1 Introduction

This paper documents the calculations used in *Swedge* to determine the safety factor of wedges formed on slopes. This involves the following series of steps:

1. Determine the wedge geometry using block theory (Goodman and Shi, 1985).
2. Determine all of the individual forces acting on a wedge, and then calculate the resultant active and passive force vectors for the wedge (section 3).
3. Determine the sliding direction of the wedge (section 4).
4. Determine the normal forces on each wedge plane (section 5).
5. Compute the resisting forces due to joint shear strength (section 6).
6. Calculate the safety factor (section 7).

2 Wedge Geometry

The orientations of 2 distinct joint planes must always be defined for an *Swedge* analysis. Using block theory, *Swedge* determines if a removable wedge can be formed by the intersection of the 2 joint planes and the slope face. An optional tension crack can also be included in the analysis.

The method used for determining the wedges is described in the text by Goodman and Shi, "Block Theory and Its Application to Rock Engineering" (1985).

In general, the wedges which are formed are tetrahedral in nature (i.e. the 2 joint planes make up 2 sides of a tetrahedron, and the third and fourth sides are formed by the slope and upper faces). If a tension crack is included in the analysis, the tetrahedron will be truncated by the tension crack plane and in general a five-sided wedge will result.

When the wedge coordinates have been determined, the geometrical properties of a wedge can be calculated, including:

- Wedge volume
- Wedge face areas
- Normal vectors for each wedge plane

3 Wedge Forces

All forces on the wedge can be classified as either Active or Passive. In general, Active forces represent driving forces in the safety factor calculation, whereas Passive forces represent resisting forces.

The individual force vectors are computed for each quantity (e.g. wedge weight, bolt force, water force etc), and then the resultant Active and Passive force vectors are determined by a vector summation of the individual forces.

3.1 Active Force Vector

The resultant Active force vector is comprised of the following components:

$$A = W + C + X + U + E + B_a$$

$A =$ resultant active force vector
$W =$ wedge weight vector
$C =$ external force vector
$X =$ active pressure force vector
$U =$ water force vector
$E =$ seismic force vector
$B_a =$ active bolt force vector

3.1.1 Wedge Weight Vector

The wedge weight is usually the primary driving force in the analysis.

$$W = (\gamma_r V) \bullet \hat{g}$$

$W =$ wedge weight vector
$\gamma_r =$ unit weight of rock
$\mathrm{V} =$ wedge volume
$\hat{g} =$ gravity direction

3.1.2 External Force Vector

User-defined external loads can be added on the slope face or upper face.

$$C = \sum_{i=1}^{n} c_i$$

$C =$ total external force vector
$c_i =$ individual external force vectors

3.1.3 Pressure Force (Active) Vector

Pressure force is applied with the **Pressure** option in the Support menu, and can be defined as either active or passive.

$$X = p_s a_s \hat{n}_s + p_u a_u \hat{n}_u$$

$X =$ resultant active pressure force vector
$p_s =$ pressure magnitude on the slope face
$a_s =$ area of the wedge intersection on the slope face
$\hat{n}_s =$ unit direction corresponding to the trend/plunge of the slope pressure
$p_u =$ pressure magnitude on the upper face
$a_u =$ area of the wedge intersection on the upper face
$\hat{n}_u =$ unit direction corresponding to the trend/plunge of the upper face pressure

3.1.4 Water Force Vector

In *Swedge* there are two different methods for defining the existence of water pressure on the joint planes – Constant or Gravitational.

3.1.4.1 Constant pressure on each joint

$$U = \sum_{i=1}^{3} u_i a_i \hat{n}_i$$

$U =$ resultant water force vector
$u_i =$ water pressure on the i^{th} joint face
$a_i =$ area of the i^{th} joint face
$\hat{n}_i =$ inward (into wedge) normal of i^{th} joint face

3.1.4.2 Gravitational pressure on each joint

In *Swedge* it is assumed that the water table is a planar surface which is oriented parallel with the upper face. The line of intersection of the water table and the slope surface is a line which is parallel with the crest of the slope. The elevation of the water table surface varies between the toe of the wedge on the slope surface (0 percent filled) and the elevation of the upper face (100 percent filled). *Swedge* allows you to define the elevation by letting you define a percent filled variable between 0 and 100.

