Unwedge

Geometry and Stability Analysis of Underground Wedges

Sample Problems

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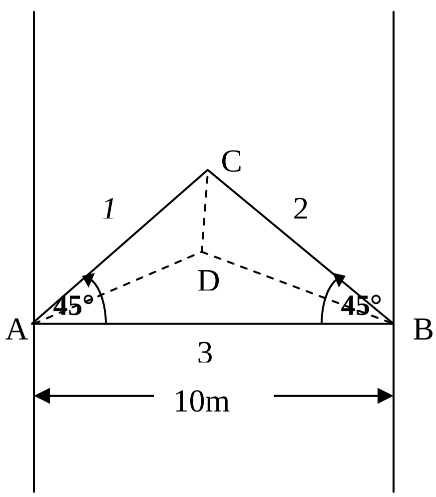
UNWEDGE SAMPLE PROBLEM #1

Calculate the weight of the maximum wedge formed in the roof of a 10m wide square excavation. The tunnel axis has a plunge/trend of 0/0 degrees. The three planes have the following orientation:

Plane	Dip	Dip Direction
1	70	315
2	50	45
3	60	180

Using *Unwedge*, verify this value.

SOLUTION:



Set up Cartesian Coordinate System:

A(0,0,0); B(10, 0, 0); C(5, 5, 0)

Calculate D by intersecting the 3 planes:

⇒ we need the equations of 3 planes

using the plane equation, and noting that (u_1, v_1, w_1) is the normal vector to plane 1;

plane 1: $u_1x + v_1y + w_1z + d_1 = 0$
plane 2: $u_2x + v_2y + w_2z + d_2 = 0$
plane 3: $u_3x + v_3y + w_3z + d_3 = 0$

find the normal vector to each of the planes using the following equations:
$u = \sin(\text{dip})\bullet\sin(\text{dip direction})$
$v = \sin(\text{dip})\bullet\cos(\text{dip direction})$
$w = \cos(\text{dip})$

Plane 1
$u_1 = \sin(70)\bullet\sin(315) = -0.66446$
$v_1 = \sin(70)\bullet\cos(315) = 0.66446$
$w_1 = \cos(70) = 0.34202$

Plane 2
$u_2 = \sin(50)\bullet\sin(45) = 0.541675$
$v_2 = \sin(50)\bullet\cos(45) = 0.541675$
$w_2 = \cos(50) = 0.642788$

Plane 3

u_3 = sin(60)•sin(180) = 0
v_3 = sin(60)•cos(180)= -0.866
w_3 = cos(60) = 0.5

$d_1 = d_3 = 0$ since planes 1 and 3 pass through the origin

find d_2:
$d_2 = -u_2x - v_2y - w_2z$
substituting in point B into the equation yields:
d_2 = -(0.542)(10)
$\Rightarrow d_2 = -5.42$

$\therefore$ we have 3 equations and 3 unknowns, and are now able to solve the system

$$-0.664x + 0.664y + 0.342z = 0$$
$$0.542x + 0.542y + 0.643z = 5.42$$
$$0x - 0.866y + 0.500z = 0$$

$\Rightarrow$ at point D:
x = 3.825; y = -2.021; z = -3.501

$$Volume = \frac{1}{3}A \bullet h = \frac{1}{3}\left(\frac{1}{2}(10)(5)\right)(3.501) = 29.177m^3$$
$$Weight = \gamma V = 0.026MN/m^3 \bullet 29.177m^3 = 0.758602MN$$
$$\Rightarrow Weight = 75.9tonnes$$

Using *Unwedge*, the maximum weight is 75.861 tonnes.

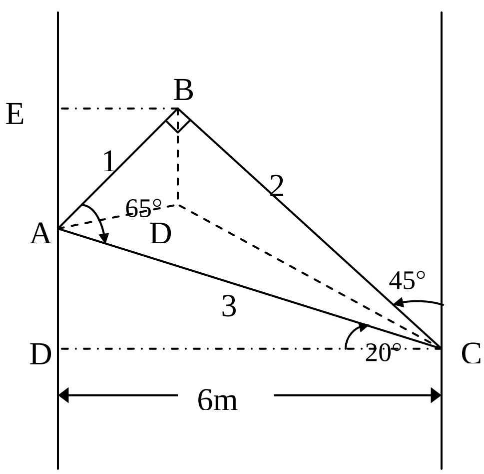
UNWEDGE SAMPLE PROBLEM #2

a) Calculate the maximum volume and weight of a tetrahedral rock wedge formed in the roof of a 6 meter wide square tunnel. The tunnel axis has a plunge/trend of 0/0 degrees. The three planes have the following orientation:

Plane	Dip	Dip Direction
1	80	315
2	65	45
3	40	200

Assume a rock unit weight of 2.6tonnes/m^3. Use *Unwedge* to verify the calculated value.

SOLUTION:



Set up Cartesian Coordinate System

A(0, 0, 0); B(?, ?, 0); C(6, ?, 0)

AC: cos(20)=6/CA ⇒ CA=6.385
AD: tan(20)=AD/6 ⇒ AD=2.184
AB: cos(65)=AB/AC ⇒ AB=2.698
EA: cos(45)=EA/AB ⇒ EA=1.908 = EB

⇒ A(0, 0, 0); B(1.908, 1.908, 0);
C(6, -2.184, 0);

Plane 1

u_1 = sin(80)•sin(315) = -0.696
v_1 = sin(80)•cos(315) = 0.696
w_1 = cos(80) = 0.174

Plane 2

u_2 = sin(65)•sin(45) = 0.641
v_2 = sin(65)•cos(45) = 0.641
w_2 = cos(65) = 0.423

Plane 3

u_3 = sin(40)•sin(200) = -0.220

v_3 = sin(40)•cos(200) = -0.604
w_3 = cos(40) = 0.766

$d_1 = d_3 = 0$, find d_2:

$$d_2 = -0.641x - 0.641y - 0.423z = 0.641(1.908) - 0.641(1.908)$$
$$\Rightarrow d_2 = -2.446$$

Solving the linear system:

$$-0.696x + 0.696y + 0.174z = 0$$
$$0.641x + 0.641y + 0.423z = 2.446$$
$$-0.220x - 0.604y + 0.766z = 0$$
$\Rightarrow$ D(1.611, 1.249, 1.448)

$$Volume = \frac{1}{3}A \bullet h = \frac{1}{3}\left(\frac{1}{2}AB \bullet BC\right)(z_D) = \frac{1}{6}(2.698)(\sqrt{6.385^2 - 2.698^2})(1.448)$$
$$\Rightarrow V = 3.77m^3$$

$$W = \gamma V = (2.6tonnes/m^3)(3.77m^3) = 9.80tonnes$$

Using *Unwedge*, the maximum weight is 9.799 tonnes

b) If the capacity of a mechanically anchored rock bolt is 10 tonnes, determine the number of bolts required to achieve a factor of safety of 2.0.

