Examine ***3D***

Energy Balance Utility

Developed for the
Canadian Rockburst Research Program

by

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Introduction

The purpose of this project is to develop the methodology for the calculation of energy components from a 3-D elastic analysis of excavations in an infinite media; and then incorporate this methodology into an Examine-3D utility.

Examine-3D is a three-dimensional boundary element program. It uses the direct boundary integral method, with 3 noded and 6 noded triangular conforming elements, to calculate elastic stresses and displacements around underground excavations in rock. Closed form integration techniques are used for singular and nearly singular influence functions, resulting in accurate displacements on, and close to, the mined surface. As will be seen, this is crucial for the accurate calculation of certain energy components.

Background

As a result of mining, a transition between two equilibrium states occurs. During this transition, a transfer of energy takes place within the rock mass. Work is done by the external (far field) forces, energy is stored within the surrounding rock mass, energy is given up by the mined material, work is done by all newly excavated faces and support elements, and energy is released. Each of these components is part of the energy balance first introduced by Cook (1967) and later refined by Salamon (1974,1984). This balance is defined as:

$$W + Um = Uc + Ws + Wr \qquad [1]$$

where,
- W = total work done by the external (far field) forces
- Um = stored strain energy in the material mined
- Uc = increase in stored strain energy in the surrounding rock mass
- Ws = increase in stored strain energy in the backfill and other support
- Wr = released energy

As well, the released energy is defined as follows:

$$Wr = Wk + Um \qquad [2]$$

where,
- Wk = The kinetic energy released into the rock mass

The kinetic energy is the energy released into the rock mass due to instantaneous excavation. This energy is expressed as a spherical stress wave that propagates away from the opening. According to Salamon (1984) and Brady and Brown (1985), it is equivalent to the work done by the induced displacements over the excavated surface. By substitution of [2] into [1], and assuming an unsupported excavation (Ws=0), the following equation results:

$$Uc = W - Wk \qquad [3]$$

Salamon (1984) presents a comprehensive discussion of each of the energy components and derives a set of basic mathematical definitions for each term. It is these definitions that are used to calculate the various energy components within Examine-3D. The following is a summary of the components as presented in Salamon (1984).

$$W = \int_S (T_i^{(p)} + \tfrac{1}{2} T_i^{(i)}) u_i^{(i)} dS \qquad [4]$$

$$W_k = \tfrac{1}{2} \int_{Sm} T_i^{(p)} u_i^{(i)} dS \qquad [5]$$

$$U_m = \tfrac{1}{2} \int_{Sm} T_i^{(p)} u_i^{(p)} dS \qquad [6]$$

where,

$T_i^{(p)}$ = primitive stress or traction vector acting on a surface

$T_i^{(i)}$ = induced stress or traction vector acting on a surface

$u_i^{(p)}$ = primitive displacement vector

$u_i^{(i)}$ = induced displacement vector

S = far field surface enclosing all excavations

Sm = Surface mined

The Wr and Uc components can be calculated using the relationships in [2] and [3]. More in depth discussions of energy changes as a result of mining are given in Salamon (1984), Brady and Brown (1985) and Jaeger and Cook (1976).

Results

Salamon (1984) and Hedley (1993) both derive a set of inter-relationships between the various energy components. As Hedley mentions in his report, the relationships developed by Salamon are incorrect. Hedley then proceeds to derive a new set of relationships based on the closed form energy component equations derived by Itasca for a circular tunnel under hydrostatic stresses. These equations where then compared to numerical results using FLAC. The results provided good agreement between the closed form solutions and the numerical results. But there still remains the question of whether these inter-relationships can be used for any mining geometry under any far field stress state.

To test the hypothesis that the energy inter-relationships developed by Hedley hold for all mining conditions, two simple closed form calculations were performed. The first, found in appendix A, calculates the energy components for a sphere under hydrostatic far field stresses. To make the calculations somewhat easier, the sphere is excavated in one step. The solution for W, Wk and Uc for a spherical cavity, can also be found in Brady and Brown (1985). The second solution, found in appendix B, involves a circular tunnel with a biaxial stress field. Both of these solutions have been tested and verified using Examine-3D, with the results being tabulated in appendix C. It is immediately clear from both solutions, that the energy inter-relationships given by Hedley, are not universally applicable. Hedley's equations (below) are correct for a circular tunnel in a hydrostatic stress field, but they do not apply to all mining situations. From these results, clearly, the inter-relationships depend on the geometry, and the far field stress state. As a result, these energy inter-relationships are dependent on the mining conditions. Given below is a table of the relationships for a circular tunnel and a spherical cavity in a hydrostatic stress field, and a circular tunnel in a biaxial stress field.