The water pressure is assumed to be 0 along all edges of the wedge that lie on the slope or upper face. In the case of no tension crack, the maximum pressure lies at a point midway along the line of intersection of the two joint planes. This midpoint is half-way between the toe of the wedge and the point at which the line of intersection daylights on the upper face. The magnitude of the pressure is determined based on the vertical distance from the midpoint to where the line of intersection intersects the upper face. The maximum water pressure has a value of $\frac{1}{2}\gamma_w H_w$. H_w is the vertical distance between the wedge toe and the point at which the line of intersection daylights on the upper face. The actual pressure distribution on each face is a tetrahedron with volume Ab/3 where A is the face area of the slip plane and b is the maximum water pressure. In the case of a tension crack, the maximum pressure is at the base of the tension crack. The magnitude of the pressure is based on the distance from the base of the tension crack to the upper face.

No Tension Crack:

$$U = \frac{1}{6}\sum_{i=1}^{2} p^3 \gamma_w H_w a_i \hat{n}_i$$

U = resultant water force vector

p = proportion filled = percent filled/100

γ_w = unit weight of water

H_w = vertical height of the line of intersection of joint 1 and 2

a_i = area of the i^{th} joint face

$\hat{n}_i$ = inward (into wedge) normal of i^{th} joint face

With Tension Crack:

$$U = \frac{1}{3}\sum_{i=1}^{3} p^3 \gamma_w H_w a_i \hat{n}_i$$

U = resultant water force vector
p = proportion filled = percent filled/100
γ_w = unit weight of water
H_w = heigth of upper face above base of tension crack
a_i = area of the ith joint face
$\hat{n}_i$ = inward (into wedge) normal of ith joint face

3.1.5 Seismic Force Vector

This determines the seismic force vector if the **Seismic** option is applied. The trend and plunge of the direction of the seismic force is first converted to a vector in cartesian coordinates.

$$E = (k\gamma_r V) \bullet \hat{e}$$

E = seismic force vector
k = seismic coefficient
γ_r = unit weight of rock
V = wedge volume
$\hat{e}$ = direction of seismic force

3.1.6 Active Bolt Force Vector

Swedge contains a very simple bolt model. The bolt force is taken as the capacity of the bolt with a direction defined by the user (trend/plunge). The force is assumed to go through the centroid of the wedge. The user must determine the capacity based on the tensile strength of the bolt, bond strength, and plate capacity. The efficiency of the bolt depending on its orientation is not accounted for.

$$B_a = \sum_{i=1}^{n} c_i \hat{e}_i$$

c_i = capacity of the ith bolt
$\hat{e}_i$ = unit direction vector of the ith bolt

3.2 Passive Force Vector

The resultant Passive Force Vector is the sum of the bolt, shotcrete and pressure (passive) support force vectors.

$$P = H + Y + B_p$$

P = resultant passive force vector
H = shotcrete shear resistance force vector
Y = passive pressure force vector
B_p = resultant passive bolt force vector

3.2.1 Pressure Force (Passive) Vector

Pressure force is applied with the **Pressure** option in the Support menu, and can be defined as either active or passive.

$$Y = p_s a_s \hat{n}_s + p_u a_u \hat{n}_u$$

Y = resultant passive pressure force vector
p_s = pressure magnitude on the slope face
a_s = area of the wedge intersection on the slope face
$\hat{n}_s$ = unit direction corresponding to the trend/plunge of the slope pressure
p_u = pressure magnitude on the upper face
a_u = area of the wedge intersection on the upper face
$\hat{n}_u$ = unit direction corresponding to the trend/plunge of the upper face pressure

