difference codes, FLAC and FLAC^{3D} respectively (software products of HC Itasca).

The first flow rule is for very low confining stresses. In this stress range, rock masses are typically subjected to unconfined compression and hence exhibit large rates of volumetric expansion associated with axial splitting and wedging effects. At very high stresses, another flow rule is introduced to describe the point at which the rock mass no longer dilates during yield (i.e., a constant-volume flow rule). A composite flow rule was also created as an interpolation, according to the stress condition, between the two stresses previously mentioned.

The softening parameter in Cundall et al. (2003) is taken as the plastic strain, e_3^p. This parameter takes into consideration the theory presented in Carranza-Torres et al. (2002) (including the strength loss parameter, β) as well as a drop modulus, $2G\eta$ (where G is the shear modulus and η controls the slope of the softening curve). The parameter K_ψ is also introduced and is related to the instantaneous dilation angle ψ. The softening parameter, e_3^p, is essentially used to gradually reduce the value of β from zero (peak parameters), down to the post-peak value of β that occurs at a critical strain level. Again it must be stated, that this model applies β to the peak values of σ_{ci} and m_b, but not s or a, which is inconsistent with the industry leaders discussion.

Note that as well as allowing softening behaviour, FLAC also permits modelling of brittle behaviour by allowing the user to freely input both peak and post-peak values for the Hoek-Brown parameters σ_{ci}, m_b, s, and a. Parameters that are entered in FLAC are independent of each other and there is no limitation to values that can be assigned (Carranza-Torres 2004, Personal Communication).

Dr. Evert Hoek in his communication with Rocscience said that it is necessary to refine how post-failure characteristics are defined and that this can only be done “by back analyses of a large number of well documented case histories.” This type of approach into post-peak behaviour of field-scale rock masses is the subject of research at the University of Toronto. The approach involves matching field data with results from numerical analyses of stresses and strains in underground mining operations in order to create rock mass stress-strain curves and determine post-peak parameters.

The field data comes from monitoring the performance of mining reinforcement, which is critical in improving and optimizing reinforcement designs. One ground support element widely used in the mining industry is the cable bolt. The recently developed SMART (Stretch Measurement for Assessment of Reinforcement Tension) cables are instrumented to accurately assess the deformations that occur in the cable support elements due to deformation in the surrounding rock mass (Bawden and Lausch 2000). The MPBX (Multi-Point Borehole Extensometer) is a flexible six-point borehole extensometer with an integrated electronic readout head that measures the deformation of the rock mass itself. These two types of instruments then can provide direct readings of displacements of the rock mass in both the elastic and post-peak regimes.

The approach begins by creating three-dimensional boundary element models using the Rocscience boundary element program, *Examine*^{3D}. A plane, or cross-section, of stress is then analysed through the location of a SMART or MPBX instrument. The “far-field” elastic stresses computed by *Examine*^{3D} are then imported, along with the geometry, into *Phase*^{2} for plastic analysis. *Phase*^{2} allows the cable support to be added and analysed. Known peak (elastic) rock mass parameters are entered along with estimated post-peak parameters. The simulation is run and the calculated displacements are compared to those obtained in the field. If the displacements from the simulation and field vary considerably, then the post-peak parameters are varied until valid displacements are obtained. From this approach, the strains are known from the field, and the valid stresses are calculated, hence creating one point on a stress-strain curve in the post-peak region. As well, Hoek-Brown parameters for that point are also known. This whole process is then repeated at other given times for which displacements are known and new sets of stresses, strains, and post-peak parameters are obtained.

It is envisioned at this point, that if this type of analysis is completed for several different mines and stress regimes, that at least initially, empirical relationships may be formed for softening behaviour of large field-scale rock masses along with the appropriate degradation of the Hoek-Brown parameters.

Conclusions

In *Phase*^{2}, version 5, the Hoek-Brown m and s parameters were the only post-peak strength parameters that were available. It was clear from communications with the industrial leaders that the Geological Strength Index (*GSI*) and the Disturbance factor (*D*) are index parameters that are used to determine reasonable estimates of the peak strength parameters and should not enter into the determination of post-peak strength. Similarly, the unconfined compressive strength of intact rock samples, σ_{ci}, and Hoek-Brown parameter m_i are both properties of intact specimens determined from laboratory testing and hence should not be altered for determination of post-peak rock mass parameters. Thus, Rocscience has incorporated into *Phase*^{2} Version 6 (soon to be released) the option to enter peak and post-peak (elastic and plastic) values for m_b, s, and a.

Knowledge of post-peak behaviour is significant to several aspects of rock engineering design, including: rock mass support, stope design and mine sequencing, and prediction of underground opening behaviour under seismic loads. The ultimate aim of research at the University of Toronto is to explicitly model ground support and rock mass behaviour and to predict strains in fractured rock masses so that better designs can be implemented which will in due course increase safety and reduce rehabilitation costs.

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