Circular Tunnel (Hedley 1993)*	Spherical Cavity*	Circular Tunnel Biaxial Loading A=K+1,B=K-1 K=stress ratio
$W = \frac{W_k + U_m}{1-\nu}$	$W = \frac{\frac{2}{3}(W_k + U_m)(1+\nu)}{1-\nu}$	$W = \frac{(A^2 + B^2(3-4\nu))(W_k + U_m)}{(1-\nu)(A^2 + 2B^2)}$
$U_c = \frac{\nu W_k + U_m}{1-\nu}$	$U_c = \frac{\frac{1}{3}(W_k + U_m)(1+\nu)}{(1-\nu)}$	$W = \frac{(A^2 + B^2(3-4\nu))(W_k + U_m)}{2(1-\nu)(A^2 + 2B^2)}$
$W_r = W(1-\nu)$	$W_r = \frac{W(1-\nu)}{\frac{2}{3}(1+\nu)}$	$W_r = \frac{W(1-\nu)(A^2 + 2B^2)}{(A^2 + B^2(3-4\nu))}$

* hydrostatic far field stresses

From the above discussion, it is clear that the circular tunnel energy inter-relationships can not be used to calculate the energy components for arbitrary three-dimensional mining geometry's. In Examine-3D, the Wk, Um, and thus the Wr energy components can be calculated directly from the induced displacements, far field stresses and volume of excavation. The energy components are calculated based on total excavation in one step. As a result, the strain energy in the material mined is due only to the strain caused by the far field stresses. Thus, if the volume mined is known, then Um can be calculated from the following equation for strain energy density in Jaeger and Cook (1976).

$$U_m = \frac{1}{2E}\left\{ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3\right) \right\} V_m \qquad [7]$$

where,

E = youngs's modulus
ν = poisson ratio
$\sigma_1, \sigma_2, \sigma_3$ = principal stresses
V_m = Volume mined

From equation [2], the released energy Wr can then be calculated. This leaves the calculation of W, the work done by the external (far field) forces, and Uc, the increase in strain energy in the rock mass. To calculate W within Examine-3D, an exterior boundary would have to be placed at a remote location to all excavations. Accurate displacements would then have to be calculated on this boundary and integrated over the surface of this boundary to yield the amount of work (see equation [4]) The problem with doing this is that the displacements tend to zero as you get farther from the excavations. Machine accuracy difficulties can create problems in accurately calculating displacements on this boundary. This is not an insurmountable problem, but one that might require modifications to Compute-3D. In the case of Uc, the boundary integral representation would still require this integration over the remote boundary. Otherwise, a volume discretization could be performed. The increase in strain energy density could then be calculated within the volume, and integrated over the volume to yield the increase in strain energy in the rock mass. Stresses would have to be calculated on a dense grid that extends a considerable amount from the openings. This would require a great deal of computational effort, while calculating the boundary integral would be much more efficient.

In the case of both a spherical opening and the circular tunnel, if the excavation is done in one step, then the following relationships exist between W,Wk, and Uc.

W = 2Wk [8]

Uc = Wk [9]

In the case of a sequential excavation, the energy transfer process is more complicated. Energy equations reported in Hedley (1993) for the excavation of an annulus of material around a circular tunnel, indicate that the energy relationships are not as simple as indicated by equations [8] and [9]. In the current implementation of the energy utility, the calculation of the energy balance is based on a one stage process. In view of this, the utility for calculating the energy components uses equation [8] and [9] for calculating W and Uc.

Conclusions

So far a good basis has been derived for the calculation of energy components through mining. Both theoretical and practical aspects of energy theory have been explored, and a simple model has been proposed for the calculation of energy changes from results obtained from Examine-3D. To further add to the value of the work presented here, the following items should be added to the energy calculation algorithm.