Since the maximum weight is 9.8 tonnes, and the capacity of the rock bolt is 10 tonnes, to achieve a factor of safety of 2 requires the use of 2 bolts. This can be confirmed with *Unwedge* by adding 2 spot bolts to the roof wedge and reading the corresponding factor of safety.

UNWEDGE SAMPLE PROBLEM #3

a) An 8 meter wide square tunnel is to be constructed as part of a hydroelectric installation in southern Ontario. The tunnel axis has a plunge of 0° and a trend of 120°. The geologist's field surveys have determined the existence of three major joint planes having the following orientations:

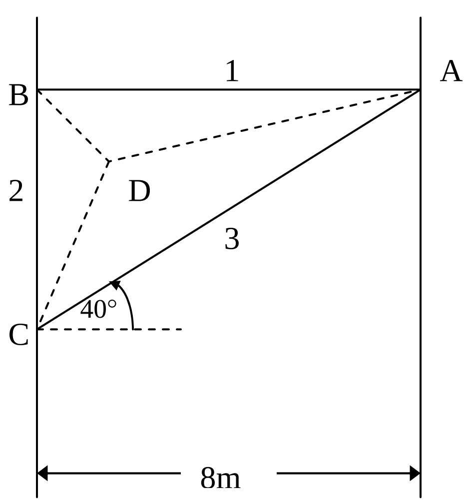
Plane	Dip	Dip Direction
1	30	120
2	25	30
3	50	260

Calculate the maximum volume and weight of a tetrahedral rock wedge formed in the roof of the tunnel, assuming a rock unit weight of 2.7 tonnes/m^3. Verify this value using *Unwedge.*

SOLUTION:

Instead of evaluating the tunnel oriented at 120°, rotate the tunnel to a trend of 0°. To translate the dip direction of the individual planes, subtract 120° from each.

Plane	Dip	Dip Direction	Altered
1	30	120	0
2	25	30	270
3	50	260	140



Set up a Cartesian Coordinate System:

BC: tan(50)=8/BC⇒ BC=6.713

A(8, 6.713, 0); B(0, 6.713, 0); C(0, 0, 0)

Plane 1

$u_1 = \sin(30) \bullet \sin(0) = 0$

$v_1 = \sin(30) \bullet \cos(0) = 0.5$

$w_1 = \cos(30) = 0.866$

Plane 2

$u_2 = \sin(25) \bullet \sin(270) = -0.423$

$v_2 = \sin(25) \bullet \cos(270) = 0$

$w_2 = \cos(25) = 0.906$

Plane 3

$u_3 = \sin(50) \bullet \sin(140) = 0.492$

$v_3 = \sin(50) \bullet \cos(140) = -0.587$

$w_3 = \cos(50) = 0.643$

$d_2 = d_3 = 0$

$find\ d_1 :$

$d_1 = 0 - 0.5y - 0.866z = -0.5(6.713)$

$\Rightarrow d_1 = -3.36$

Solving the resulting system of equations:

$$\begin{aligned} 0x + \quad 0.5y + 0.866z &= 3.36 \\ -0.423x + \quad 0y + 0.906z &= 0 \\ 0.492x - 0.587y + 0.643z &= 0 \end{aligned}$$

$\Rightarrow$D(3.11, 4.20, 1.45)

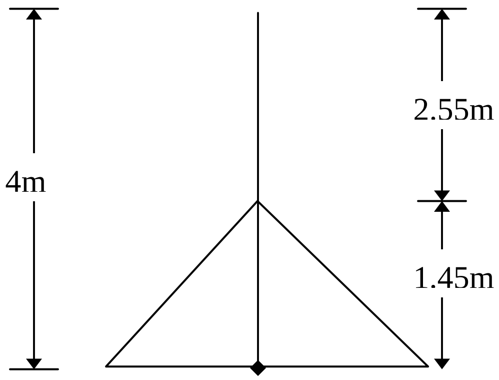
$$Volume = \frac{1}{3} A \bullet h = \frac{1}{3}\left(\frac{1}{2} BC \bullet AB\right)(z_D) = \frac{1}{6}(6.713)(8)(1.45) = 12.98m^3$$

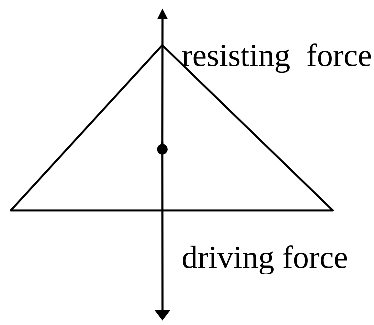
$$Weight = \gamma V = 2.7 tonnes / m^3 \bullet 12.98m^3 = 35.05 tonnes$$

Using *Unwedge*, the maximum weight is 35.061 tonnes.

b) A single, 4 meter long, fully grouted cable bolt is to be installed vertically through the base of the wedge so as to pass through its apex. The cable bond strength is 15 tonnes/m, and faceplates are not to be used. Will the roof wedge fall? If so, how does the bolt fail?

SOLUTION:





$$driving\ \ forces = W = 35.05 tonnes$$

$$resisting\ \ forces = 1.45m \bullet 15\frac{t}{m} = 21.75 tonnes$$

$$factor\ of\ \ safety = \frac{resisting\ \ forces}{driving\ \ forces} = \frac{21.75}{35.05}$$

$$\Rightarrow FS = 0.621$$

$\therefore$ Yes, the roof wedge will fail as its factor of safety is 0.621. Since there is no face plate on the bolt, and there is a greater length of bolt imbedded in the surrounding rock than there is in the wedge, the wedge shears off the grout on the bolt, leaving the complete bolt in the roof.

c) If, instead of a cable, a layer of shotcrete is added to the roof of the excavation, what thickness is required for a factor of safety of 2? The shear strength of the shotcrete is 200 tonnes/m^2. Confirm this number with Unwedge using shotcrete of unit weight 1 tonne/m^3.

SOLUTION:

$$factor\ of\ safety = \frac{resisting\ force}{driving\ force} = \frac{resisting\ force}{Weight\ of\ wedge} = \frac{resisting\ force}{35.05} = 2$$

$$\Rightarrow resisting\ force = 70.1 tonnes$$

$$resisting\ force = strength = 70.1 = 200\frac{t}{m^2} \bullet l \bullet x$$

$$l = 8 + \frac{8}{\tan(50)} + \frac{8}{\sin(50)} = 25.15m$$

$$x = \frac{70.1}{200 \bullet 25.15} = 0.014m$$

$$\Rightarrow x = 1.4cm$$

Therefore, for a factor of safety of 2, a 1.4cm layer of shotcrete should be added to the roof of the excavation.