3.2.2 Shotcrete Shear Resistance

The shotcrete support model in *Swedge* assumes that the shotcrete fails by punching shear. Basically the wedge shears through the shotcrete on the slope face. The shear zones are along the line of intersections of joint 1 and joint 2 on the slope face. If you want to account for adhesion failure coupled with bending failure, then you will have to factor the shear strength accordingly.

$$Y = (L_1 + L_2) t \tau_s \hat{n}_s$$

$L_1 =$ trace length of joint 1 on slope face
$L_2 =$ trace length of joint 2 on slope face
$t =$ shotcrete thickness
$\tau_s =$ shotcrete shear strength
$\hat{n}_s =$ slope face unit normal pointing into slope

3.2.3 Passive Bolt Force Vector

Swedge contains a very simple bolt model. The bolt force is taken as the capacity of the bolt with a direction defined by the user (trend/plunge). The force is assumed to go through the centroid of the wedge. The user must determine the capacity based on the tensile strength of the bolt, bond strength, and plate capacity. The efficiency of the bolt depending on its orientation is not accounted for.

$$B_p = \sum_{i=1}^{n} c_i \hat{e}_i$$

$c_i =$ capacity of the i^{th} bolt
$\hat{e}_i =$ unit direction vector of the i^{th} bolt

4 Sliding Direction

Next, the sliding direction of the wedge must be determined. The sliding (deformation) direction is computed by considering active forces only (**A** vector). Passive forces (**P** vector) DO NOT influence sliding direction.

The calculation algorithm is based on the method presented in chapter 9 of "Block Theory and its application to rock engineering", by Goodman and Shi (1985).

In *Swedge*, there are 4 possible directions ($\hat{s}_0, \hat{s}_1, \hat{s}_2, \hat{s}_{12}$). These represent the modes of: falling / lifting ($\hat{s}_0$), sliding on a single joint plane ($\hat{s}_1, \hat{s}_2$), or sliding along the line of intersection of two joint planes ($\hat{s}_{12}$).

Calculation of the sliding direction is a two step process: 1) compute all possible sliding directions and 2) determine which one of the possible sliding directions is the actual valid direction.

4.1 Step 1 – Compute list of 4 possible sliding directions

4.1.1 Falling (or Lifting)

$$\hat{s}_0 = \hat{a} = A / \|A\|$$

$\hat{s}_0$ = falling or lifting direction
$\hat{a}$ = unit direction of the resultant active force
A = active force vector

4.1.2 Sliding on a single face i

$$\hat{s}_i = \frac{(\hat{n}_i \times A) \times \hat{n}_i}{\|(\hat{n}_i \times A) \times \hat{n}_i\|}$$

$\hat{s}_i$ = sliding direction on joint i
$\hat{n}_i$ = normal to joint face i directed into wedge
A = active force vector

4.1.3 Sliding on two faces i and j

$$\hat{s}_{ij} = \frac{\hat{n}_i \times \hat{n}_j}{\left\| \hat{n}_i \times \hat{n}_j \right\|} sign\left(\left(\hat{n}_i \times \hat{n}_j \right) \bullet A \right)$$

$\hat{s}_{ij}$ = sliding direction on joint i and j (along line of intersection)

$\hat{n}_i$ = normal to joint face i directed into wedge

$\hat{n}_j$ = normal to joint face j directed into wedge

A = active force vector

4.2 Step 2 – Compute which of the possible sliding directions is valid

For the following 8 tests, whichever satisfies the given inequalities is the sliding direction of the wedge. If none of these tests satisfies the given inequalities, the wedge is unconditionally stable.