- Addition of an outer boundary for the calculation of total work done by external forces. This is required to check that validity of equations [8] and [9] for practical problems.
- Addition of staging for calculation of incremental energy changes.

These two enhancements would greatly add to both the theory and practical application of energy theory in mining. To make these additions, a period of two to four weeks would be required.

UTILITY MANUAL - PROGRAM ENCOMP

The utility, encomp, was written to compute the energy components associated with an Examine-3D analysis. The program requires the .ex3/.res files for an analysis. The ex3 file need not contain any stress planes or grids for calculation of stresses and displacements in the rock mass. By default, Compute-3D writes all the boundary displacement information needed for calculation of the energy components. The program calculates W,Wk,Wr,Uc,Um, and V, the excavated volume. The results are printed on the screen and written to a file with a .egy extension.

References

Cook, N.G.W., 1967. Design of Underground Excavations, 8th U.S. Rock Mech. Symp., Minnesota, pp 167-193.

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Salamon, M.D.G., 1984, Energy Considerations in Rock Mechanics: Fundamental Results, J.S.Afr. Inst. Min. Metal., Vol. 84, No. 8, pp 233-246.

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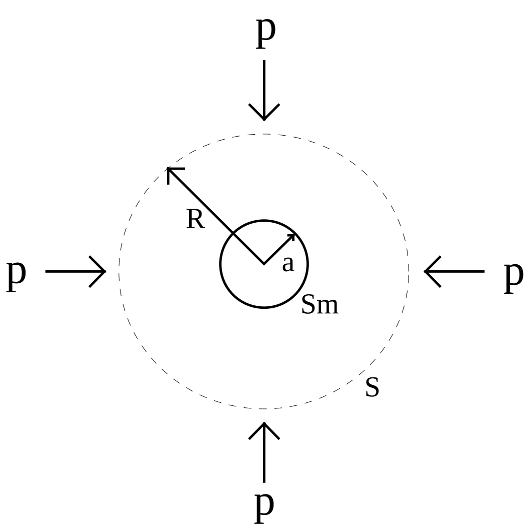
APPENDIX A

Analytical Solution of the Energy Components for the Excavation of a Spherical Cavity



E = young's modulus
ν = poisson ratio
V_m = volume mined = $\frac{4}{3}\pi a^3$

$$\sigma_{rr} = p\left(1 - \frac{a^3}{r^3}\right)$$

$$T_r^i = \sigma_r^i = -p\frac{a^3}{r^3} = \text{ induced stress / traction}$$

$$T_r^p = p = \text{ far field stress prior to excavation}$$

$$u_r = \frac{(1+\nu)pa^3}{2Er^2}$$

$$u_r^i = \frac{(1+\nu)pa^3}{2Er^2} \quad = \text{ induced displacement}$$

- Calculation of work done by external forces @ r=R

$$W = \int_S T_r^p u_r^i dS + \tfrac{1}{2}\int_S T_r^i u_i^i dS \qquad \text{(from Salamon 1984)}$$

$$= \int_S \frac{(1+\nu)p^2a^3}{2ER^2}dS + \tfrac{1}{2}\int_S \left(-\frac{pa^3}{R^3}\right)\left[\frac{(1+\nu)pa^3}{2ER^2}\right]dS$$

$$= \frac{4\pi R^2(1+\nu)p^2a^3}{2ER^2} - \frac{4\pi R^2 pa^3(1+\nu)pa^3}{2R^3ER^2}$$

$$= \frac{3(1+\nu)p^2}{4E}\left[2 - \frac{a^3}{R^3}\right]V_m$$

$$= \frac{3(1+\nu)p^2}{2E}V_m \qquad \text{as } R \to \infty$$

- Calculation of energy stored in material excavated

$$
\begin{aligned}
U_m &= \frac{1}{2E}\left\{ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3\right) \right\} V_m \\
&= \frac{1}{2E}\left\{3p^2 - 6\nu p^2\right\}V_m \\
&= \frac{3p^2}{2E}(1-2\nu)V_m
\end{aligned}
$$