Using Unwedge to verify this number, a layer of shotcrete 1.4cm thick is added to the roof of the excavation and results in a factor of safety of 1.988.

a) An underground tunnel (figure 4.1) is to be excavated as part of a hydropower installation. Geological mapping in nearby access tunnels have revealed a series of joint planes that could cause structural failure in both the roof and sidewalls of the tunnel. The geological mapping results are given in table 4.1. Using *Unwedge* and the results from mapping, determine the maximum sized wedges that can form in the roof and sidewalls of the tunnel. What are their corresponding factors of safety?

Assume c= 1.0 tonnes/m^3 and ϕ=35° for all joints.

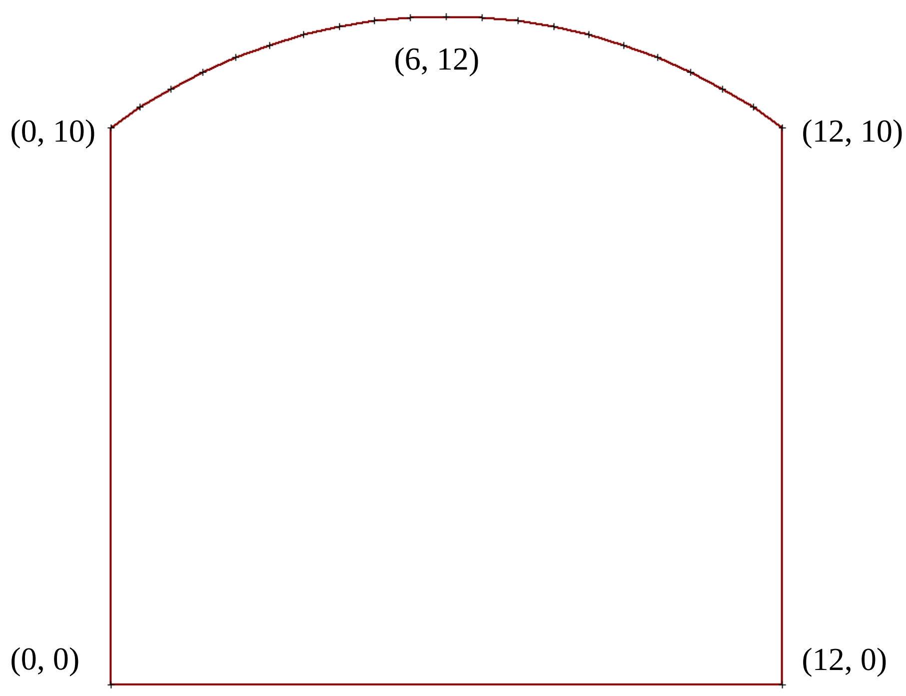


Figure 4.1: Tunnel Cross Section

Plane	Dip	Dip Direction
1	70	300
2	50	60
3	60	180

Table 4.1: Orientation of Joint Planes

SOLUTION:

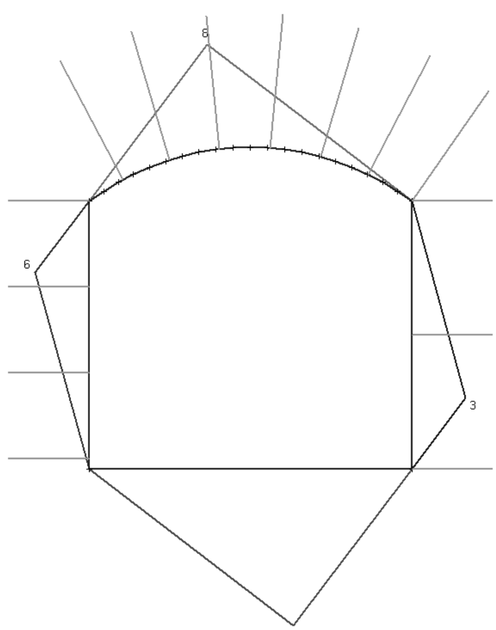
Wedge	Weight (tonnes)	Factor of Safety
floor	315.431	stable
lower right	47.924	1.82
upper left	47.924	1.001
roof	104.078	0
near end	151.375	1.691
far end	151.375	0.713

b) A square pattern of cable bolts is to be used to support the walls and ceiling of the underground tunnel. The cable bolts have a tensile capacity of 25 tonnes, plate capacity of 10 tonnes, and bond strength of 35 tonnes. Taking the individual wedge sizes into consideration, design an economic bolt pattern with a safety factor of 2.

SOLUTION:

Since the apex height of wall and ceiling wedges differ, three different patterns and bolt lengths are used to ensure an economic design.

Wedge					
#	Location	Apex Height (m)	Length	Spacing	New FS
8	roof	3.93	5	1.9x1.9	2.139
6	upper left	2.01	3	3.2x3.2	2.040
3	lower right	2.01	3	5x5	2.098



A spiral ramp is to be constructed at an underground mine to provide access between levels. To meet the size specifications of the mine, the ramp must have an inclination of 15 degrees. Structural mapping of the joints (Table 5.1) in the adjacent drifts has indicated that there are three major persistent and fairly rough joints. Analyze the excavation (Figure 5.1) at 30 different trend values to determine, by plot, the excavation trend that produces the failure wedge (FS = 0) of maximum weight. What is this weight?