4.2.1 Falling wedge

$$A \bullet \hat{n}_1 > 0$$
$$A \bullet \hat{n}_2 > 0$$
$$A \bullet W \geq 0$$

A = resultant active force vector

$\hat{n}_i$ = inward normal to joint i

W = weight vector

4.2.2 Lifting wedge

$$A \bullet \hat{n}_1 > 0$$
$$A \bullet \hat{n}_2 > 0$$
$$A \bullet W < 0$$

4.2.3 Sliding on joint 1

$$A \bullet \hat{n}_1 \leq 0$$
$$\hat{s}_1 \bullet \hat{n}_2 > 0$$

4.2.4 Sliding on joint 2

$$A \bullet \hat{n}_2 \leq 0$$
$$\hat{s}_2 \bullet \hat{n}_1 > 0$$

4.2.5 Sliding on the intersection of joint 1 and joint 2

$$\hat{s}_1 \bullet \hat{n}_2 \leq 0$$
$$\hat{s}_2 \bullet \hat{n}_1 \leq 0$$

5 Normal Force

The calculation of the normal forces on each of the two joint planes for a wedge first requires the calculation of the sliding direction. Once the sliding direction is known, the following equations are used to determine the normal forces given a resultant force vector, F. The force vector, F, is generally either the active or the passive resultant force vector.

5.1 Falling or lifting wedge

$$N_1 = 0$$
$$N_2 = 0$$

$N_i =$ normal force on the i^{th} joint

5.2 Sliding on joint 1

$$N_1 = -F \bullet \hat{n}_1$$
$$N_2 = 0$$

$N_i =$ normal force on the ith joint
$F =$ force vector
$\hat{n}_1 =$ inward (into wedge) normal of joint plane 1

5.3 Sliding on joint 2

$$N_1 = 0$$
$$N_2 = -F \bullet \hat{n}_2$$

$N_i =$ normal force on the ith joint
$F =$ force vector
$\hat{n}_2 =$ inward (into wedge) normal of joint plane 2

5.4 Sliding on joints 1 and 2

$$N_1 = -\frac{(F \times \hat{n}_2) \bullet (\hat{n}_1 \times \hat{n}_2)}{(\hat{n}_1 \times \hat{n}_2) \bullet (\hat{n}_1 \times \hat{n}_2)}$$
$$N_2 = -\frac{(F \times \hat{n}_1) \bullet (\hat{n}_2 \times \hat{n}_1)}{(\hat{n}_2 \times \hat{n}_1) \bullet (\hat{n}_2 \times \hat{n}_1)}$$

$N_i =$ normal force on the ith joint
$F =$ force vector
$\hat{n}_1 =$ inward (into wedge) normal of joint plane 1
$\hat{n}_2 =$ inward (into wedge) normal of joint plane 2

6 Shear Strength

There are 3 joint strength models available in *Swedge*: 1) Mohr-Coulomb, 2) Barton-Bandis, and 3) Power Curve.

Shear strength is computed based on the normal stress acting on each joint plane. The normal stress is computed based on the active and passive normal forces computed on the joint planes using the equations in the previous section.

6.1 Compute normal stress on each joint

First compute the stress on each joint plane based on the normal forces computed in section 5.

$$\sigma_{n_i} = N_i / a_i$$

σ_{n_i} = normal stress on the ith joint

N_i = normal force on the ith joint

a_i = area of the ith joint

6.2 Compute shear strength of each joint

Use the strength criteria defined for the joint, and the normal stress, to compute the shear strength.

6.2.1 Mohr-Coulomb Strength Criterion

$$\tau_i = c_i + \sigma_{n_i} \tan \phi_i$$

τ_i = shear strength of the ith joint

c_i = cohesion of the ith joint

σ_{n_i} = normal stress on the ith joint

ϕ_i = friction angle of the ith joint

6.2.2 Barton-Bandis Strength Criterion

$$\tau_i = \sigma_{n_i} \tan\left[JRC_i \log_{10}\left(\frac{JCS_i}{\sigma_{n_i}}\right) + \phi_{r_i} \right]$$