- Calculation of kinetic energy

$$
\begin{aligned}
W_k &= \tfrac{1}{2}\int_{Sm} T_r^p u_r^i dS \\
&= \int_{Sm} \frac{(1+\nu)p^2 a}{4E} dS \\
&= \frac{4\pi a^2 (1+\nu)p^2 a}{4E} \\
&= \frac{3(1+\nu)p^2}{4E}V_m
\end{aligned}
$$

- Calculation of energy released

$$
\begin{aligned}
W_r &= W_k + U_m \\
&= \frac{3(1+\nu)p^2}{4E}V_m + \frac{3(1-2\nu)p^2}{2E}V_m \\
&= \frac{9(1-\nu)p^2}{4E}V_m
\end{aligned}
$$

- Calculation of energy in rock mass

$$
\begin{aligned}
U_c &= W + U_m - W_r \\
&= \frac{3(1+\nu)p^2}{2E}V_m + \frac{3(1-2\nu)p^2}{2E}V_m - \frac{9(1-\nu)p^2}{4E}V_m \\
&= \frac{3(1+\nu)p^2}{4E}V_m = W_k = \tfrac{1}{2}W
\end{aligned}
$$

- Calculation of W/Wr for a sphere

$$\frac{W}{W_r} = \frac{6(1+v)}{9(1-v)} = \frac{2(1+v)}{3(1-v)}$$

$$\therefore W = \frac{2/3\,(W_k + U_m)(1+v)}{(1-v)}$$

- Calculation of Uc=f(Wk,Um) for a sphere

$$\frac{U_c}{W_r} = \frac{3(1+v)}{9(1-v)} = \frac{(1+v)}{3(1-v)}$$

$$\therefore U_c = \frac{1/3\,(W_k + U_m)(1+v)}{(1-v)}$$

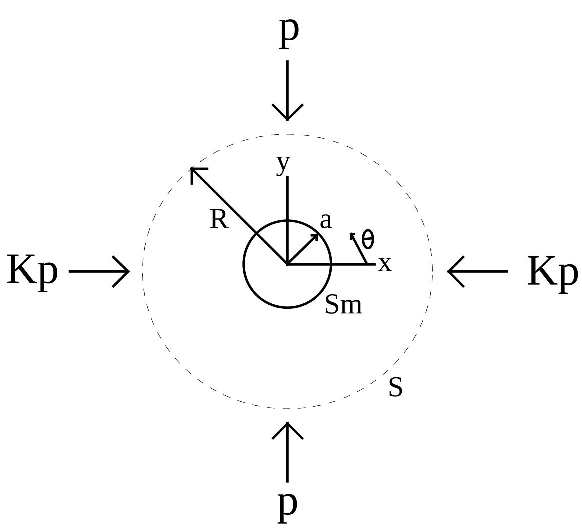
- Calculation of Wr=f(W) for a sphere

$$\frac{W}{W_r} = \frac{6(1+v)}{9(1-v)} = \frac{2(1+v)}{3(1-v)}$$

$$\therefore W_r = \frac{W(1-v)}{2/3\,(1+v)}$$

APPENDIX B

Analytical Solution of the Energy Components for the Excavation of a Circular Tunnel with Biaxial Loading



G = shear modulus = $E/2(1+\nu)$

E = young's modulus

ν = poisson ratio

A = K + 1

B = K - 1

V_m = volume mined / unit length = πa^2

$$u_r^i = -\frac{pa^2}{4Gr}\left[A + B \;\; 4(1-\nu) - \frac{a^2}{r^2} \;\; \cos 2\theta\right]$$

$$u_\theta^i = \frac{pa^2}{4Gr}\left[B\left\{2(1-2\nu) + \frac{a^2}{r^2}\right\}\sin 2\theta\right]$$

$$u_x^i = u_r\cos\theta - u_\theta\sin\theta$$

$$= -\frac{pa^2}{4Gr}\left[A + B\left\{4(1-\nu) - \frac{a^2}{r^2}\right\}\cos 2\theta\right]\cos\theta - \frac{pa^2}{4Gr}\left[B\left\{2(1-2\nu) + \frac{a^2}{r^2}\right\}\sin 2\theta\right]\sin\theta$$

$$u_y^i = u_r\sin\theta + u_\theta\cos\theta$$

$$= -\frac{pa^2}{4Gr}\left[A + B\left\{4(1-\nu) - \frac{a^2}{r^2}\right\}\cos 2\theta\right]\sin\theta + \frac{pa^2}{4Gr}\left[B\left\{2(1-2\nu) + \frac{a^2}{r^2}\right\}\sin 2\theta\right]\cos\theta$$