Plane	Dip	Dip Direction
1	85	310
2	70	45
3	45	190

Table 5.1: Joint Orientation

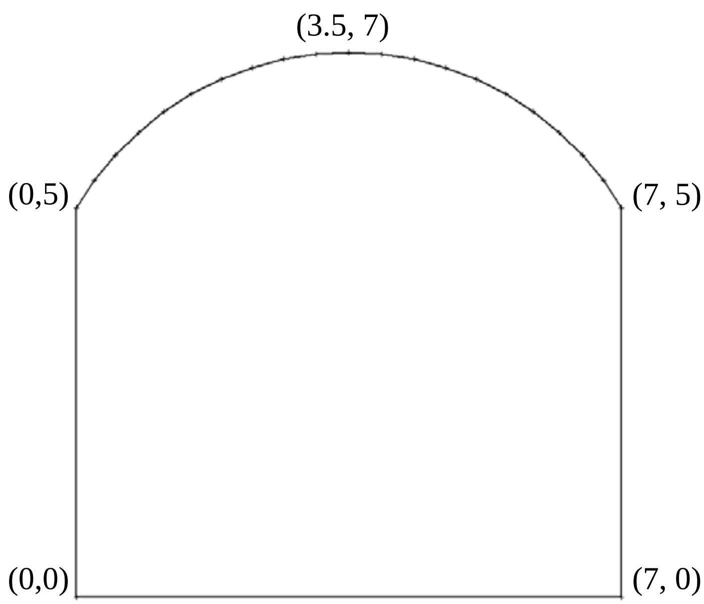
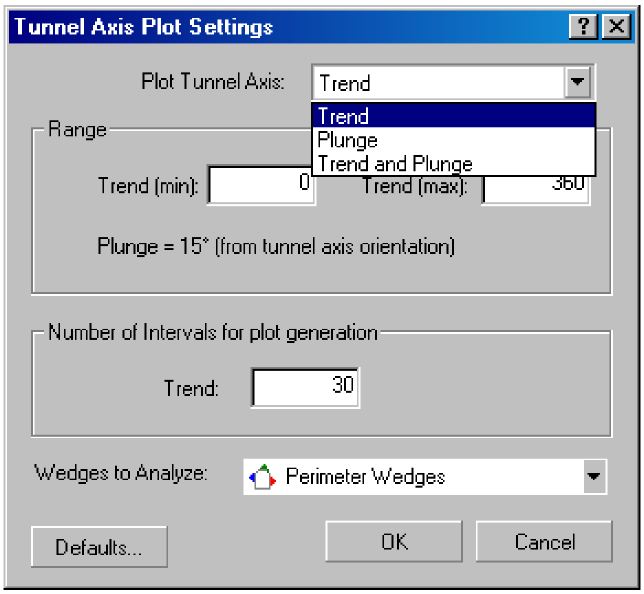


Figure 5.1: Ramp Dimensions

SOLUTION:

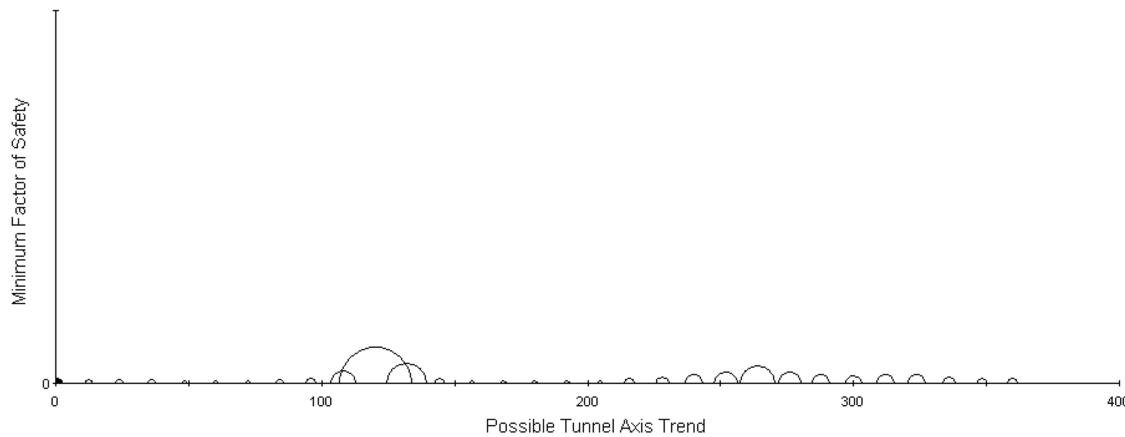
Using *Unwedge* to analyze the excavation, we can evaluate the wedges formed at twelve increments of 30° (total of 360°). First, the joint properties must be input, ensuring that the Tunnel Axis Orientation menu on the sidebar has a Plunge of 15° and a trend of 0°. By selecting "Tunnel Axis Plot Settings" in the analysis menu and choosing only the trend of the excavation, graphs capable of plotting various data results.



Tunnel Axis Plot Input

This analysis will compute the weight and factor of safety of each wedge formed at 30 intervals of 360°, as is required when designing a spiral ramp.

Selecting "Minimum Factor of Safety" as the ordinate, and "Maximum Wedge Weight" as the bubble size, the following graph is created:



Optimization for Tunnel Axis Plunge = 15°

As seen on the graph, the maximum wedge weight occurs when the tunnel is oriented at a trend of 120°. When 120° is used as the trend of the tunnel axis orientation, the weight of the wedge with FS = 0 is 114.088 tonnes.

UNWEDGE SAMPLE PROBLEM #6

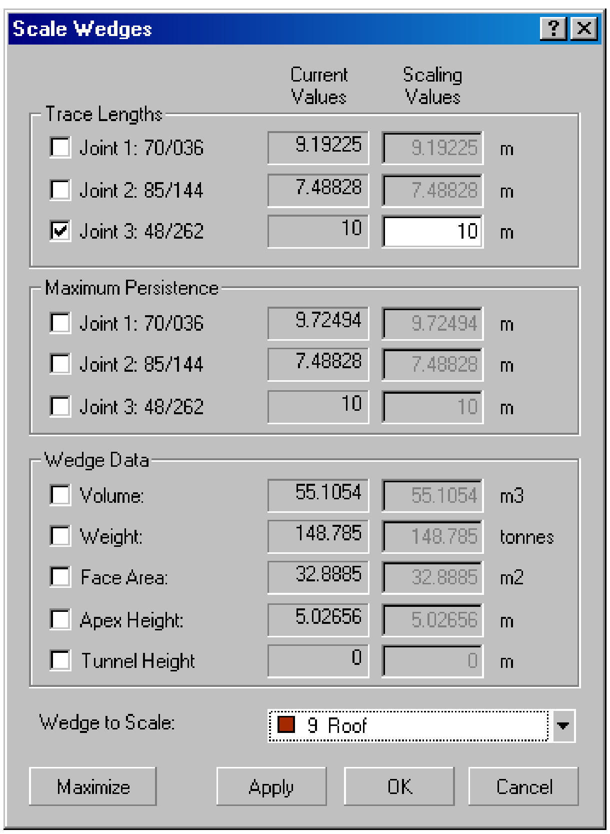
A drawpoint is to be excavated at the base of a stope to allow for the recovery of broken ore. During mapping of the surrounding tunnels, it was observed that the largest trace length underground was 10 meters in length. As miners are constantly at work in the opening, and the repercussions of a wedge failure in this area are detrimental, the design factor of safety for a drawpoint should be 2.0.

Using *Unwedge* and the *drawplan* example file, determine the weight of the roof wedge formed above the drawpoint.