$\tau_i =$ shear strength of the ith joint

$JRC_i =$ joint roughness coefficient of the ith joint

$JCS_i =$ joint compressive strength of the ith joint

$\sigma_{n_i} =$ normal stress on the ith joint

$\phi_{r_i} =$ residual friction angle of the ith joint

6.2.3 Power Curve Strength Criterion

$$\tau_i = c_i + a_i \left(\sigma_{n_i} + d_i\right)^{b_i}$$

$\tau_i =$ shear strength of the ith joint

$a_i, b_i, c_i, d_i =$ strength parameters of the ith joint

$\sigma_{n_i} =$ normal stress on the ith joint

6.3 Compute resisting force due to shear strength

Force acts in a direction opposite to the direction of sliding (deformation).

$$J_i = \tau_i a_i \cos\theta_i$$

$J_i =$ magnitude of the resisting force due to the shear strength of joint i

$\tau_i =$ shear strength of the ith joint

$a_i =$ area of the ith joint

$\theta_i =$ angle between the sliding direction and the ith joint

7 Factor of Safety

Swedge computes 3 separate factors of safety:

i. falling factor of safety
ii. unsupported factor of safety
iii. supported factor of safety

The reported factor of safety is the maximum of the above three factors of safety. The logic of this is simple - support is assumed to never decrease the factor of safety from the unsupported value. The factor of safety can never be less than if the wedge was falling with only support to stabilize it.

The equations are based on 2 joint planes making up a tetrahedral wedge.

The limit equilibrium safety factor calculations only consider force equilibrium in the direction of sliding. Moment equilibrium is not considered.

$$F = \text{Factor of Safety} = \max(F_f, F_u, F_s)$$

$$F_f = \text{Falling factor of safety}$$
$$F_u = \text{Unsupported factor of safety}$$
$$F_s = \text{Supported factor of safety}$$

7.1 Factor of safety definition

$$\text{Factor of safety} = \frac{\text{resisting forces (e.g. shear/tensile strength, support)}}{\text{driving forces (e.g. weight, seismic, water)}}$$

7.2 Falling factor of safety

The falling factor of safety assumes that only passive support and tensile strength act to resist movement. Basically the wedge is assumed to be falling so no influence of the joint planes (shear strength, failure direction) is incorporated. Driving forces are due to the active forces on the wedge as defined in section 3.1. The falling direction is calculated from the direction of the active force vector.

$$F_f = \frac{-P \bullet \hat{s}_0}{A \bullet \hat{s}_0}$$

F_f = Falling factor of safety
P = resultant passive force vector (section 3.2)
A = resultant active force vector (section 3.1)
$\hat{s}_0$ = falling direction (section 4.1.1)

7.3 Unsupported factor of safety

The unsupported factor of safety assumes that shear strength acts to resist movement. No passive support force is used.

Driving forces are due to the active forces on the wedge as defined in section 3.1. The sliding direction is calculated from the equations in section 4. The shear strength is calculated based on the normal forces from the active force vector only. Normal forces from the passive force vector are not included.

$$F_u = \frac{\sum_{i=1}^{2} J_i^u}{A \bullet \hat{s}}$$

F_u = Unsupported factor of safety
A = resultant active force vector (section 3.1)
J_i^u = magnitude of the resisting force due to the unsupported shear strength of joint i (section 6.3)
$\hat{s}$ = sliding direction (section 4)

7.4 Supported factor of safety

The supported factor of safety assumes that passive support forces and shear and tensile strength act to resist movement.

Driving forces are due to the active forces on the wedge as defined in section 3.1. The sliding direction is calculated from the equations in section 4. The shear strength is calculated based on the normal force calculated from the active force vector plus the passive force vector.

$$F_s = \frac{-P \bullet \hat{s} + \sum_{i=1}^{2} J_i^s}{A \bullet \hat{s}}$$

F_s = Supported factor of safety
P = resultant passive force vector (section 3.2)
A = resultant active force vector (section 3.1)
J_i^s = magnitude of the resisting force due to the supported shear strength of joint i (section 6.3)
$\hat{s}$ = sliding direction (section 4)