Tractions in cartesian xy space:

$$T_i^p = \left\{T_x^p \quad T_y^p\right\} = \sigma_{ij}^p n_j = \begin{bmatrix} Kp & 0 \\ 0 & p \end{bmatrix}\begin{Bmatrix} -\cos\theta \\ -\sin\theta \end{Bmatrix} = \left\{-Kp\cos\theta \quad -p\sin\theta\right\}$$

Tractions in polar rθ space:

$$T_i^p = \left\{T_r^p \quad T_\theta^p\right\} = \sigma_{ij}^p n_j = \begin{bmatrix} Kp\cos^2\theta + p\sin^2\theta & -Bp\sin\theta\cos\theta \\ -Bp\sin\theta\cos\theta & Kp\cos^2\theta - p\sin^2\theta \end{bmatrix}\begin{Bmatrix} -1 \\ 0 \end{Bmatrix}$$

$$= \left\{-Kp\cos^2\theta - p\sin^2\theta \quad Bp\sin\theta\cos\theta\right\} = \left\{-\frac{p}{2}\left(A + B\cos 2\theta\right) \quad \frac{p}{2}B\sin 2\theta\right\}$$

- Calculation of work done by external forces @ r=R

$$W = \tfrac{1}{2}\int_S T_i^p u_i^i dS \qquad ; \; dS = rd\theta$$

$$= \tfrac{1}{2}\int_S T_r^p u_r^i dS + \tfrac{1}{2}\int_S T_\theta^p u_\theta^i dS$$

$$\tfrac{1}{2}\int_S T_r^p u_r^i dS = 4\int_0^{\pi/2} \left[-\frac{p}{2}\left(A + B\cos 2\theta\right)\right]\left[-\frac{pa^2}{4GR}\left(A + B(4-4v-\tfrac{a^2}{R})\cos 2\theta\right)\right] R d\theta$$

$$= \frac{p^2 a^2}{2G}\left\{A^2\left(\tfrac{\pi}{2}\right) + B^2(4-4v-\tfrac{a^2}{R})\left(\tfrac{\pi}{4}\right)\right\} = \frac{p^2 V_m}{8G}\left\{2A^2 + 4B^2(1-v)\right\} \qquad ; \quad R \to \infty$$

$$\tfrac{1}{2}\int_S T_\theta^p u_\theta^i dS = 4\int_0^{\pi/2} \left[\frac{p}{2}\left(B\sin 2\theta\right)\right]\left[\frac{pa^2}{4GR}\left(B(2-4v+\tfrac{a^2}{R})\sin 2\theta\right)\right] R d\theta$$

$$= \frac{p^2 a^2}{2G}\left\{B^2(2-4v+\tfrac{a^2}{R})\left(\tfrac{\pi}{4}\right)\right\} = \frac{p^2 V_m}{8G}\left\{2B^2(1-2v)\right\}$$

$$\therefore W = \frac{p^2 V_m}{8G}\left\{2A^2 + 4B^2(1-v)\right\} + \frac{p^2 V_m}{8G}\left\{2B^2(1-2v)\right\} = \frac{p^2 V_m}{4G}\left\{A^2 + B^2(3-4v)\right\}$$

- Calculation of energy stored in material excavated

$$U_m = \frac{1}{2E}\left\{\ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2v\left(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3\right)\ \right\} V_m$$

$$\sigma_1 = Kp$$
$$\sigma_2 = p$$
$$\sigma_3 = v(\sigma_1 + \sigma_2) = vAp$$

$$= \frac{1}{2E}\left\{K^2 p^2 + p^2 + v^2 A^2 p^2 - 2v(vAp^2 + KAvp^2 + Kp^2\right\} V_m$$

$$= \frac{p^2 V_m}{8G}\left(A^2(1-2v) + B^2\right)$$

- Calculation of kinetic energy

$$
\begin{aligned}
W_k &= \tfrac{1}{2}\int_{Sm} T_i^{\,p} u_i^i dS \qquad ; \quad dS = r d\theta \\
&= \tfrac{1}{2}\int_{Sm} T_r^{\,p} u_r^i dS + \tfrac{1}{2}\int_{Sm} T_\theta^{\,p} u_\theta^i dS
\end{aligned}
$$