SOLUTION:

Open the drawplan.weg example file to view the structural planes surrounding the drawpoint.

The largest trace length observed underground was 10m in length, but since *Unwedge* computes the maximum wedges formed by the given joints, the wedges should be scaled to make an accurate analysis.



Scaling the roof wedge to the largest trace length

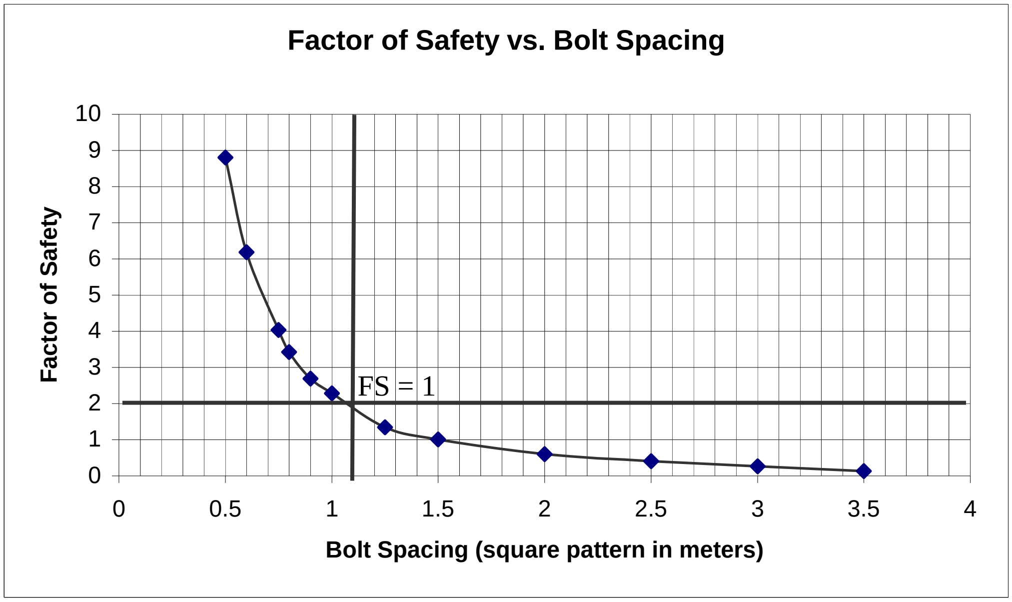
As Joint 3 has the largest trace length before scaling, it is scaled to the largest trace length observed (10m). This is repeated for each wedge, scaling the joint of largest length down to 10m.

When these changes are applied, it can be seen that the weight of the roof wedge is 148.785 tonnes.

b) Design a safe and economical square bolt pattern to support the roof of the drawpoint. The bolts available have a capacity of 10 tonnes, and are only available in 1 meter increments (3m, 4m, etc). To ensure the simple bolt does not fail, there should be at least 1m of bolt imbedded in the surrounding rock. Determine by plot which bolt spacing will give the required factor of safety.

The maximum apex height of the roof wedge is 5.03m. Since we need 1 meter in the surrounding rock at all times, and the bolts are available in increments of 1 meter, a bolt of 7 meters length is chosen.

Finding the factor of safety at different square spacings, the following graph is generated.



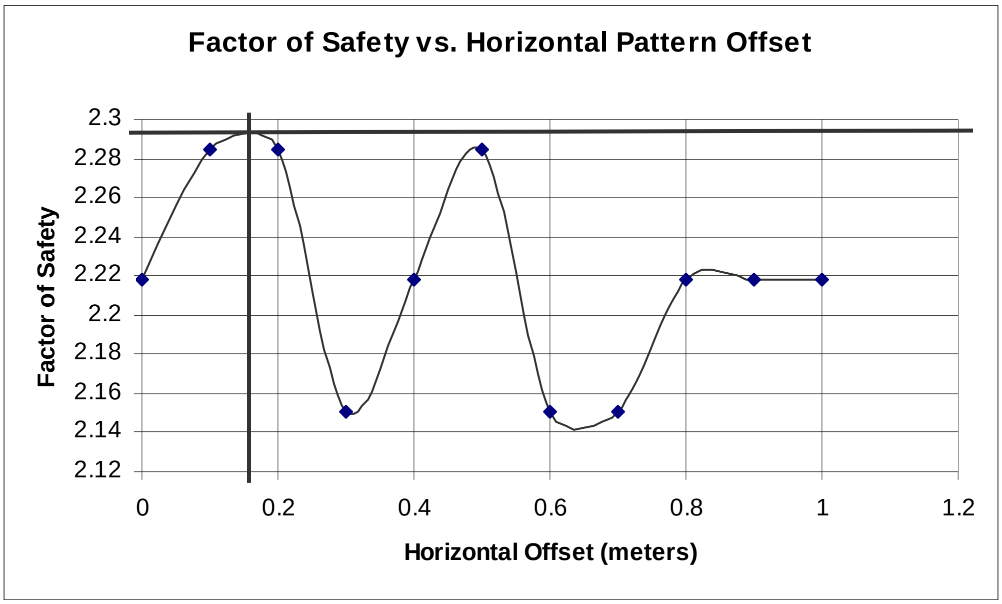
FS vs. Bolt Spacing: Bolt length of 7m

From the graph, it is determined that a spacing of approximately 1.12 will yield the required factor of safety of 2. However, to simplify the design of the bolt pattern, a 1x1 grid will be used in the drawpoint. Using a 1x1 grid gives an FS of 2.285.

c) The factor of safety of the roof will differ depending on the location of the grid origin. By offsetting the origin of the grid both horizontally and vertically, determine the ideal placement for the spacing chosen in part b. Take the lower left corner of the drawpoint in the "End Support Designer" view (coordinates: (48, 30)) as the initial analysis point.

