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Technical Note

Estimating Mohr–Coulomb Friction and Cohesion Values from the Hoek–Brown Failure Criterion

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INTRODUCTION

Hoek and Brown [1–3] published a rock mass failure criterion and suggested that the values of the material constants m and s could be estimated from Bieniawski's rock mass rating (RMR) [4]. Since this is one of the few techniques available for estimating the rock mass strength from geological data, the criterion has been widely used in rock mechanics analyses.

Most of the analyses which are currently used for the evaluation of the stability of underground excavations or for slope stability calculations are formulated in terms of the Mohr–Coulomb failure criterion. Consequently, a question which frequently arises is how to determine equivalent values for the Mohr–Coulomb friction angle ϕ_i' and the cohesive strength c_i' from the tangent to the envelope to the principal stress circles defined by the Hoek–Brown criterion.

This Technical Note sets out the equations required to calculate these values for three conditions:

1. For a specified value of the effective normal stress σ_n', a condition which generally applies in slope stability analysis.
2. For a specified minor principal effective stress (σ_3') as would be appropriate in an analysis of stresses around underground openings.
3. For a condition in which the uniaxial compressive strength of the rock mass is the same for both Hoek–Brown and Mohr–Coulomb criteria, i.e. $\sigma_{cmasshb} = \sigma_{cmassmc}$. These conditions can be used when neither the average effective normal stress σ_n' nor the minor principal effective stress σ_3' are known.

Before giving the solutions for these three conditions, the basic Hoek–Brown criterion and the relation between the constants m and s and Bieniawski's RMR value will be reviewed.

ESTIMATING MATERIAL CONSTANTS FROM ROCK MASS RATING

The Hoek–Brown failure criterion is defined by the following equation [1,2,5]:

$$\sigma_1' = \sigma_3' + \sqrt{m\sigma_c\sigma_3' + s\sigma_c^2}, \tag{1}$$

where:

σ_1' = the major principal effective stress at failure,
σ_3' = the minor principal effective stress or confining pressure,
m, s = material constants,
σ_c = the uniaxial compressive strength of the *intact* rock.

Note that the uniaxial compressive strength of the intact rock refers to the strength which would be determined on a laboratory sized specimen (say a 50 mm dia × 100 mm long core) which is free from discontinuities such as joints or bedding planes. This value is a measure of the contribution of the rock material to the overall strength of the rock mass. The uniaxial compressive strength of the *rock mass* is given by substituting $\sigma_3' = 0$ into equation (1):

$$\sigma_{cmasshb} = \sqrt{s}\sigma_c. \tag{2}$$

Similarly, substituting $\sigma_1' = 0$ into equation (1) and solving the resulting quadratic equation gives the uniaxial tensile strength of the rock or rock mass as:

$$\sigma_{tmass} = \frac{\sigma_c}{2}(m - \sqrt{m^2 + 4s}). \tag{3}$$

In order to provide a basis for linking their criterion to measurements or observations which can be carried out in the field, Hoek and Brown [3] suggested a set of relations between the rock mass rating (RMR) from Bieniawski's [4] rock mass classification and the constants m and s. Following Priest and Brown [6], the relations were presented in the form of the following equations:

disturbed rock masses:

$$\frac{m}{m_i} = \exp\left(\frac{\text{RMR} - 100}{14}\right), \tag{4}$$

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$$s = \exp\left(\frac{\text{RMR} - 100}{6}\right); \quad (5)$$

undisturbed or interlocking rock masses:

$$\frac{m}{m_i} = \exp\left(\frac{\text{RMR} - 100}{28}\right), \quad (6)$$

$$s = \exp\left(\frac{\text{RMR} - 100}{9}\right); \quad (7)$$

where m_i is the value of m for the *intact* rock, determined from the results of triaxial tests [5]. When no laboratory test data are available, the value of m_i can be estimated from Table 1.

For users who would prefer to use the Tunnelling Quality Index Q from the NGI rock mass classification by Barton *et al.* [7] the RMR value can be calculated from the following relation proposed by Bieniawski [8]:

$$\text{RMR} = 9 \log_e Q + 44 \quad (8)$$

FIND c_i', ϕ_i', σ_{cmassmc} AND σ_{tmass} FOR A SPECIFIED VALUE OF THE NORMAL STRESS σ_n'

From Hoek and Brown [3], the calculation sequence is as follows:

$$h = 1 + \frac{16(m\sigma_n' + s\sigma_c)}{3m^2\sigma_c}, \quad (9)$$

$$\theta = \frac{1}{3}\left(90 + \arctan\frac{1}{\sqrt{h^3 - 1}}\right), \quad (10)$$

$$\phi_i' = \arctan\frac{1}{\sqrt{4h\cos^2\theta - 1}}, \quad (11)$$

$$\tau = (\cot\phi_i' - \cos\phi_i')\frac{m\sigma_c}{8}, \quad (12)$$

$$c_i' = \tau - \sigma_n' \tan\phi_i'. \quad (13)$$

The uniaxial compressive strength σ_{cmassmc} of rock mass with these ϕ_i' and c_i' values is given by:

$$\sigma_{\text{cmassmc}} = \frac{2c_i'\cos\phi_i'}{(1 - \sin\phi_i')}. \quad (14)$$

The uniaxial tensile strength of the rock mass is obtained from the Hoek–Brown criterion and is calculated from equation (3). This value can be used as a Mohr–Coulomb "tension cut-off".