$$
\begin{aligned}
\tfrac{1}{2}\int_{Sm} T_r^{\,p} u_r^i dS &= 2\int_0^{\pi/2} \left[-\frac{p}{2}\left(A + B\cos 2\theta\right) \right] \left[-\frac{pa^2}{4G}\left(A + B(3-4v)\cos 2\theta\right) \right] d\theta \\
&= \frac{p^2 a^2}{4G}\left\{ A^2\left(\tfrac{\pi}{2}\right) + B^2 (3-4v)\left(\tfrac{\pi}{4}\right) \right\} = \frac{p^2 V_m}{8G}\left\{ A^2 + \frac{B^2}{2}(3-4v) \right\}
\end{aligned}
$$

$$
\begin{aligned}
\tfrac{1}{2}\int_{Sm} T_\theta^{\,p} u_\theta^i dS &= 2\int_0^{\pi/2} \left[\frac{p}{2}\left(B\sin 2\theta\right) \right] \left[\frac{pa^2}{4G}\left(B(3-4v)\sin 2\theta\right) \right] d\theta \\
&= \frac{p^2 a^2}{4G}\left\{ B^2 (3-4v)\left(\tfrac{\pi}{4}\right) \right\} = \frac{p^2 V_m}{8G}\left\{ \frac{B^2}{2}(3-4v) \right\}
\end{aligned}
$$

$$
\therefore W_k = \frac{p^2 V_m}{8G}\left\{ A^2 + \frac{B^2}{2}(3-4v) \right\} + \frac{p^2 V_m}{8G}\left\{ \frac{B^2}{2}(3-4v) \right\} = \frac{p^2 V_m}{8G}\left\{ A^2 + B^2 (3-4v) \right\}
$$

- Calculation of energy released

$$
\begin{aligned}
W_r &= W_k + U_m \\
&= \frac{p^2 V_m}{8G}\left\{ A^2 + B^2 (3-4v) \right\} + \frac{p^2 V_m}{8G}\left\{ A^2 (1-2v) + B^2 \right\} \\
&= \frac{p^2 V_m}{4G}\left\{ A^2 (1-v) + 2B^2 (1-v) \right\}
\end{aligned}
$$

- Calculation of energy in rock mass

$$
\begin{aligned}
U_c &= W - W_k \\
&= \frac{p^2 V_m}{4G}\left\{ A^2 + B^2 (3-4v) \right\} - \frac{p^2 V_m}{8G}\left\{ A^2 + B^2 (3-4v) \right\} \\
&= \frac{p^2 V_m}{8G}\left\{ A^2 + B^2 (3-4v) \right\} = W_k = \tfrac{1}{2} W
\end{aligned}
$$

- Calculation of W/Wr for a sphere

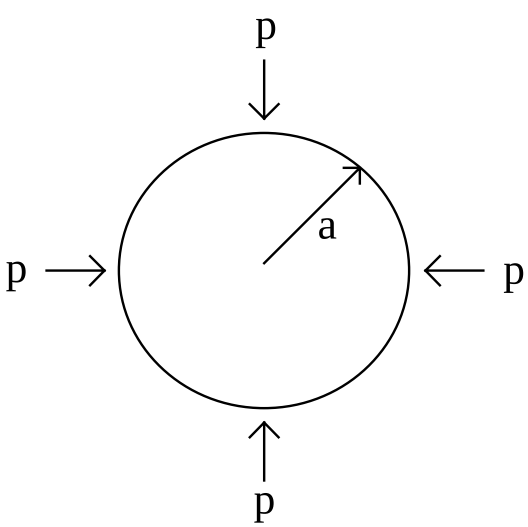
$$\frac{W}{W_r} = \frac{\frac{p^2 V_m}{4G}\left\{A^2 + B^2(3-4v)\right\}}{\frac{p^2 V_m}{4G}\left\{A^2(1-v) + 2B^2(1-v)\right\}} = \frac{A^2 + B^2(3-4v)}{(1-v)A^2 + 2B^2}$$