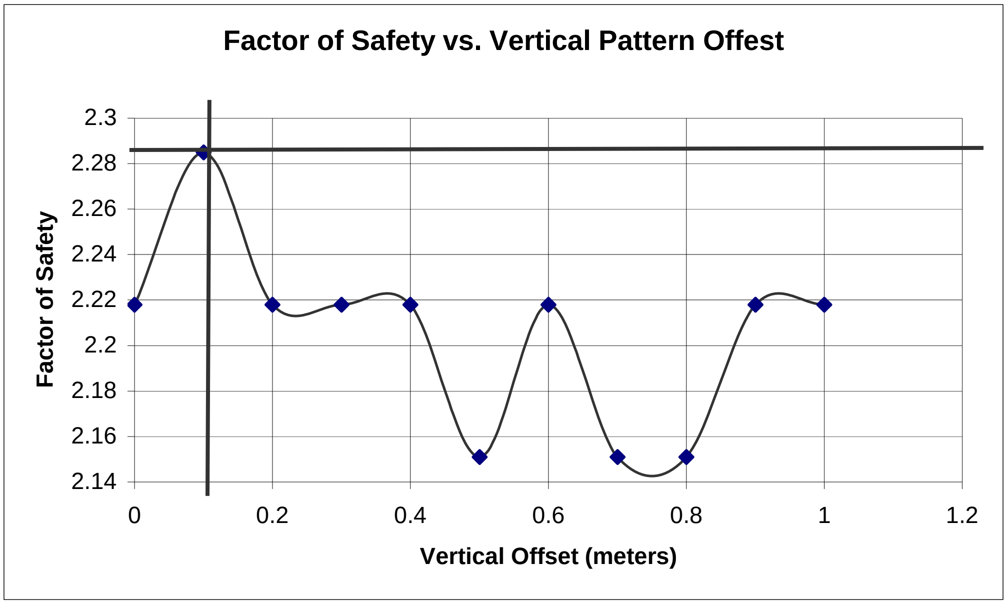
Taking the point (48, 30) as the origin, a bolt spacing of 1x1, and calculating the factor of safety at all points 0.1m away from this point, the pattern can be optimized to achieve the highest factor of safety for any given spacing.

First, right click on the bolt pattern, select the "edit the bolt pattern" option and then click on the "adjust the installation point" button. To obtain more accurate results, enter the desired coordinates into the navigation box on the bottom tool bar. If the horizontal offset is to be examined first, evaluate the FS at the point (48.1, 30). Repeating this procedure until one period has been evaluated yeilds the following results:



From the graph generated, it is seen that the factor of safety is maximized when the grid is set 0.15m from the lower left corner of the drawpoint. Another fact to be noted is that the Factor of Safety never dips below 2, so any placement of the bolt pattern will be up to safety standard.

Repeating this same procedure for the vertical offset, (48, 30.1) would be the first point evaluated.



From this graph, it is revealed that the factor of safety is highest when the pattern is started 0.1m from the lower left corner of the drawpoint.

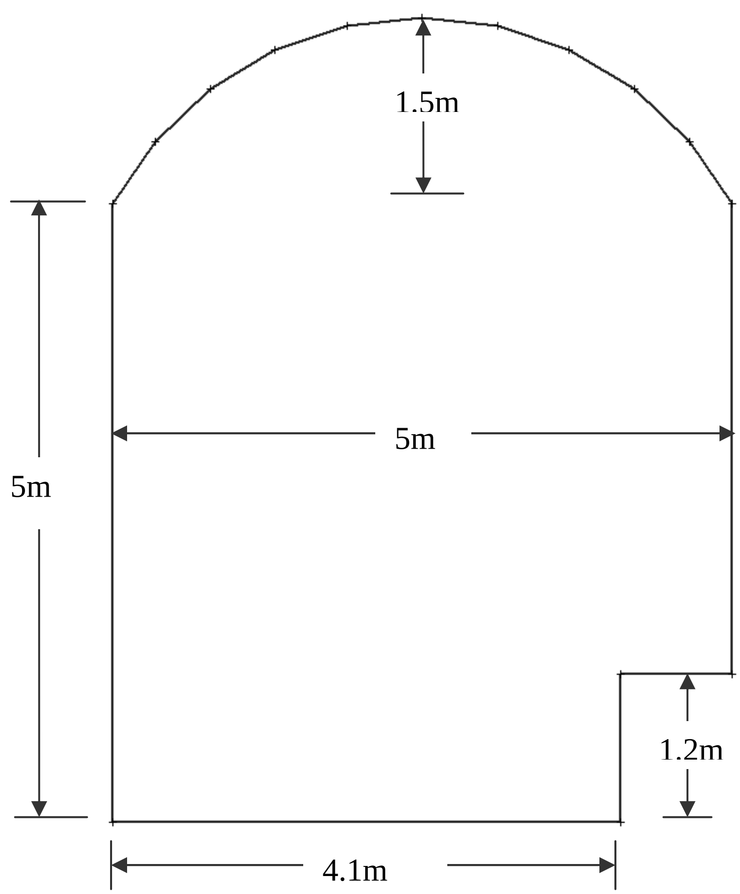
Therefore, to obtain the highest factor of safety for a bolt pattern spacing of 1x1, the origin of the grid should be placed at (48.15, 30.1) or any point on the drawpoint with the coordinate (x.15, y.1).

UNWEDGE SAMPLE PROBLEM #7

In addition to wedge geometry and the strength characteristics of discontinuity planes that create the wedge, stresses within the rock mass also play a role in wedge stability.

If a bolt pattern were to be designed to support the following tunnel to a safety factor of 1.5, how would a design need to change if field stresses are included compared to when they are omitted? What conclusions can be made about field stresses?

The tunnel is oriented at a trend of 45° and a plunge of 0°. The rock mass has a unit weight of 0.0265MN/m^3, a Young's Modulus of 5000MPa, and a Poisson's ratio of 0.25. The joints involved with the following excavation all have the same Mohr-Coulomb strength parameters: zero cohesion and a friction angle of 30°.