FIND c_i', ϕ_i', σ_{cmassmc} AND σ_{tmass} FOR A GIVEN VALUE OF σ_3'

Assuming that the value of σ_3' is specified, the corresponding value of σ_1' for failure is calculated from equation (1). The remaining calculation follows:

$$\sigma_n' = \sigma_3' + \frac{(\sigma_1' - \sigma_3')^2}{2(\sigma_1' - \sigma_3') + \frac{1}{2}m\sigma_c}, \quad (15)$$

$$\tau = (\sigma_n' - \sigma_3')\sqrt{1 + \frac{m\sigma_c}{2(\sigma_1' - \sigma_3')}}, \quad (16)$$

$$\phi_i' = 90 - \arcsin\left(\frac{2\tau}{(\sigma_1' - \sigma_3')}\right). \quad (17)$$

The cohesive strength c_i' is calculated from equation (13) while the uniaxial compressive and tensile strengths of the rock mass are calculated from equations (14) and (3).

FIND c_i' AND ϕ_i' FOR A CONDITION IN WHICH THE UNIAXIAL COMPRESSIVE STRENGTH OF THE ROCK MASS IS THE SAME FOR BOTH THE MOHR–COULOMB AND HOEK–BROWN FAILURE CRITERION

The uniaxial compressive strength of the rock mass is the same for both criteria, i.e. $\sigma_{\text{cmassmc}} = \sigma_{\text{cmasshb}}$, and is given by equation (2). The rest of the calculation is as follows:

$$\sigma_n' = \frac{2s\sigma_c}{4\sqrt{s} + m}, \quad (18)$$

$$\tau = \sigma_n'\sqrt{1 + \frac{m}{2\sqrt{s}}}, \quad (19)$$

$$\phi_i' = 90 - \arcsin\left(\frac{2\tau}{\sqrt{s}\sigma_c}\right). \quad (20)$$

The cohesive strength c_i' is calculated from equation (13).

EXAMPLE CALCULATIONS

The following examples are given to illustrate the application of the solutions presented earlier and to provide users with a set of values against which to check their own calculations.

Assume a sandstone rock mass in which the uniaxial compressive strength of the intact rock material is

Table 1. Approximate values of m_i for different rock types

Carbonate rocks with well-developed crystal cleavage (dolomite, limestone and marble):	$m_i = 7$
Lithifield argillaceous rocks [mudstone, shale and slate (normal to cleavage)]:	$m_i = 10$
Arenaceous rocks with strong crystals and poorly-developed crystal cleavage (sandstone and quartzite):	$m_i = 15$
Fine grained polymineriallic igneous crystalline rocks (andesite, dolerite, diabase and rhyotite):	$m_i = 17$
Coarse grained polymineriallic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite, norite and granodiorite):	$m_i = 25$

$\sigma_c = 60$ MPa. From Table 1, the value of the material constant $m_i = 15$. A classification of the rock mass using the Tunnelling Quality Index by Barton *et al.* [7] gives a value of $Q = 0.8$. Equation (8) gives an equivalent rock mass rating of RMR = 42.

A slope has been excavated into this sandstone rock mass and additional excavations are to be created in the rock mass close to the slope. Under these circumstances it can be assumed that the stress levels in the rock mass have been reduced by excavation of the slope and that some movement along discontinuities in the rock mass will have occurred. Consequently, equations (4) and (5) for a *disturbed* rock mass will be used to calculate the constants m and s.

From equation (4), $m = 0.238$ and, from equation (5), $s = 0.000063$. Substitution of these values into equations (2) and (3) give the uniaxial compressive and uniaxial tensile strengths of the *rock mass* as $\sigma_{cmasshb} = 0.476$ MPa and $\sigma_{tmass} = -0.0159$ MPa.

Solution for a specified normal effective stress σ'_n

Assume an effective normal stress $\sigma'_n = 0.5$ MPa. Substitution of this value into equations (9–13) gives the following values:

$$h = 1.1927,$$
$$\theta = 46.7174°,$$
$$\phi'_i = 41.896°,$$
$$\tau = 0.6610 \text{ MPa},$$
$$c'_i = 0.2124 \text{ MPa}.$$

The uniaxial compressive strength of the rock mass corresponding to the linear failure envelope defined by $\phi'_i = 41.896°$ and $c'_i = 0.2124$ MPa is given by equation (14) as $\sigma_{cmassmc} = 0.9519$ MPa. The uniaxial tensile strength of the rock mass is given by equation (3) as $\sigma_{tmass} = -0.0159$ MPa.

Solution for a specified minor principal effective stress σ'_3

Assume a minor principal effective stress $\sigma'_3 =$ 0.25 MPa. The corresponding value of the major principal effective stress at failure is given by equation (1) as $\sigma_1 = 2.1985$ MPa.

Substitution of these values into equations (15–17) give $\sigma'_n = 0.5940$, $\tau = 0.7429$ MPa, $\phi'_i = 40.31°$ and $c'_i = 0.239$ MPa.

The uniaxial compressive strength of the rock mass corresponding to this linear failure envelope is given by equation (14) as $\sigma_{cmassmc} = 1.0322$ MPa. The uniaxial tensile strength of the rock mass is given by equation (3) as $\sigma_{tmass} = -0.0159$ MPa.

Solution for a the condition in which the uniaxial compressive strength of the rock mass is the same for the Hoek–Brown and Mohr–Coulomb criteria

The uniaxial compressive and uniaxial tensile strengths of the *rock mass* are $\sigma_{cmassmc} = \sigma_{cmasshb} = 0.476$ MPa and $\sigma_{tmass} = -0.0159$ MPa, from equations (2) and (3).

Substitution of the uniaxial compressive strength of the *intact* rock, $\sigma_c = 60$ MPa, and the values of $m = 0.238$ and $s = 0.000063$ into equations (18–20) give $\sigma'_n = 0.028$ MPa, $\tau = 0.112$ MPa and $\phi'_i = 61.94°$. The corresponding value of $c'_i = 0.059$ MPa, from equation (14).

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