note: $W = \frac{1}{(1-v)}$; for $B = 0, K = 1$ or $v = 0.25$

APPENDIX C

Summary of Equations and Tabulated Results

Summary of Energy Equations for a Spherical Cavity



E = young's modulus

ν = poisson ratio

V_m = volume mined = $\frac{4}{3}\pi a^3$

$$W = \frac{3(1+\nu)p^2}{2E}V_m$$

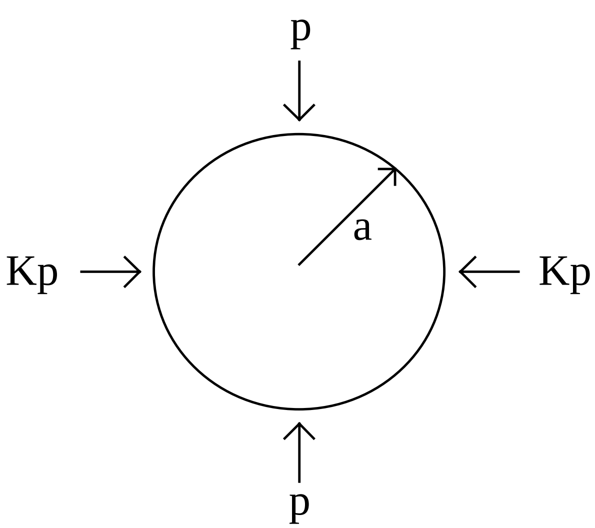
$$U_m = \frac{3(1-2\nu)p^2}{2E}V_m$$

$$U_c = \frac{3(1+\nu)p^2}{4E}V_m$$

$$W_r = \frac{9(1-\nu)p^2}{4E}V_m$$

$$W_k = \frac{3(1+\nu)p^2}{4E}V_m$$

Summary of Energy Components for a Circular Tunnel with Biaxial Loading



G = shear modulus = $E/2(1+\nu)$

E = young's modulus

ν = poisson ratio

A = K + 1

B = K - 1

V_m = volume mined / unit length = πa^2

$$W = \frac{p^2 V_m}{4G}\left\{ A^2 + B^2(3-4\nu)\right\}$$

$$U_m = \frac{p^2 V_m}{8G}\left\{ A^2(1-2\nu) + B^2\right\}$$

$$U_c = \frac{p^2 V_m}{8G}\left\{ A^2 + B^2(3-4\nu)\right\}$$

$$W_r = \frac{p^2 V_m}{4G}\left\{(1-\nu)(A^2 + 2B^2)\right\}$$

$$W_k = \frac{p^2 V_m}{8G}\left\{ A^2 + B^2(3-4\nu)\right\}$$

Comparison of Energy Components for a Spherical Cavity (in MJoules)

sphere with,
a =1.0 m
p=5.0 MPa
ν=0.25
E=10000 MPa

	Analytical Solution	Examine-3D
W	0.0196	0.0193
Um	0.00785	0.00779
Uc	0.00982	0.00963
Wr	0.0177	0.01741
Wk	0.00982	0.00963
W/Wr	1.111	1.106

note: Examine-3D calculates W using the relationships W=2Wk. W can also be calculated using the relationship W=pΔv , where Δv is the volumetric closure and p is the far hydrostatic stress. Examine-3D also calculates this relationship and for a spherical cavity W=0.0192 MJ.

Comparison of Energy Components for a Biaxial Loaded Cylinder (in MJoules)

cylinder with,
a =1.0 m
p=5.0 MPa
ν=0.25
E=10000 MPa
length=20.0 m

	K=0		K=1		K=2	
	Examine-3D	Analytical	Examine-3D	Analytical	Examine-3D	Analytical
W	0.287	0.294	0.384	0.393	1.06	1.08
Um	0.073	0.074	0.098	0.098	0.268	0.270
Uc	0.144	0.147	0.192	0.196	0.528	0.540
Wr	0.217	0.221	0.290	0.295	0.796	0.810
Wk	0.144	0.147	0.192	0.196	0.528	0.540
W/Wr	1.325	1.333	1.333	1.333	1.326	1.333
W/Wk	2.00	2.00	2.00	2.00	2.00	2.00

note: Examine-3D calculates W using the relationships W=2Wk.