Joint	Dip	Dip Direction
1	30	5
2	43	120
3	50	270

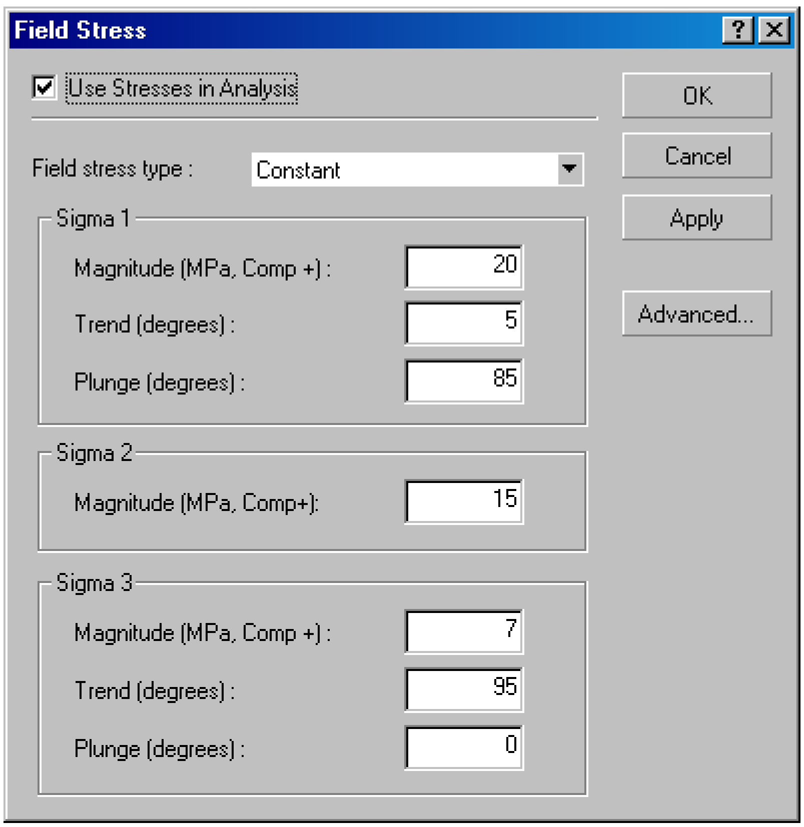
Joint Plane Orientations

Principal Stress	Magnitude (MPa)	Orientation	
		Trend	Plunge
σ_1	20	5°	85°
σ_2	15	185°	5°
σ_3	7	95°	0°

In situ stresses in rock mass

SOLUTION:
(Wedge numbers correspond to those assigned in *Unwedge*)

Choosing Field Stress from the Analysis menu, the *in situ* stresses in the rock mass can be included in the wedge stability analysis by inputting the stress data in the Field Stress dialogue.



Field Stress Analysis Data Input

	Factor of Safety	
Wedge	Stresses Omitted	Stresses Included
1	stable	stable
3	1.277	3.111
4	1.000	1.000
6	0.619	3.461
8	0.000	0.750

As seen in the table above, the factors of safety increase in wedges 3, 6 and 8, remains the same in wedge 4 and does not need to be considered in wedge 1.

The factor of safety in wedge #4 remains the same, as *Unwedge* does not allow the stressed value to be lower than the unstressed value.

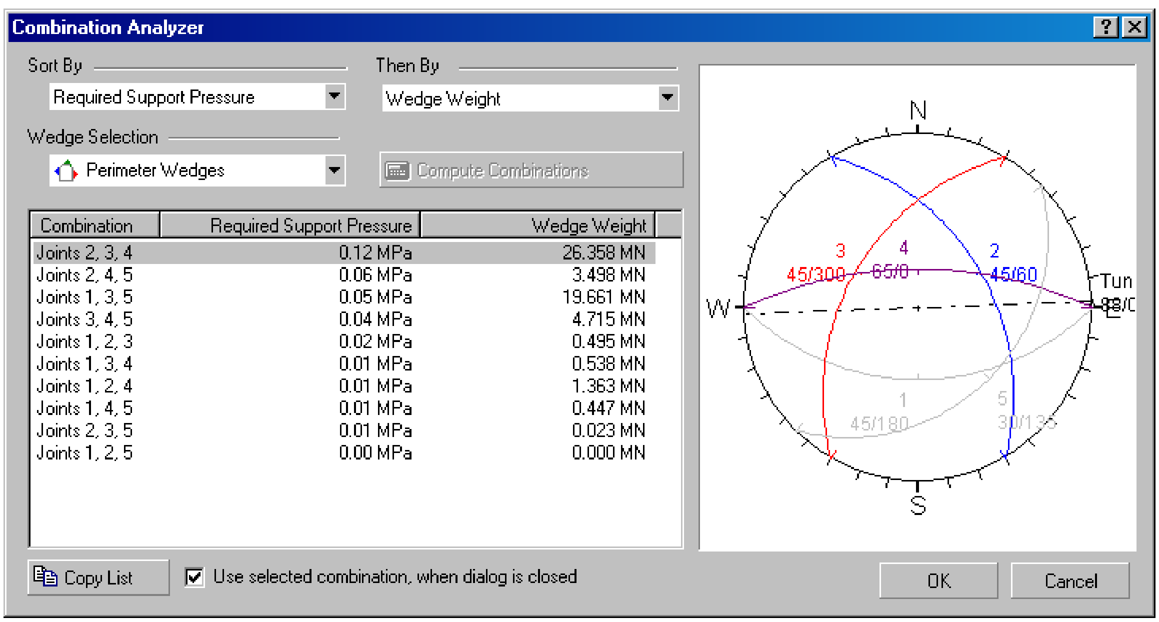
Omitting the effects of stress clamping results in very conservative estimates for certain cases. In this example, instead of wedges 3, 4, 6, and 8 requiring support to increase their factor of safety above the required value, only 4 and 8 need to be supported once field stresses are considered.

It can be concluded that stresses, even relatively small ones, can considerably alter the factor of safety values computed for underground wedges.

a) A tunnel is to be constructed as part of a hydroelectric installation in southern Ontario. The geologist's field surveys have determined the existence of five major joint planes in the excavation area. Using the *Unwedge* input file titled *combinations*, determine the weight of the wedge that will require the greatest support pressure. Which planes intersect to form this most critical joint combination?

SOLUTION:

Using the Combination Analyzer in the Analysis menu, the maximum wedges for all joint combinations can be computed and sorted according to selected criteria. When the analyzer is run for the *combinations* sample file, and then sorted first by required support pressure, then by wedge weight, the results are as follows:



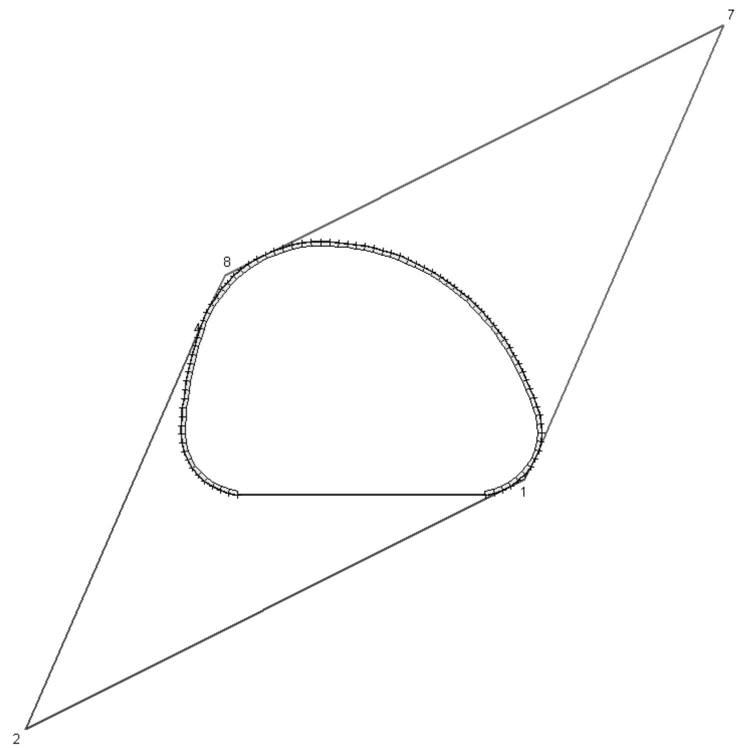
Combination Analyzer Results

From here, it can be seen that the wedge which is formed by the intersection of joints 2, 3 and 4 requires the most support pressure, 0.12MPa. This wedge has a weight of 26.358MN.

b) The excavation proposal indicates that a factor of safety of 1.5 is required for each of the wedges. If a layer of shotcrete with shear strength 2MPa and a unit weight of 0.026MN/m^3 is used to support this tunnel, how thick must the layer be to meet design specifications?

SOLUTION:

Adding an 18cm thick layer of shotcrete to the tunnel walls and ceiling increases the factor of safety of all wedges to a value over 1.5.



UNWEDGE SAMPLE PROBLEM #9

It is possible to determine the stability of an underground wedge by performing a statistical analysis of the variables affecting the factor of safety. When material properties can be assigned any value from a range of data, the factor of safety must be computed taking these variations into account. The Point Estimate Method (PEM), first presented by Rosenblueth [1], is a direct computational procedure that obtains moment estimates for a random variable. As the shape of the probability density function (pdf) is not critical to the analysis, a distribution may be assumed. This is due to the fact that the pdf is represented by a mean and two hypothetical point masses located at plus and minus one standard deviation from the mean.

The tunnel is 3m square and has an axis that plunges at zero degrees and trends exactly north. Three joint planes have a dip and dip direction of 45/0, 45/60, and 45/300 and all three have zero tensile strength.

Additional Information:

ϕ = 35°, V(ϕ) = 10%, c = 10 tonnes/m^2, V(c) = 40% and γ = 2.7tonnes/m^3

a) Using Unwedge to calculate the differing Factors of Safety, and the Point Estimate Method developed by Rosenblueth [1], obtain the expected values and the coefficients of variation for the factor of safety for correlation factors (ρ) = -1, 0, +1. What happens to these values as the correlation coefficient increases?

SOLUTION:

Using the PEM developed by Rosenblueth [1]:

All parameters but ϕ and *c* will be considered constant.

$$c+ = c + (V(c) \bullet c) = 10 + 0.4 \bullet 10 = 14$$
$$c- = c - (V(c) \bullet c) = 10 - 0.4 \bullet 10 = 6$$
$$\phi+ = \phi + (V(\phi) \bullet \phi) = 35° + 0.10 \bullet 35° = 38.5°$$
$$\phi- = \phi - (V(\phi) \bullet \phi) = 35° - 0.10 \bullet 35° = 31.5°$$

Summarized

c+	14	ϕ+	38.5
c-	6	ϕ -	31.5

Calculate the Point-mass Weights for each value of correlation coefficient.

	ρ	-1	0	1
p++, p--	(1+ρ)/4	0	0.25	0.5
p+-, p-+	(1- ρ)/4	0.5	0.25	0

Using the given equation, find the factor of safety for each combination of varying attribute, for each value of correlation coefficient.

Factor of Safety

The following factor of safety values were calculated using Unwedge:

	ϕ	c	FS
FS + +	38.5	14	12.77
FS + -	38.5	6	5.927
FS - +	31.5	14	12.587
FS - -	31.5	6	5.745

First Moment

Sample calculations:

$$FS++_{(\rho=-1)} = p++_{(\rho=-1)} \bullet FS(c+,\phi+) = 0 \bullet 12.77 = 0$$

$$E[FS] = \sum FS_{(\rho=-1)} = 0 + 2.9635 + 6.2935 + 0 = 9.257$$

			ρFS		
FS(ϕ, c)	**FS**	ρ =	**-1**	**0**	**1**
FS + +	12.77		0	3.1925	6.385
FS + -	5.927		2.9635	1.48175	0
FS - +	12.587		6.2935	3.14675	0
FS - -	5.745		0	1.43625	2.8725
		E[FS] =	9.257	9.25725	9.2575

Second Moment

Sample Calculations:

$$FS^2++_{(\rho=1)} = \left(FS++_{(\rho=1)}\right)^2 = 12.77^2 = 163.0729$$

$$E[FS^2] = \sum FS^2_{(\rho=1)} = 0 + 17.56466 + 79.21628 + 0 = 96.78095$$

			ρFS^2		
FS(φ, c)	FS^2	ρ	-1	0	1
FS + +	163.0729		0	40.76823	81.53645
FS + -	35.12933		17.56466	8.782332	0
FS - +	158.4326		79.21628	39.60814	0
FS - -	33.00503		0	8.251256	16.50251
		$E[FS^2]$ =	96.78095	97.40996	98.03896

Expected Values and Coefficient of Variance

Sample Calculations:

$$V[FS] = E[FS^2] - (E[FS])^2 = 96.78095 - (9.257^2) = 11.0889$$

$$\sigma[FS] = \sqrt{V[FS]} = \sqrt{11.0889} = 3.33$$

$$V(FS)\% = \frac{\sigma[FS]}{E[FS]} \times 100 = \frac{3.33}{9.257} \times 100 = 119.7893$$

	σ [FS]	V[FS]
ρ = -1	3.33	11.0889
ρ = 0	3.422467	11.71328
ρ = +1	3.5125	12.33766

From these calculations, it can be seen that as the correlation coefficient increases, the expected values and the coefficients of variation also increase.

b) If FS = 1 represents failure, calculate the probability of failure for the results obtained in part (a).

SOLUTION:

Since the first and second moments are calculated, a normal distribution is assumed for the factor of safety.

Sample Calculations:

Standardize values to fit normal curve:

$$z = \frac{E(FS) - FS}{\sigma(FS)} = \frac{9.257 - 1}{3.33} = 2.47957958$$

To find the area under the normal curve which represents the probability that FS ≤ 1:

$$P(FS \leq 1) = \frac{1}{2} - \Phi(2.48) = \frac{1}{2} - 0.493431 = 0.00657$$

$$R(\%) = \left(1 - P(FS \leq 1)\right) \times 100 = (1 - 0.00657) \times 100 = 99.34231\%$$

ρ	Probility of Failure	R (%)
-1	0.006577	99.34231
0	0.007918	99.20817
1	0.009364	99.06357

REFERENCES

1. Rosenblueth, E. (1975*b*): “Point Estimates for Probability Moments,” *Proc. Nat. Acad. Sci. USA*, vol.72, no.10.